EL-MOASSER

By a group of supervisors

THE MAIN BOOK

2nd PREP.

FIRST TERM

Maths

GPS

AL TALABA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
e-mail: info@elmoasserbooks.com
15014 www.elmoasserbooks.com

Contents

Notes

The notes found at the margin of some pages in **geometry** and referred to by (*) are theorems and corollaries have been studied before

First Algebra and Statistics

UNIT 1

Real Numbers

Lessons of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers $\grave{\mathbb{Q}}$
3. The set of real numbers $\mathbb{R}$ – Ordering numbers in $\mathbb{R}$
4. Intervals.
5. Operations on the real numbers.
6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

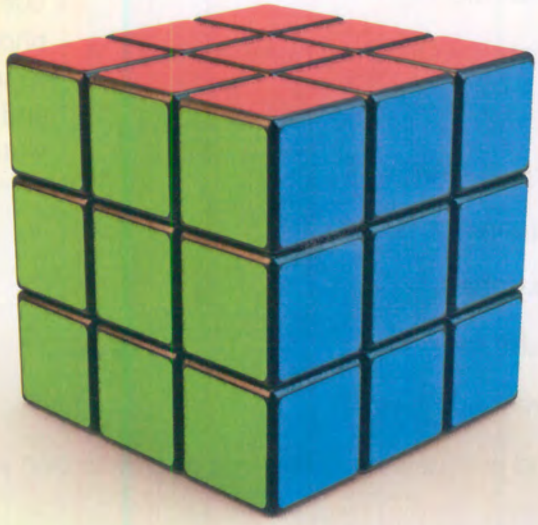
Unit Objectives :

By the end of this unit, student should be able to :

- recognize the cube root of a rational number.
- find the cube root of a rational number.
- recognize the set of irrational numbers.
- represent the irrational number on the number line.
- recognize the set of real numbers.
- perform the operations on the intervals.
- perform the arithmetic operations on the real numbers.
- solve equations and inequalities of the first degree in one variable in $\mathbb{R}$
- perform the operations on the square roots and the cube roots.
- recognize two conjugate numbers.
- apply what he studied in the real numbers to find the volumes and the areas of some of the solids.

LESSON 1

The cube root of a rational number



Remember the square root of the perfect square rational number

The square root of the perfect square rational number (a) is the rational number whose square equals (a)

The symbol $\sqrt{\ }$ means the positive square root of a number

For example:

25 has two square roots which are 5 and – 5

Because : $(5)^2 = 25$, $(-5)^2 = 25$

and we write $\sqrt{25} = 5$, $-\sqrt{25} = -5$, $\pm\sqrt{25} = \pm 5$

- $\sqrt{16} = 4$, $-\sqrt{16} = -4$, $\pm\sqrt{16} = \pm 4$
- $\sqrt{0} = 0$
- $\sqrt{\text{negative number}}$ is meaningless
- $\sqrt{a^2} = |a|$

Notice that :
The two square roots of the rational number , each of them is the additive inverse of the other and their sum = zero.

For example:

$\sqrt{3^2} = |3| = 3$, $\sqrt{(-6)^2} = |-6| = 6$

- Sometimes , you need to factorize a number to its prime factors to facilitate finding its square root , then you take a factor from each two equal factors , then the product of these taken factors is the square root of this number.

For example:

$$\because 441 = \underbrace{3 \times 3}_{3} \times \underbrace{7 \times 7}_{7}$$

$$\therefore \sqrt{441} = 3 \times 7$$

$$= 21$$

$$\begin{array}{r|l} 441 & 3 \\ 147 & 3 \\ 49 & 7 \\ 7 & 7 \\ 1 & \end{array}$$

You can use your calculator to check your answer.

The cube root of a rational number

- The product of a number by itself three times is the cube of that number.

For example: 64 is the cube of 4 because $4 \times 4 \times 4 = 64$

- The reverse of finding the cube is finding the cube root.
- Finding the cube root of a number is finding another number if multiplied by itself three times , we get the first number.

For example: 4 is the cube root of 64 because $64 = 4 \times 4 \times 4$

Definition

The cube root of the number "a" is the number whose cube equals a

- The symbol $\sqrt[3]{}$ (read as "the cube root of ") is used to designate the cube root.

For example: $\sqrt[3]{64}$ designates the cube root of 64

- The cube root of a positive number is positive and the cube root of a negative number is negative.

For example: $\sqrt[3]{64} = 4$ and $\sqrt[3]{-64} = -4$

i.e. The cube root of any number has the same sign of this number.

Finding the cube root of a rational number (representing a perfect cube)

- The perfect cube rational number is the number which can be written as a cube of a rational number **i.e.** $(\text{rational number})^3$ as the numbers : $8 = 2^3$, $-27 = (-3)^3$
- The cube root of a perfect cube rational number is also a rational number.

For example: $\sqrt[3]{8} = 2$, $\sqrt[3]{-27} = -3$

- If a number is not a perfect cube , then you indicate its cube root by using the cube root symbol.

For example: The cube root of 4 is $\sqrt[3]{4}$ because 4 is not a perfect cube.

- $\sqrt[3]{a^3} = a$ **For example:** $\sqrt[3]{5^3} = 5$, $\sqrt[3]{(-5)^3} = -5$
- $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$ **For example:** $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$

UNIT 1

• You can use factorization to find the cube root of a perfect cube number , as in the following example.

Example 1 **Find each of the following :**

WATCH VIDEO

1 $\sqrt[3]{216}$ **2** $\sqrt[3]{\frac{-8}{125}}$ **3** $\sqrt[3]{0.064}$

Solution

1 $\sqrt[3]{216} = 2 \times 3 = 6$

$$\begin{array}{r|l} 216 & 2 \\ 108 & 2 \\ 54 & 2 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

2 3

2 $\sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$

$$\begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \qquad \begin{array}{r|l} 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

2 5

3 $\sqrt[3]{0.064} = \sqrt[3]{\frac{64}{1000}} = \frac{2 \times 2}{2 \times 5}$

$= \frac{4}{10} = 0.4$

$$\begin{array}{r|l} 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1000 & 2 \\ 500 & 2 \\ 250 & 2 \\ 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

2 2 2 5

Example 2 **Choose the correct answer from those given :**

1 $\sqrt[3]{\cdots\cdots} = -5$

ⓐ 125 ⓑ 25 ⓒ – 25 ⓓ – 125

2 $\sqrt{4} - \sqrt[3]{-8} = \cdots\cdots$

ⓐ – 4 ⓑ – 2 ⓒ 4 ⓓ 8

3 $\sqrt{(-7)^2} - \sqrt[3]{(-7)^3} = \cdots\cdots$

ⓐ – 14 ⓑ zero ⓒ 7 ⓓ 14

4 If $\sqrt[3]{x} = \sqrt{4}$, then $x = \cdots\cdots$

ⓐ 2 ⓑ 4 ⓒ 8 ⓓ ± 8

5 $\sqrt{x^4} = \sqrt[3]{\dots\dots}$

ⓐ x ⓑ x^2 ⓒ x^4 ⓓ x^6

Solution

1 **(d) The reason :** $(-5)^3 = -125$

2 **(c) The reason :** $\sqrt{4} - \sqrt[3]{-8} = 2 - (-2) = 2 + 2 = 4$

3 **(d) The reason :** $\sqrt{(-7)^2} - \sqrt[3]{(-7)^3} = 7 - (-7) = 7 + 7 = 14$

4 **(c) The reason :** $\because \sqrt[3]{x} = \sqrt{4}$ $\therefore \sqrt[3]{x} = 2$ $\therefore x = 2^3 = 8$

5 **(d) The reason :** $\because \sqrt{x^4} = x^2$ $\therefore (x^2)^3 = x^6$ $\therefore \sqrt{x^4} = \sqrt[3]{x^6}$

TRY by yourself 1

Complete the following :

1 $\sqrt[3]{512} = \dots\dots$ **2** $\sqrt{64} - \sqrt[3]{64} = \dots\dots$

3 $\sqrt[3]{27} = \sqrt{\dots\dots}$

4 If $\sqrt[3]{x} = 6$, then $x = \dots\dots$

Final answers of try by yourself questions are at the end of each lesson to check your answer.

Solving equations in $\mathbb{Q}$

- If "a" is a perfect cube number ,
  then the equation : $x^3 = a$ has a unique solution in $\mathbb{Q}$, which is $\sqrt[3]{a}$

For example :

- The equation : $x^3 = 8$ has a unique solution in $\mathbb{Q}$ which is $\sqrt[3]{8} = 2$
- The equation : $x^3 = 9$ has no solution in $\mathbb{Q}$ because 9 is not a perfect cube.

Example 3 **Solve each of the following equations in $\mathbb{Q}$:**

1 $40\,x^3 - 1 = -136$ **2** $(y - 2)^3 = -343$

Solution

1 $\because 40\,x^3 - 1 = -136$ $\therefore 40\,x^3 = -136 + 1$

$\therefore 40\,x^3 = -135$ $\therefore x^3 = -\frac{135}{40}$

$\therefore x^3 = -\frac{27}{8}$ $\therefore x = \sqrt[3]{-\frac{27}{8}}$

$\therefore x = -\frac{3}{2}$

2 $\because (y-2)^3 = -343$

Taking the cube root of each side :

$\therefore \sqrt[3]{(y-2)^3} = \sqrt[3]{-343}$ $\therefore y-2=-7$

$\therefore y=-7+2$ $\therefore y=-5$

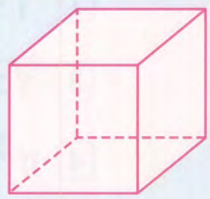
TRY by yourself 2 ***Find in $\mathbb{Q}$ the S.S. of each of the following equations :***

1 $27x^3-2=62$ 2 $(5x-3)^3-2=6$

Applications

Remember that

- The volume of a cube = the edge length × itself × itself
- The area of one face of a cube = the edge length × itself
- The lateral area of a cube = the area of one face × 4
- The total area of a cube = the area of one face × 6



For example: If the volume of a cube is 8 cm³ , then :

- The edge length $= \sqrt[3]{8} = 2$ cm.
- The area of one face $= 2 \times 2 = 4$ cm²
- The lateral area $= 4 \times 4 = 16$ cm²
- The total area $= 4 \times 6 = 24$ cm²

Example 4 **Find each of the following :**

1 The length of the inner edge of a vessel in the shape of a cube if its capacity = 8 litres.

2 The radius length of a sphere of volume $\frac{36}{125}\pi$ cm³

Knowing that : The volume of the sphere $= \frac{4}{3}\pi r^3$

where r is the radius length of the sphere , π is the ratio between the circumference of the circle and its diameter length.

3 The diameter length of a sphere of volume equals 38808 cm³ $\left(\pi \approx \frac{22}{7}\right)$

Solution

1 $\because$ The capacity of the vessel = 8 litres = $8 \times 1000 = 8000$ cm^3.

$\therefore$ The inner edge length $= \sqrt[3]{8000}$

$= 20$ cm.

Remember that

1 litre = 1000 cm^3.

2 $\because$ The volume of the sphere $= \frac{4}{3} \pi r^3$

$\therefore \frac{4}{3} \cancel{\pi} r^3 = \frac{36}{125} \cancel{\pi}$ $\qquad \therefore \frac{4}{3} r^3 = \frac{36}{125}$

$\therefore r^3 = \frac{36}{125} \times \frac{3}{4}$ $\qquad \therefore r^3 = \frac{27}{125}$

$\therefore r = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$ cm. $\qquad \therefore$ The radius length of the sphere $= \frac{3}{5}$ cm.

3 $\because$ The volume of the sphere $= \frac{4}{3} \pi r^3$

$\therefore \frac{4}{3} \pi r^3 = 38808$ $\qquad \therefore \frac{4}{3} \times \frac{22}{7} r^3 = 38808$

$\therefore \frac{88}{21} r^3 = 38808$ $\qquad \therefore r^3 = 38808 \times \frac{21}{88}$

$\therefore r^3 = 9261$ $\qquad \therefore r = \sqrt[3]{9261}$

9261	3
3087	3
1029	3
343	7
49	7
7	7
1	

3

7

$\therefore r = 3 \times 7 = 21$ cm.

$\therefore$ The diameter length $= 21 \times 2 = 42$ cm.

Notice that : You can use the calculator to find $\sqrt[3]{9261}$ directly.

TRY by yourself 3

1. Find the length of the inner edge of a vessel in the shape of a cube with capacity equals 27 litres.
2. Find the length of the diameter of a sphere of volume 36π cm^3.
(Knowing that : the volume of the sphere $= \frac{4}{3} \pi r^3$)

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

Answers of try by yourself

1 [1] 8 [2] 4 [3] 9 [4] 216

2 [1] $\left\{\frac{4}{3}\right\}$ [2] $\{1\}$

3 [1] 30 cm. [2] 6 cm.

LESSON 2

The set of irrational numbers $\mathbb{Q}'$

The sets of numbers

You had studied before the following sets of numbers :

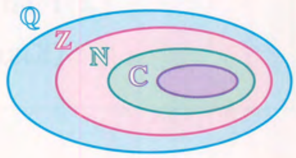
- The set of counting numbers : $\mathbb{C} = \{1, 2, 3, 4, ...\}$
- The set of natural numbers : $\mathbb{N} = \{0, 1, 2, 3, ...\} = \mathbb{C} \cup \{0\}$
- The set of integers : $\mathbb{Z} = \{..., 3, 2, 1, 0, -1, -2, -3, ...\}$
- The set of positive integers : $\mathbb{Z}_+ = \{1, 2, 3, ...\} = \mathbb{C}$
- The set of negative integers : $\mathbb{Z}_- = \{-1, -2, -3, ...\}$
- The set of rational numbers : $\mathbb{Q} = \left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}$

Examples of rational numbers : $\frac{2}{3}$, $-\frac{1}{2}$, zero, 3, –5, 0.2, 25%, ...

Notice that :

$\mathbb{C} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

The opposite figure shows that.



Prelude

* You studied before that a rational number is the number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$, and the set of rational numbers is denoted by $\mathbb{Q}$

* **Based on the previous , you know that :**

All integers are rational numbers For example: 3 is a rational number because it can be expressed as $\frac{3}{1}$ or $\frac{6}{2}$ or ...	**All decimals** are rational numbers For example: 2.5 is a rational number because it can be expressed as $\frac{25}{10}$ or $\frac{5}{2}$ or ...	**All percentages** are rational numbers For example: 15 % is a rational number because it can be expressed as $\frac{15}{100}$ or $\frac{150}{1000}$ or ...

The square root of a perfect square rational number is a rational number For example: $\sqrt{36}$, $\sqrt{\frac{4}{25}}$, $\sqrt{0.09}$ are all rational numbers where $\sqrt{36}=6$, $\sqrt{\frac{4}{25}}=\frac{2}{5}$, $\sqrt{0.09}=\sqrt{\frac{9}{100}}=\frac{3}{10}$	**The cube root of a perfect cube rational number** is a rational number For example: $\sqrt[3]{8}$, $\sqrt[3]{-64}$, $\sqrt[3]{\frac{27}{1000}}$ are all rational numbers where $\sqrt[3]{8}=2$, $\sqrt[3]{-64}=-4$, $\sqrt[3]{\frac{27}{1000}}=\frac{3}{10}$

Irrational numbers

The square root of a rational number which is not a perfect square is not a rational number For example: $\sqrt{2} \notin \mathbb{Q}$ because there is no rational number whose square is 2 , so $\sqrt{2}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$	**The cube root of a rational number which is not a perfect cube** is not a rational number For example: $\sqrt[3]{4} \notin \mathbb{Q}$ because there is no rational number whose cube is 4 , so $\sqrt[3]{4}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$
π is not a rational number (However $\frac{22}{7}$, 3.14 and 3.142 are rational numbers , each of them represents an approximating value of π)	**Other examples** of numbers not rational $\sqrt{5}+1$, $1-\sqrt[3]{7}$, $2\sqrt{7}$, $-\frac{\sqrt[3]{9}}{5}$

The set of irrational numbers is denoted by $\grave{\mathbb{Q}}$

Notice that : $\mathbb{Q}$ and $\grave{\mathbb{Q}}$ are disjoint sets. **i.e.** $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

UNIT 1

! Remarks

- $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a$, where $a \geq 0$ For example: $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$
- $(\sqrt[3]{a})^3 = \sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$, where $a \in \mathbb{Q}$ For example: $(\sqrt[3]{-7})^3 = \sqrt[3]{-7} \times \sqrt[3]{-7} \times \sqrt[3]{-7} = -7$

WATCH VIDEO

Example 1

Show which of the following numbers belongs to $\mathbb{Q}$ and which of them belongs to $\grave{\mathbb{Q}}$:

1. $\sqrt{0.49}$
2. $\sqrt[3]{-0.064}$
3. $\sqrt{\frac{25}{49}}$
4. $\sqrt[3]{\frac{25}{49}}$
5. $\sqrt{25} + \sqrt[3]{16}$

Solution

1. $\because \sqrt{0.49} = 0.7 = \frac{7}{10}$ $\therefore \sqrt{0.49} \in \mathbb{Q}$
2. $\because \sqrt[3]{-0.064} = -0.4 = -\frac{4}{10}$ $\therefore \sqrt[3]{-0.064} \in \mathbb{Q}$
3. $\because \sqrt{\frac{25}{49}} = \sqrt{\left(\frac{5}{7}\right)^2} = \frac{5}{7}$ $\therefore \sqrt{\frac{25}{49}} \in \mathbb{Q}$
4. $\because \sqrt[3]{\frac{25}{49}} \notin \mathbb{Q}$ because there is no rational number whose cube is $\frac{25}{49}$
   $\therefore \sqrt[3]{\frac{25}{49}} \in \grave{\mathbb{Q}}$
5. $\because \sqrt{25} + \sqrt[3]{16} = 5 + \sqrt[3]{16}$ $\because$ There is no rational number whose cube is 16
   $\therefore \sqrt[3]{16} \notin \mathbb{Q}$ $\therefore (5 + \sqrt[3]{16}) \notin \mathbb{Q}$ $\therefore (\sqrt{25} + \sqrt[3]{16}) \in \grave{\mathbb{Q}}$

TRY by yourself 1

Complete using one of the symbols $\mathbb{Q}$ or $\grave{\mathbb{Q}}$:

1. $3 \in$ ....
2. $\sqrt{3} \in$ ....
3. $9 \in$ ....
4. $\sqrt{9} \in$ ....
5. $-8 \in$ ....
6. $\sqrt[3]{-8} \in$ ....
7. $5 \in$ ....
8. $\sqrt[3]{5} \in$ ....
9. $\sqrt[3]{-9} \in$ ....

Solving equations in $\grave{\mathbb{Q}}$

Example 2 If $x \in \grave{\mathbb{Q}}$, find the S.S. of each of the following equations :

1. $x^2 = 5$
2. $x^3 = 7$
3. $\frac{2}{5} x^2 = \frac{4}{25}$
4. $64 x^3 - 2 = -29$
5. $(x^2 - 10)(x^3 - 4) = 0$

Solution

1. $\because x^2 = 5$ $\quad\therefore x = \pm\sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5} , -\sqrt{5}\}$

2. $\because x^3 = 7$ $\quad\therefore x = \sqrt[3]{7}$

$\therefore$ The S.S. $= \{\sqrt[3]{7}\}$

3. $\because \frac{2}{5} x^2 = \frac{4}{25}$ $\quad\therefore x^2 = \frac{4}{25} \times \frac{5}{2}$

$\therefore x^2 = \frac{2}{5}$ $\quad\therefore x = \pm\sqrt{\frac{2}{5}}$

$\therefore$ The S.S. $= \left\{\sqrt{\frac{2}{5}} , -\sqrt{\frac{2}{5}}\right\}$

4. $\because 64 x^3 - 2 = -29$ $\quad\therefore 64 x^3 = -29 + 2$ $\quad\therefore 64 x^3 = -27$

$\therefore x^3 = -\frac{27}{64}$ $\quad\therefore x = \sqrt[3]{-\frac{27}{64}}$ $\quad\therefore x = -\frac{3}{4}$

$\because -\frac{3}{4} \in \mathbb{Q}$ $\quad\therefore -\frac{3}{4} \notin \grave{\mathbb{Q}}$ $\quad\therefore$ The S.S. $= \varnothing$

5. $\because (x^2 - 10)(x^3 - 4) = 0$

$\therefore x^2 - 10 = 0$ or $x^3 - 4 = 0$

$\therefore x^2 = 10$ $\quad\therefore x^3 = 4$

$\therefore x = \pm\sqrt{10}$ $\quad\therefore x = \sqrt[3]{4}$

$\therefore$ The S.S. $= \{\sqrt{10} , -\sqrt{10} , \sqrt[3]{4}\}$

Notice that :
We used the concept of the square root to find the value of x according to the following remark :
If $x^2 = a$, then $x = \pm\sqrt{a}$

Remember that
For any two numbers x , y :
If x y = zero , then
x = zero or y = zero

TRY by yourself 2 *Find the S.S. in $\grave{\mathbb{Q}}$ for each of the following :*

1. $2 x^3 - 7 = 3$
2. $\frac{1}{2} x^2 - 5 = 3$

Finding an approximated value of an irrational number

If you use the calculator to find the values of some irrational numbers , you will find that :

$\sqrt{2} \simeq$ 1.4142 ... , $\sqrt{3} \simeq$ 1.73205 ... , $\sqrt{5} \simeq$ 2.236 ...

i.e. The irrational number is represented by an infinite decimal and not recurring.

And you can deduce an approximated value of the irrational number without using the calculator.

For example:

You can deduce an approximated value of the irrational number $\sqrt{5}$ as follows :

$\because 4 < 5 < 9$ (notice that we chose 4 and 9 because each of them is a perfect square , and the number 5 includes between them) and by taking the square root for all the terms.

$\therefore \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\quad\therefore 2 < \sqrt{5} < 3$

i.e. $\sqrt{5} = 2 +$ decimal less than 1

To find an approximated value of the number $\sqrt{5}$, you search for the values of the following numbers : $(2.1)^2$, $(2.2)^2$ and $(2.3)^2$

, then you find that $(2.1)^2 = 4.41$, $(2.2)^2 = 4.84$, $(2.3)^2 = 5.29$

$\because 4.84 < 5 < 5.29$ $\quad\therefore \sqrt{4.84} < \sqrt{5} < \sqrt{5.29}$ $\quad\therefore 2.2 < \sqrt{5} < 2.3$

We can say that 2.2 and 2.3 are approximated values of $\sqrt{5}$ and thus we can get more accurate values for the irrational number $\sqrt{5}$ and we can use the calculator to check the approximated value of the number $\sqrt{5}$

! Remark

Each irrational number lies between two rational numbers.

Example 3

Prove that :

1. $\sqrt{3}$ lies between 1.7 and 1.8
2. $\sqrt[3]{12}$ lies between 2.2 and 2.3

Solution

1. $\because (\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = 3$, $(1.7)^2 = 2.89$, $(1.8)^2 = 3.24$

$\because 2.89 < 3 < 3.24$ $\quad\therefore \sqrt{2.89} < \sqrt{3} < \sqrt{3.24}$ $\quad\therefore 1.7 < \sqrt{3} < 1.8$

i.e. $\sqrt{3}$ lies between 1.7 and 1.8

You can solve the problem using the calculator as follows :

$\because \sqrt{3} \simeq 1.73$, $\because 1.7 < 1.73 < 1.8$

$\therefore 1.7 < \sqrt{3} < 1.8$ $\quad\therefore \sqrt{3}$ lies between 1.7 and 1.8

2 $\because \left(\sqrt[3]{12}\right)^3 = \sqrt[3]{12} \times \sqrt[3]{12} \times \sqrt[3]{12} = 12$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

, $\because 10.648 < 12 < 12.167$ $\quad \therefore \sqrt[3]{10.648} < \sqrt[3]{12} < \sqrt[3]{12.167}$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$

i.e. $\sqrt[3]{12}$ lies between 2.2 and 2.3

You can solve the problem using the calculator as follows :

$\sqrt[3]{12} \simeq 2.289$ $\qquad \because 2.2 < 2.289 < 2.3$

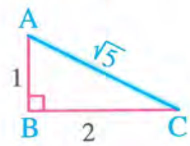
$\therefore 2.2 < \sqrt[3]{12} < 2.3$ $\qquad \therefore \sqrt[3]{12}$ lies between 2.2 and 2.3

TRY by yourself 3

1. Find two consecutive integers such that $\sqrt{13}$ lies between them.
2. Prove that : $\sqrt{7}$ lies between 2.6 and 2.7

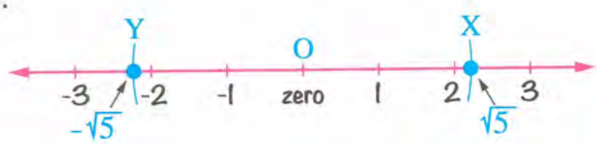
Representing an irrational number on the number line

- If you draw the right-angled triangle ABC at B such that :
  AB = 1 length unit , BC = 2 length units , then according to Pythagoras' theorem you find :
  $(AC)^2 = (AB)^2 + (BC)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$
  $\therefore AC = \sqrt{5}$ length unit.



i.e. The length of $\overline{AC}$ represents the irrational number $\sqrt{5}$

- If you draw the number line and you open the compasses with a distance equal to the length of $\overline{AC}$ and using O which represents zero as a centre and draw an arc cutting the number line at the point X on the right of the point O , then the point X represents the number $\sqrt{5}$ on the number line.



- And with the same length of $\overline{AC}$, if you use O as a centre and draw an arc cutting the number line at the point Y on the left side of O , then the point Y represents the number $-\sqrt{5}$ on the number line.

Generally

Each irrational number can be represented by a point on the number line.

! Remark

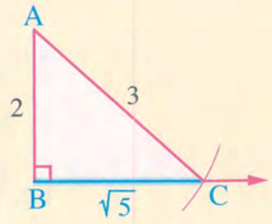
If you draw the right-angled triangle ABC at B such that

AB = 2 length units , AC = 3 length units ,

then $(BC)^2 = (AC)^2 - (AB)^2 = (3)^2 - (2)^2 = 9 - 4 = 5$

i.e. $BC = \sqrt{5}$ length unit, then you can use the length of $\overline{BC}$

to determine the point which represents $\sqrt{5}$ or $-\sqrt{5}$



From the previous , we deduce that :

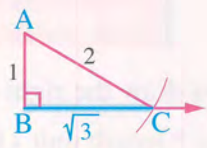
To get a line segment with length that equals the irrational number $\sqrt{a}$, we search for two numbers , the sum of their squares or the difference between their squares = a , then we use them to draw a right-angled triangle.

The following figures can help you to get two numbers such that the difference between their squares equals the square of the irrational number.

- **To draw a line segment with length $\sqrt{3}$ length unit ,**

  then the length of one of the two sides of the right-angle $= \frac{3-1}{2} = 1$ length unit.

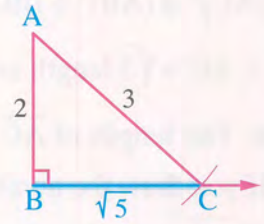
  and the length of the hypotenuse $= \frac{3+1}{2} = 2$ length units.



- **To draw a line segment with length $\sqrt{5}$ length unit ,**

  then the length of one of the two sides of the right-angle $= \frac{5-1}{2} = 2$ length units.

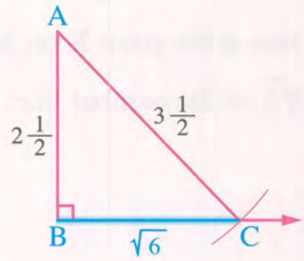
  and the length of the hypotenuse $= \frac{5+1}{2} = 3$ length units.



- **To draw a line segment with length $\sqrt{6}$ length unit ,**

  then the length of one side of the right-angle $= \frac{6-1}{2} = 2\frac{1}{2}$ length units.

  and the length of the hypotenuse $= \frac{6+1}{2} = 3\frac{1}{2}$ length units.



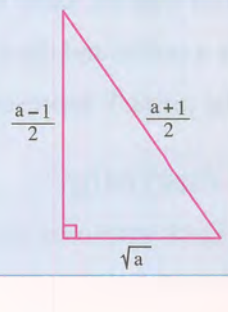
Generally

To draw a line segment with length $\sqrt{a}$ length unit where a > 1

, draw a right-angled triangle in which

the length of one side of the right-angle $= \frac{a-1}{2}$ length unit.

and the length of the hypotenuse $= \frac{a+1}{2}$ length unit.



Example 4 **Draw a line segment with length $\sqrt{7}$ length unit , then use it to determine the points which represent the following numbers on the number line :**

1 $\sqrt{7}$	**2** $-\sqrt{7}$	**3** $1+\sqrt{7}$
4 $2-\sqrt{7}$	**5** $2\sqrt{7}$	

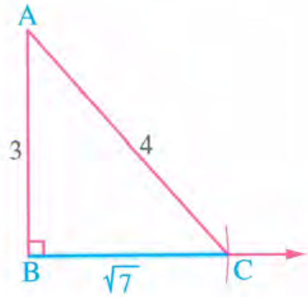
Solution **Draw the right-angled triangle ABC at B such that :**

$AB = \frac{7-1}{2} = 3$ length units.

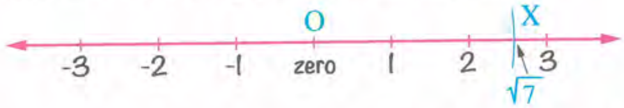
, $AC = \frac{7+1}{2} = 4$ length units.

, then $(BC)^2 = (AC)^2 - (AB)^2 = 16 - 9 = 7$

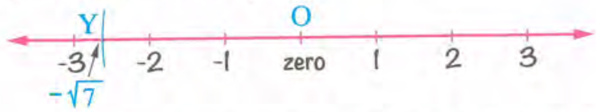
$\therefore BC = \sqrt{7}$ length unit.



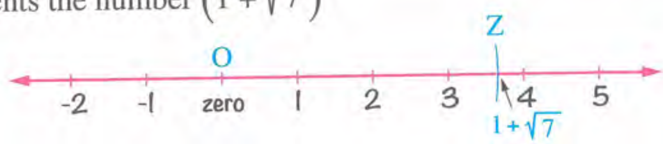
1 Using the compasses with a distance equal to the length of $\overline{BC}$ taking O as a centre , draw an arc to cut the number line on the right side of O at the point X , then X is the point which represents $\sqrt{7}$



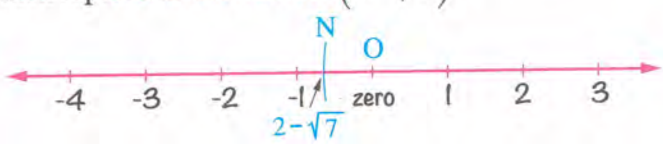
2 Using the same previous distance and taking O as a centre , draw an arc to cut the number line on the left side of O at the point Y, then Y is the point which represents the number $-\sqrt{7}$



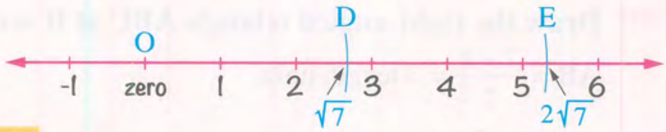
3 Using the same previous distance and taking the point which represents the number 1 on the number line as a centre, draw an arc to cut the number line on the right side of the previous point at Z , then Z represents the number $\left(1+\sqrt{7}\right)$



4 Using the same previous distance and taking the point which represents the number 2 on the number line as a centre , draw an arc to cut the number line on the left side of this point at the point N , then N is the point which represents the number $\left(2-\sqrt{7}\right)$

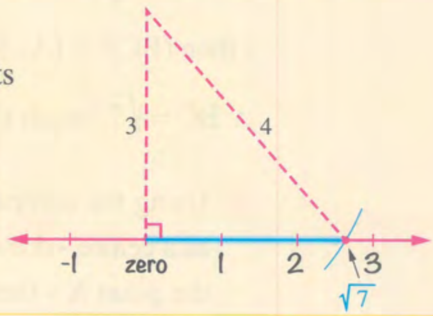


5 Using the same previous distance and taking the point O on the number line as a centre, draw an arc to cut the number line on the right side of O at the point D, then taking D as a centre and with the same previous distance in the same direction, draw an arc to cut the number line at E, then E is the point which represents the number $2\sqrt{7}$



Remark

In the previous example we can determine the point which represents the number $\sqrt{7}$ on the number line by drawing the right-angled triangle directly on the number line as in the opposite figure.



Example 5

Find the length of the diagonal of a square whose area = 5 cm².

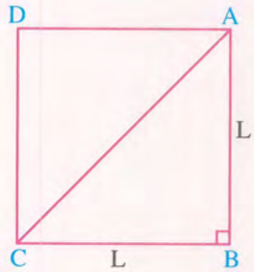
Solution

Let the side length of the square be L cm.

$\therefore$ The area of the square $= L \times L = L^2$ cm².

$\therefore L^2 = 5$ $\qquad\qquad$ $\therefore L = \sqrt{5}$ cm.

(**Notice that :** The square side length must be positive, so L equals $\sqrt{5}$ not $-\sqrt{5}$)



$\because \Delta ABC$ is a right-angled triangle at B,
(from the properties of the square ABCD)

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = \left(\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2 = 5 + 5 = 10$

$\therefore AC = \sqrt{10}$ cm.

$\therefore$ The length of the diagonal of the square $= \sqrt{10}$ cm.

TRY by yourself 4

Determine the point which represents the number $\sqrt{11}$ on the number line.

Answers of try by yourself

1 (1) ∈ (2) ∉ (3) ∈ (4) ∈ (5) ∈ (6) ∈ (7) ∈ (8) ∉ (9) ∉

2 (1) $\{\sqrt[3]{5}\}$ (2) $\varnothing$

3 (1) 3, 4 (2) prove by yourself.

4 determine by yourself.

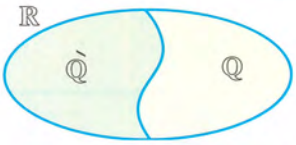
LESSON 3

The set of real numbers $\mathbb{R}$ and ordering numbers in $\mathbb{R}$

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

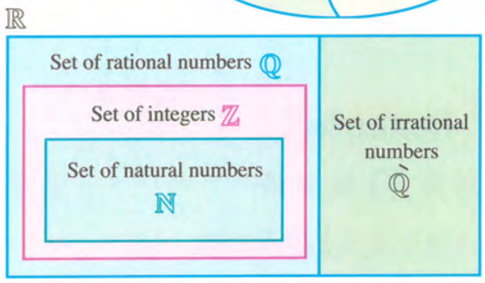


Noticing that : $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

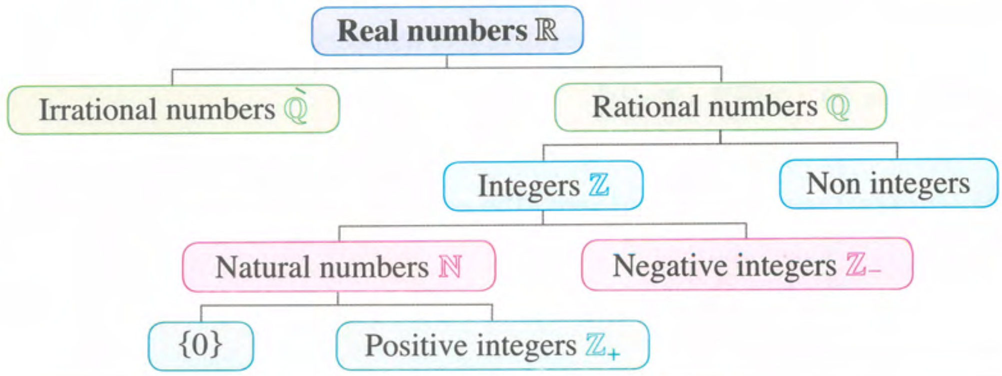
- **The opposite Venn diagram shows that :**

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

and $\grave{\mathbb{Q}} \subset \mathbb{R}$



The following diagram shows the relation among the sets of numbers that we studied till now :



Ordering numbers in $\mathbb{R}$

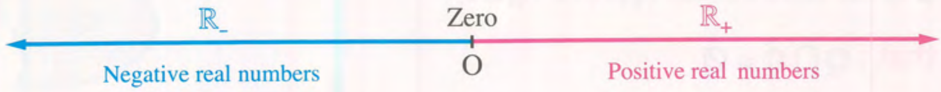
- Each real number is represented by a unique point on the number line.
- The set of real numbers is an ordered set.
- If the point representing the number x on the number line lies on the left of the point representing the number y as shown in the figure , then $x < y$ or $y > x$

- Each real number represented by a point lying on the right side of the origin O is greater than zero , and all these numbers form a set called "the set of the positive real numbers" denoted by $\mathbb{R}_+$

$$\mathbb{R}_+ = \{x : x \in \mathbb{R} , x > \text{zero}\}$$

- Each real number represented by a point lying on the left side of the origin O is less than zero and all these numbers form a set called "the set of the negative real numbers" denoted by $\mathbb{R}_-$

$$\mathbb{R}_- = \{x : x \in \mathbb{R} , x < \text{zero}\}$$



! Remarks

- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- The number zero is neither positive nor negative.
- $\mathbb{R}_+ \cup \{0\} = \{x : x \in \mathbb{R} , x \geq 0\}$
  and it is called the set of the non-negative real numbers.
- $\mathbb{R}_- \cup \{0\} = \{x : x \in \mathbb{R} , x \leq 0\}$
  and it is called the set of the non-positive real numbers.
- The set of real numbers without zero (The non-zero real numbers) is denoted by $\mathbb{R}^*$

  i.e. $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$

Example 1 **Arrange the following numbers ascendingly :**

$\sqrt{75}$, $\sqrt{68}$, $-\sqrt{45}$, -8 , 7 and $-\sqrt{32}$

Solution

- **Arrange the positive numbers which are** $\sqrt{75}$, $\sqrt{68}$ and 7

$\because 7 = \sqrt{49}$

, $\because 49 < 68 < 75$ $\quad\quad \therefore \sqrt{49} < \sqrt{68} < \sqrt{75}$

i.e. $7 < \sqrt{68} < \sqrt{75}$

- **Arrange the negative numbers which are** $-\sqrt{45}$, -8 and $-\sqrt{32}$

$\because 8 = \sqrt{64}$

, $\because 64 > 45 > 32$ $\quad\quad \therefore \sqrt{64} > \sqrt{45} > \sqrt{32}$

$\therefore -\sqrt{64} < -\sqrt{45} < -\sqrt{32}$

i.e. $-8 < -\sqrt{45} < -\sqrt{32}$

$\therefore$ The ascending order is : -8 , $-\sqrt{45}$, $-\sqrt{32}$, 7 , $\sqrt{68}$ and $\sqrt{75}$

! Remark

You can use the calculator to get the solution by finding approximated values of the roots.

Example 2 **Write three irrational numbers included between 11 and 12**

Solution

$\because (11)^2 = 121$, $(12)^2 = 144$

$\therefore$ 125 , 126 and 130 are three integers included between 121 and 144

$\therefore 121 < 125 < 126 < 130 < 144$

$\therefore \sqrt{121} < \sqrt{125} < \sqrt{126} < \sqrt{130} < \sqrt{144}$

$\therefore$ The required irrational numbers are : $\sqrt{125}$, $\sqrt{126}$ and $\sqrt{130}$

(Notice that : There are other irrational numbers included between 11 and 12)

Example 3 **Find the S.S. in $\mathbb{R}$ for each of the following equations :**

1 $3x^2 + 125 = 221$ **2** $\frac{1}{6}x^3 - 8 = 28$ **3** $2x^2 + 6 = 4$

Solution

1 $\because 3x^2 + 125 = 221$ $\therefore 3x^2 = 221 - 125$ $\therefore 3x^2 = 96$

$\therefore x^2 = \frac{96}{3}$ $\therefore x^2 = 32$ $\therefore x = \pm\sqrt{32}$

$\therefore$ The S.S. $= \left\{\sqrt{32}, -\sqrt{32}\right\}$

2 $\because \frac{1}{6}x^3 - 8 = 28$ $\therefore \frac{1}{6}x^3 = 36$ $\therefore x^3 = 6 \times 36$

$\therefore x^3 = 216$ $\therefore x = \sqrt[3]{216}$ $\therefore x = 6$

$\therefore$ The S.S. $= \{6\}$

3 $\because 2x^2 + 6 = 4$ $\therefore 2x^2 = 4 - 6$ $\therefore 2x^2 = -2$

$\therefore x^2 = -\frac{2}{2}$ $\therefore x^2 = -1$ $\therefore x = \pm\sqrt{-1}$

$, \because \sqrt{-1} \notin \mathbb{R}, -\sqrt{-1} \notin \mathbb{R}$ $\therefore$ The S.S. $= \varnothing$

TRY by yourself

Complete each of the following using one of the symbols > or < :

1 $\sqrt{2}$ 1	**2** $-\sqrt{3}$ .... -1	**3** $\sqrt[3]{9}$ 3
4 $-\sqrt[3]{7}$ .... -2	**5** $\sqrt{7}$ 2.6	**6** $-\sqrt[3]{16}$ .... -2.52

Answers *of try by yourself*

1 > **2** < **3** <

4 > **5** > **6** >

LESSON 4 | Intervals

Prelude Through your previous study , you knew different methods to express a subset of the set of natural numbers and a subset of the set of integers and you learnt how to represent them on the number line.

For example:

If X = the set of integers which are greater than or equal to – 3 and less than 2

* Then you can express the set X by the description method as follows : → $X = \{a : a \in \mathbb{Z} , -3 \le a < 2\}$
* You can also express it by listing method as follows : → $X = \{-3 , -2 , -1 , 0 , 1\}$
* The set X is represented on the number line as shown in the figure : →

• **And now the question is : Is it possible to use the same previous methods to express a subset of the set of real numbers and represent it on the number line ?**

Assuming that : K = the set of real numbers that are greater than or equal to – 3 and less than 2

* You can express the set K by the description method as follows : → $K = \{a : a \in \mathbb{R} , -3 \le a < 2\}$
* But it is impossible to express the set K by listing method because there are an infinity of real numbers existing between – 3 and 2
* For the same reason , it is impossible to represent this set K by separate points on the number line as shown in the previous figure therefore we use another method to express a subset of the set of real numbers , which is the intervals.

• **In the following , we will show the types of intervals :**

UNIT 1

First Limited intervals

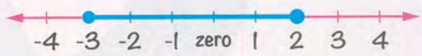
A Closed interval

> **Notice that :** The smaller number must be written first when you write the interval.

- The set $\{x : x \in \mathbb{R} , -3 \le x \le 2\}$ expresses the set of real numbers which consists of the two numbers -3 and 2 and all the real numbers included between them.

  We denote it by $[-3 , 2]$ and it is called a «closed interval».

- It is represented on the number line as shown in the figure :



Notice that : $-3 \in [-3 , 2]$, $2 \in [-3 , 2]$

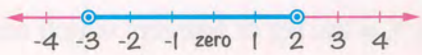
We express this by drawing two shaded circles at the two points representing the two numbers -3 and 2

B Opened interval

- The set $\{x : x \in \mathbb{R} , -3 < x < 2\}$ expresses the set of real numbers included between the two numbers -3 and 2 such that the two numbers -3 and 2 are not contained in this set.

  We denote this set by $]-3 , 2[$ and it is called an «opened interval».

- It is represented on the number line as in the figure :



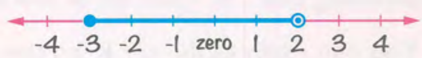
Notice that : $-3 \notin]-3 , 2[$ and $2 \notin]-3 , 2[$

We express this by drawing two unshaded circles at the two points representing the two numbers -3 and 2

C Half opened interval (Half closed interval)

1 • The set $\{x : x \in \mathbb{R} , -3 \le x < 2\}$ expresses the number -3 and all the real numbers included between -3 and 2 without the number 2 , we denote it by $[-3 , 2[$ and it is called a «half opened interval» or «half closed interval».

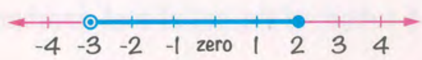
- It is represented on the number line as in the figure :



Notice that : $-3 \in [-3 , 2[$, $2 \notin [-3 , 2[$

2 • The set $\{x : x \in \mathbb{R} , -3 < x \le 2\}$ expresses the number 2 and all the real numbers included between -3 and 2 without the number -3 , we denote it by $]-3 , 2]$ and it is called a «half opened interval» or «half closed interval».

- It is represented on the number line as in the figure :



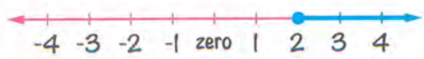
Notice that : $-3 \notin]-3 , 2]$, $2 \in]-3 , 2]$

Second Unlimited intervals

1 • The set $\{x : x \in \mathbb{R} , x \geq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are greater than 2 with no end.

It is denoted by $[2 , \infty[$ where the symbol «∞» is read as positive infinity and it doesn't represent a real number

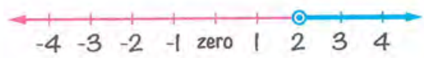
• It is represented on the number line as shown in the figure :



Notice that : $2 \in [2 , \infty[$

2 • The set $\{x : x \in \mathbb{R} , x > 2\}$ expresses the set of all real numbers which are greater than the number 2 with no end. It is denoted by $]2 , \infty[$

• It is represented on the number line as shown in the figure :

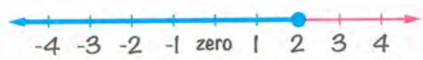


Notice that : $2 \notin]2 , \infty[$

3 • The set $\{x : x \in \mathbb{R} , x \leq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are smaller than the number 2 with no end.

It is denoted by $]-\infty , 2]$ where the symbol «– ∞» is read as negative infinity and it doesn't represent a real number.

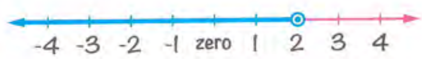
• It is represented on the number line as shown in the figure :



Notice that : $2 \in]-\infty , 2]$

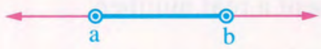
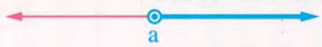
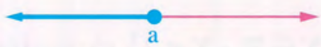
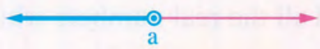
4 • The set $\{x : x \in \mathbb{R} , x < 2\}$ expresses the set of all real numbers which are smaller than the number 2 with no end. It is denoted by $]-\infty , 2[$

• It is represented on the number line as shown in the figure :



Notice that : $2 \notin]-\infty , 2[$

• We can express the previous symbolically in the following table assuming that :
$a \in \mathbb{R}$, $b \in \mathbb{R}$ and $a < b$

Types of intervals		The interval	Expression by distinguished property	Representation on the number line	Notice that
The limited intervals	Closed	$[a , b]$	$\{x : x \in \mathbb{R} , a \leq x \leq b\}$		• $a \in [a , b]$ • $b \in [a , b]$
	Opened	$]a , b[$	$\{x : x \in \mathbb{R} , a < x < b\}$		• $a \notin]a , b[$ • $b \notin]a , b[$
	half opened (half closed)	$[a , b[$	$\{x : x \in \mathbb{R} , a \leq x < b\}$		• $a \in [a , b[$ • $b \notin [a , b[$
		$]a , b]$	$\{x : x \in \mathbb{R} , a < x \leq b\}$		• $a \notin]a , b]$ • $b \in]a , b]$
The unlimited intervals		$[a , \infty[$	$\{x : x \in \mathbb{R} , x \geq a\}$		$a \in [a , \infty[$
		$]a , \infty[$	$\{x : x \in \mathbb{R} , x > a\}$		$a \notin]a , \infty[$
		$]-\infty , a]$	$\{x : x \in \mathbb{R} , x \leq a\}$		$a \in]-\infty , a]$
		$]-\infty , a[$	$\{x : x \in \mathbb{R} , x < a\}$		$a \notin]-\infty , a[$

! Remarks

- $\mathbb{R} =]-\infty , \infty[$
- $\mathbb{R}_+ =]0 , \infty[$
- $\mathbb{R}_- =]-\infty , 0[$
- The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$
- The set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$

Example 1 **Write each of the following sets in the form of an interval , then represent it on the number line :**

1. $\{x : x \in \mathbb{R} , -3 < x \leq 0\}$
2. $\{a : a \in \mathbb{R} , 1 \geq a \geq -2\}$
3. $\{x : x \in \mathbb{R} , x > 0\}$
4. $\{y : y \in \mathbb{R} , -1 \geq y\}$

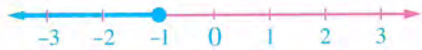
Solution

1 $]-3 , 0]$

2 $[-2 , 1]$

3 $]0 , \infty[$

4 $]-\infty , -1]$



TRY by yourself 1

1 ***Write each of the following in an interval form , then represent it on the number line :***

1 $\{x : x \in \mathbb{R} , -4 < x \le 2\}$

2 $\{y : y \in \mathbb{R} , y \ge -5\}$

2 ***Represent each of the following on the number line and express it by the description method :***

1 $]-3 , 0[$

2 $]-\infty , 2]$

Example 2

Choose the correct answer from those given :

1 $4 \in$ ....

(a) $]4 , 7[$ (b) $[-4 , 4[$ (c) $]2 , 5[$ (d) $[-11 , -4]$

2 $\sqrt[3]{-8}$ .... $[-8 , -2[$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

3 $\{1 , 6\}$ .... $]1 , 6]$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

4 If $x \in [-5 , \infty[$, then ....

(a) $x > -5$ (b) $x \ge -5$ (c) $x < -5$ (d) $x \le -5$

5 The sum of the real numbers in the interval $[-3 , 3[$ is ....

(a) -6 (b) -3 (c) zero (d) 6

Solution

1 **(c)**

2 **(b) The reason :** $\because \sqrt[3]{-8} = -2 , [-8 , -2[$ is open at -2

$\therefore -2 \notin [-8 , -2[$

3 (d) **The reason :** $1 \notin]1, 6]$ because the interval is open at 1

4 (b)

5 (b) **The reason :** Each number belongs to the interval has its additive inverse except -3 because $3 \notin [-3, 3[$

TRY by yourself 2

Complete using one of the symbols $\in$, $\notin$, $\subset$ or $\not\subset$:

1 -1 .... $[-4, -1[$

2 $\frac{1}{2}$ .... $]0, 1[$

3 $\{2, 4\}$ .... $[2, 5[$

4 $\sqrt{7}$ .... $]2, 3]$

5 $\{-1, 0, 1\}$ .... $[0, 1]$

6 $|-5|$ .... $]-\infty, 0]$

Operations on intervals

You studied before the sets and how to carry out the operations of intersection , union , difference and complement on them.

For example:

If $X = \{1, 2, 3, 4\}$, $Y = \{3, 4, 5, 6\}$, then :

- $X \cap Y$ = the set of elements which are common in X and Y $= \{3, 4\}$
- $X \cup Y$ = the set of all elements in X or Y without repeating $= \{1, 2, 3, 4, 5, 6\}$
- $X - Y$ = the set of elements which are in X and not in Y $= \{1, 2\}$
- $Y - X$ = the set of elements which are in Y and not in X $= \{5, 6\}$
- If the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$,

  then the complement of X which is denoted by $\grave{X} = U - X$

  i.e. $\grave{X}$ = the set of elements which are in U and not in X $= \{5, 6, 7\}$

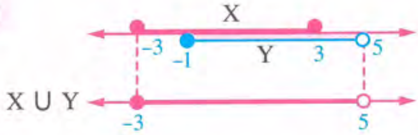
The following examples show how to carry out the operations of intersection , union and difference on intervals :

Example 3 If $X = [-3\ ,\ 3]$ and $Y = [-1\ ,5\ [$, find using the number line :

1. $X \cup Y$
2. $X \cap Y$
3. $X - Y$
4. $Y - X$

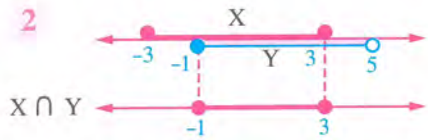
Solution

1.



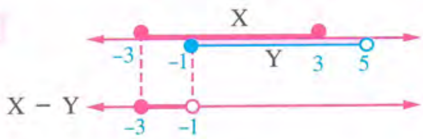
$X \cup Y = [-3\ ,5[$

2.



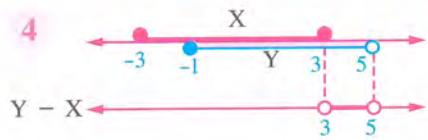
$X \cap Y = [-1\ ,3]$

3.



$X - Y = [-3\ ,-1[$

4.



$Y - X =]3\ ,5[$

Example 4 Find each of the following :

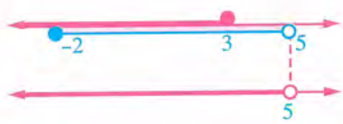
1. $]-\infty, 2\ ] \cap\ ]-3\ ,\infty\ [$
2. $]-\infty, 3\ ] \cup [-2\ ,5\ [$
3. $[5\ ,\infty\ [\ -\ ]\ 5\ ,\infty\ [$
4. $[2\ ,\infty\ [\ \cap\ ]-\infty\ ,2\ [$

Solution

1.

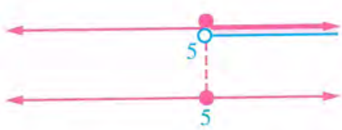
$]-\infty\ ,2\ ] \cap\ ]-3\ ,\infty[\ =\ ]-3\ ,2\ ]$

2.



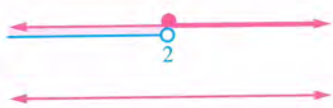
$]-\infty\ ,3\ ] \cup [-2\ ,5\ [\ =\ ]-\infty\ ,5\ [$

3.



$[5\ ,\infty[\ -\ ]\ 5\ ,\infty\ [\ = \{5\}$

4.



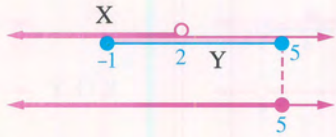
$[2\ ,\infty[\ \cap\ ]-\infty\ ,2[\ = \varnothing$

Example 5

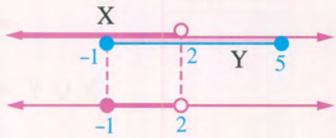
If $X =]-\infty , 2[$ and $Y = [-1 , 5]$, find using the number line :

1. $X \cup Y$
2. $X \cap Y$
3. $X - Y$
4. $Y - X$
5. $\grave{X}$
6. $\grave{Y}$

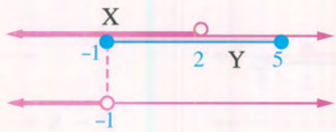
Solution

1.


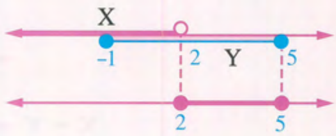
$X \cup Y =]-\infty , 5]$

2.


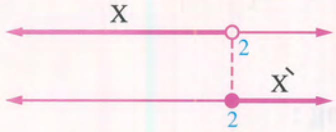
$X \cap Y = [-1 , 2[$

3.


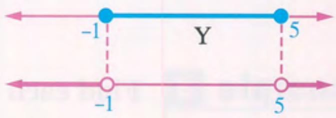
$X - Y =]-\infty , -1[$

4.


$Y - X = [2 , 5]$

5.


$\grave{X} = [2 , \infty[$

6.


$\grave{Y} =]-\infty , -1[\cup]5 , \infty[$
$= \mathbb{R} - [-1 , 5]$

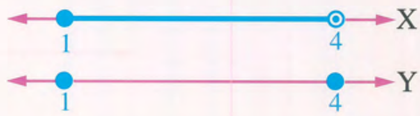
Example 6

If $X = [1 , 4[$ and $Y = \{1 , 4\}$, find :

1. $X \cap Y$
2. $X \cup Y$
3. $X - Y$
4. $Y - X$

Solution

1. $X \cap Y = \{1\}$
2. $X \cup Y = [1 , 4]$
3. $X - Y =]1 , 4[$
4. $Y - X = \{4\}$



TRY by yourself 3 *If X = [- 1 , 3 [and Y =]0 , 4] , find using the number line :*

1 $X \cap Y$	2 $X \cup Y$	3 $X - [0 , \infty[$
4 $]0 , \infty[- Y$	5 $X`$	6 $X \cap \{-2 , -1 , 0 , 1 , 2 , 3\}$

Answers of try by yourself

1 1 (1) $]-4 , 2]$

(2) $[-5 , \infty[$

2 (1) $\{x : x \in \mathbb{R} , -3 < x < 0\}$

(2) $\{x : x \in \mathbb{R} , x \leq 2\}$

2 1 $\notin$ 2 $\in$ 3 $\subset$ 4 $\in$ 5 $\not\subset$ 6 $\notin$

3 1 $]0 , 3[$ 2 $[-1 , 4]$ 3 $[-1 , 0[$ 4 $]4 , \infty[$ 5 $]-\infty , -1[\cup [3 , \infty[= \mathbb{R} - [-1 , 3[$ 6 $\{-1 , 0 , 1 , 2\}$

LESSON 5 Operations on the real numbers

First Addition

- We know that $2x$ and $3x$ are two like algebraic terms and their sum is an algebraic term like them.

  Where $2x + 3x = (2 + 3)x = 5x$

  Then we deduce that : $2\sqrt{5} + 3\sqrt{5} = (2 + 3)\sqrt{5}$

  $= 5\sqrt{5}$

Remember that

The real number $2\sqrt{5}$ is produced by multiplying the rational number 2 by the irrational number $\sqrt{5}$

- We know that $2x$ and $3y$ are two unlike algebraic terms and we express their sum by an algebraic expression whose simplest form is $2x + 3y$

  Therefore we deduce that :

  The two real numbers $2\sqrt{3}$ and $3\sqrt{2}$, their sum is expressed by a real number whose simplest form is $2\sqrt{3} + 3\sqrt{2}$

Properties of addition of real numbers

Closure

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

i.e. The sum of any two real numbers is a real number, therefore we say $\mathbb{R}$ is closed under addition.

For example: $\sqrt{5} \in \mathbb{R}$ and $2\sqrt{5} \in \mathbb{R}$, we find that : $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5} \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

For example: $5\sqrt[3]{2} + 4\sqrt[3]{2} = 9\sqrt[3]{2}$, $4\sqrt[3]{2} + 5\sqrt[3]{2} = 9\sqrt[3]{2}$

i.e. $5\sqrt[3]{2} + 4\sqrt[3]{2} = 4\sqrt[3]{2} + 5\sqrt[3]{2}$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

For example: $\left(\sqrt{3} + 2\sqrt{3}\right) + 5\sqrt{3} = 3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$,

$\sqrt{3} + \left(2\sqrt{3} + 5\sqrt{3}\right) = \sqrt{3} + 7\sqrt{3} = 8\sqrt{3}$

i.e. $\left(\sqrt{3} + 2\sqrt{3}\right) + 5\sqrt{3} = \sqrt{3} + \left(2\sqrt{3} + 5\sqrt{3}\right)$

The additive neutral

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example: $\sqrt{2} + 0 = 0 + \sqrt{2} = \sqrt{2}$, $-\sqrt[3]{5} + 0 = 0 + \left(-\sqrt[3]{5}\right) = -\sqrt[3]{5}$

The additive inverse of every real number

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

For example:
- The additive inverse of $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + \left(-\sqrt{3}\right) = 0$
- The additive inverse of $2 + \sqrt{5}$ is $-\left(2 + \sqrt{5}\right)$ and equals $-2 - \sqrt{5}$
- The additive inverse of $3 - \sqrt{2}$ is $-\left(3 - \sqrt{2}\right)$ and equals $\sqrt{2} - 3$
- The additive inverse of zero is itself.

! Remark

Since every real number has an additive inverse , then the subtraction operation is possible entirely in $\mathbb{R}$, and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a - b = a + (-b)$

i.e. The subtraction operation $(a - b)$ means adding the number a to the additive inverse of the number b

And we can deduce that :

Subtraction operation in $\mathbb{R}$ is not commutative and it is not associative.

Example 1 **Choose the correct answer from those given :**

1. $\sqrt{7}+\sqrt{7}=$ ....

   (a) $\sqrt{14}$ (b) $2\sqrt{7}$ (c) 7 (d) 14

2. $2\sqrt{2}-3\sqrt{2}=$ ....

   (a) -1 (b) $-\sqrt{2}$ (c) $\sqrt{2}$ (d) $5\sqrt{2}$

3. $4+\sqrt{3}-7-\sqrt{3}=$ ....

   (a) $-3-2\sqrt{3}$ (b) $-3+2\sqrt{3}$ (c) -3 (d) 3

4. If $x=9\sqrt{5}$, $y=5\sqrt{3}$, then $x-y=$ ....

   (a) $4\sqrt{2}$ (b) $4\sqrt{5}$ (c) $4\sqrt{3}$ (d) $9\sqrt{5}-5\sqrt{3}$

5. The additive inverse of $\sqrt{7}-\sqrt{5}$ is ....

   (a) $-\sqrt{7}-\sqrt{5}$ (b) $\sqrt{7}+\sqrt{5}$ (c) $\sqrt{5}-\sqrt{7}$ (d) $\sqrt{7}-\sqrt{5}$

6. If $\sqrt{2}+x=0$, then $x-\sqrt{2}=$ ....

   (a) zero (b) $-\sqrt{2}$ (c) $-2\sqrt{2}$ (d) $2\sqrt{2}$

Solution

1. **(b)**
2. **(b) The reason :** $2\sqrt{2}-3\sqrt{2}=(2-3)\sqrt{2}=-\sqrt{2}$
3. **(c) The reason :** $4+\sqrt{3}-7-\sqrt{3}=(4-7)+\left(\sqrt{3}-\sqrt{3}\right)=-3+0=-3$
4. **(d) The reason :** $x-y=9\sqrt{5}-5\sqrt{3}$ and this is the simplest form of the difference.
5. **(c) The reason :** The additive inverse of $\sqrt{7}-\sqrt{5}$ is $-\left(\sqrt{7}-\sqrt{5}\right)$

   which is $-\sqrt{7}+\sqrt{5}$ or $\sqrt{5}-\sqrt{7}$
6. **(c) The reason :** x is the additive inverse of $\sqrt{2}$ which is $-\sqrt{2}$

   $\therefore x-\sqrt{2}=-\sqrt{2}-\sqrt{2}=-2\sqrt{2}$

TRY by yourself 1

1 ***Write the additive inverse for each of the following numbers :***

$\sqrt{2}$, $-\sqrt[3]{5}$, $\sqrt{2}+\sqrt{7}$, $\sqrt[3]{5}-3$, $-\sqrt{6}-\sqrt[3]{7}$

2 ***Simplify to the simplest form :***

1. $2+2\sqrt{7}-1-5\sqrt{7}$
2. $3\sqrt{5}+\sqrt{3}-3\sqrt{5}+5\sqrt{3}$

Second Multiplication

- We know that : $3 \times 2\,X = (3 \times 2)\,X = 6\,X$

  Therefore we find that : $3 \times 2\sqrt{3} = (3 \times 2)\sqrt{3} = 6\sqrt{3}$
- We know also $2X \times 5X = (2 \times 5)\,(X \times X) = 10\,X^2$

  Therefore we find that : $2\sqrt{3} \times 5\sqrt{3} = (2 \times 5) \times \left(\sqrt{3} \times \sqrt{3}\right) = 10\left(\sqrt{3}\right)^2 = 10 \times 3 = 30$

Example 2 **Find the result of each of the following :**

1 $-2 \times 3\sqrt{5}$ 2 $4\sqrt{2} \times \sqrt{2}$ 3 $-2\sqrt{7} \times 4\sqrt{7}$

Solution

1 $-2 \times 3\sqrt{5} = (-2 \times 3)\sqrt{5} = -6\sqrt{5}$

2 $4\sqrt{2} \times \sqrt{2} = 4\left(\sqrt{2}\right)^2 = 4 \times 2 = 8$

3 $-2\sqrt{7} \times 4\sqrt{7} = (-2 \times 4) \times \left(\sqrt{7}\right)^2 = -8 \times 7 = -56$

Properties of multiplication of real numbers

Closure

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$

i.e. The product of any two real numbers is a real number therefore we say :

$\mathbb{R}$ is closed under multiplication.

For example: $\sqrt{3} \in \mathbb{R}$ and $2\sqrt{3} \in \mathbb{R}$

We find that : $\sqrt{3} \times 2\sqrt{3} = 2 \times 3 = 6 \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$

For example: $2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$, $3\sqrt{5} \times 2\sqrt{5} = 6 \times 5 = 30$

i.e. $2\sqrt{5} \times 3\sqrt{5} = 3\sqrt{5} \times 2\sqrt{5}$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$

For example: $\left(2\sqrt{7} \times 4\sqrt{7}\right) \times \sqrt{7} = 56 \times \sqrt{7} = 56\sqrt{7}$,

$2\sqrt{7} \times \left(4\sqrt{7} \times \sqrt{7}\right) = 2\sqrt{7} \times 28 = 56\sqrt{7}$

i.e. $\left(2\sqrt{7} \times 4\sqrt{7}\right) \times \sqrt{7} = 2\sqrt{7} \times \left(4\sqrt{7} \times \sqrt{7}\right)$

The multiplicative neutral

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$

i.e. One is the multiplicative neutral in $\mathbb{R}$

For example: $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

For example:

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$ because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $-\frac{\sqrt{2}}{5}$ is $-\frac{5}{\sqrt{2}}$
- The multiplicative inverse of 1 is itself and also the multiplicative inverse of -1 is itself.

Notice that :

- Both the number and its multiplicative inverse have the same sign.
- There is no multiplicative inverse for zero because $\frac{1}{\text{zero}}$ is meaningless (i.e. Undefined)

Remark

Since each non-zero real number has a multiplicative inverse , then the division operation by any real number does not equal zero is possible in $\mathbb{R}$ and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

i.e. The division operation $(a \div b)$ means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

And we can deduce that :

Division operation in $\mathbb{R}$ is not commutative and it is not associative.

Example 3 **Find the result of :** $\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}} \div \frac{1}{3\sqrt{2}}$

Solution

$$\left(\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}}\right) \div \frac{1}{3\sqrt{2}} = \frac{5}{15\sqrt{2}} \div \frac{1}{3\sqrt{2}} = \frac{1}{3\sqrt{2}} \times 3\sqrt{2} = 1$$

Example 4 **Write each of the following such that the denominator is an integer :**

1 $\frac{9}{\sqrt{3}}$ 2 $-\frac{3}{\sqrt{2}}$ 3 $\frac{5}{3\sqrt{5}}$

Solution

1 Multiplying the two terms of $\frac{9}{\sqrt{3}}$ by $\sqrt{3}$

, we get $\frac{9}{\sqrt{3}} = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$

Notice that : $\frac{\sqrt{3}}{\sqrt{3}} = 1$ "The multiplicative neutral"

2 $-\frac{3}{\sqrt{2}} = -\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-3\sqrt{2}}{2}$

3 $\frac{5}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{3 \times 5} = \frac{\sqrt{5}}{3}$

Another solution :

$\because \sqrt{5} \times \sqrt{5} = 5$ $\therefore \frac{5}{3\sqrt{5}} = \frac{\sqrt{5} \times \sqrt{5}}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$

Example 5 **Choose the correct answer from those given :**

1 The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is ....

(a) $\sqrt{10}$ (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $-2\sqrt{5}$

2 The additive inverse of $\frac{7}{\sqrt{7}}$ is ....

(a) $\frac{\sqrt{7}}{7}$ (b) 7 (c) $-\sqrt{7}$ (d) -7

3 The multiplicative inverse of $\frac{3\sqrt{2}}{4}$ equals $\frac{\cdots\cdots\cdots\cdots}{3}$

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $\frac{4}{3\sqrt{2}}$

Solution

1 **(c) The reason :** The multiplicative inverse of $\frac{\sqrt{5}}{10}$

is $\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$

2 **(c) The reason :** The additive inverse of $\frac{7}{\sqrt{7}}$

is $-\frac{7}{\sqrt{7}} = -\frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = -\frac{7\sqrt{7}}{7}$

$= -\sqrt{7}$

3 **(b) The reason :** The multiplicative inverse of $\frac{3\sqrt{2}}{4}$

is $\frac{4}{3\sqrt{2}} = \frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6}$

$= \frac{2\sqrt{2}}{3}$

TRY by yourself 2

1 ***Find each of the following :***

1 $\sqrt{5} \times \frac{1}{\sqrt{5}} \times \sqrt{5}$

2 $\frac{\sqrt{3}}{3} \times \frac{4\sqrt{5}}{20} \times \frac{5\sqrt{3}}{\sqrt{5}}$

2 ***Make the denominator an integer :***

1 $\frac{3}{\sqrt{7}}$

2 $\frac{9}{2\sqrt{6}}$

Distributing multiplication on addition and subtraction

WATCH VIDEO

For any three real numbers a , b and c it will be :

- $a(b \pm c) = ab \pm ac$
- $(b \pm c)a = ba \pm ca$

Example 6 **Find each of the following :**

1 $2\sqrt{3}\left(5\sqrt{3}-4\right)$

2 $\left(2+\sqrt{3}\right)\left(\sqrt{3}+7\right)$

3 $\left(7\sqrt{2}-5\right)\left(7\sqrt{2}+5\right)$

4 $\left(5\sqrt{3}-2\right)^2$

Solution

1 $2\sqrt{3}\left(5\sqrt{3}-4\right)=2\sqrt{3}\times5\sqrt{3}+2\sqrt{3}\times(-4)$

$=10\times3-8\times\sqrt{3}=30-8\sqrt{3}$

2 $\left(2+\sqrt{3}\right)\left(\sqrt{3}+7\right)=2\left(\sqrt{3}+7\right)+\sqrt{3}\left(\sqrt{3}+7\right)$

$=2\times\sqrt{3}+2\times7+\sqrt{3}\times\sqrt{3}+\sqrt{3}\times7$

$=2\sqrt{3}+14+3+7\sqrt{3}$

$=\left(2\sqrt{3}+7\sqrt{3}\right)+(14+3)=9\sqrt{3}+17$

3 $\left(7\sqrt{2}-5\right)\left(7\sqrt{2}+5\right)=98+35\sqrt{2}-35\sqrt{2}-25=73$

Another solution by multiplying by inspection :

$\left(7\sqrt{2}-5\right)\left(7\sqrt{2}+5\right)=\left(7\sqrt{2}\right)^2-(5)^2$

$=7^2\times\left(\sqrt{2}\right)^2-5^2$

$=49\times2-25=98-25=73$

Notice that :
$(a+b)(a-b)=a^2-b^2$

4 Multiplying by inspection

$\therefore\left(5\sqrt{3}-2\right)^2=\left(5\sqrt{3}\right)^2-2\times5\sqrt{3}\times2+(-2)^2$

$=5^2\times\left(\sqrt{3}\right)^2-20\sqrt{3}+4$

$=25\times3-20\sqrt{3}+4$

$=75-20\sqrt{3}+4=79-20\sqrt{3}$

Notice that :
- $(a+b)^2=a^2+2ab+b^2$
- $(a-b)^2=a^2-2ab+b^2$

Example 7 If $x=5\sqrt{3}-2$, $y=5\sqrt{3}+2$

, find the value of the expression : $x^2+2xy+y^2$

Solution From multiplying by inspection , we find that :

$(x+y)^2=x^2+2xy+y^2$

$\therefore x^2+2xy+y^2=\left(5\sqrt{3}-2+5\sqrt{3}+2\right)^2$

$=\left(10\sqrt{3}\right)^2=(10)^2\times\left(\sqrt{3}\right)^2=100\times3=300$

Example 8 Give an estimation for the result of :

$\left(5+\sqrt{10}\right)\left(3-\sqrt[3]{7}\right)$, then check your answer using the calculator.

Solution **First :** The estimation of $\sqrt{10}$ is 3 $\left(\text{because } \sqrt{9}=3\right)$

$\therefore$ The estimation of $\left(5+\sqrt{10}\right)$ is $5+3=8$

, the estimation of $\sqrt[3]{7}$ is 2 $\left(\text{because } \sqrt[3]{8}=2\right)$

$\therefore$ The estimation of $\left(3-\sqrt[3]{7}\right)$ is $3-2=1$

$\therefore$ The estimation of $\left(5+\sqrt{10}\right)\left(3-\sqrt[3]{7}\right)$ is $8\times 1=8$

Second : By using the calculator , we find that the result approximated to the nearest thousandths is 8.873

i.e. The estimation is accepted.

TRY by yourself 3

1 ***Find the result of each of the following in the simplest form :***

1 $5\sqrt{2}\left(3\sqrt{2}-2\right)$ 2 $\left(2\sqrt{3}-3\right)\left(2\sqrt{3}+3\right)$

2 If $x=2\sqrt{3}-1$ and $y=2\sqrt{3}+1$

, find the value of the expression : $x^2-2xy+y^2$

3 ***Give an estimation for the result of :*** $\left(1+\sqrt{15}\right)\left(4-\sqrt{8}\right)$

, then check your answer by using the calculator.

Answers *of try by yourself*

1 1 $-\sqrt{2}$, $\sqrt[3]{5}$, $-\sqrt{2}-\sqrt{7}$, $-\sqrt[3]{5}+3$, $\sqrt{6}+\sqrt[3]{7}$

2 (1) $1-3\sqrt{7}$ (2) $6\sqrt{3}$

2 1 (1) $\sqrt{5}$ (2) 1

2 (1) $\frac{3\sqrt{7}}{7}$ (2) $\frac{3\sqrt{6}}{4}$

3 1 (1) $30-10\sqrt{2}$ (2) 3 2 4 3 5 (Check by yourself)

LESSON **6**

Operations on the square roots

If a and b are two non negative real numbers , then :

1 $\sqrt{a} \times \sqrt{b} = \sqrt{a\,b}$

For example: • $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$ • $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

2 $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ (where $b \neq 0$)

For example: • $\dfrac{\sqrt{8}}{\sqrt{2}} = \sqrt{\dfrac{8}{2}} = \sqrt{4} = 2$ • $\sqrt{\dfrac{16}{49}} = \dfrac{\sqrt{16}}{\sqrt{49}} = \dfrac{4}{7}$

3 $\dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{\sqrt{a}}{\sqrt{b}} \times \dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{\sqrt{ab}}{b}$ (where $b \neq 0$)

This operation is carried out to make the denominator an integer.

For example: • $\dfrac{\sqrt{2}}{\sqrt{5}} = \dfrac{\sqrt{2}}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{10}}{5}$ • $\sqrt{\dfrac{3}{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}}{2}$

Remarks

1. $\sqrt{a^2+b^2} \neq a+b$, $\sqrt{a^2-b^2} \neq a-b$

For example:

- $\sqrt{6^2+8^2} \neq 6+8$ because $\sqrt{6^2+8^2} = \sqrt{100} = 10$
- $\sqrt{25-9} \neq 5-3$ because $\sqrt{25-9} = \sqrt{16} = 4$

2. $a\sqrt{b} = \sqrt{a^2 b}$

For example:

- $2\sqrt{\frac{1}{2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$
- $15\sqrt{\frac{1}{3}} = 5 \times 3\sqrt{\frac{1}{3}} = 5\sqrt{9 \times \frac{1}{3}} = 5\sqrt{3}$

Example 1

Write each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :

1. $\sqrt{27}$
2. $5\sqrt{54}$
3. $3\sqrt{\frac{2}{3}}$
4. $\frac{\sqrt{84}}{\sqrt{7}}$

Solution

1. $\sqrt{27} = \sqrt{9 \times 3}$

$= \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

2. $5\sqrt{54} = 5\sqrt{9 \times 6} = 5 \times \sqrt{9} \times \sqrt{6}$

$= 5 \times 3 \times \sqrt{6} = 15\sqrt{6}$

3. $3\sqrt{\frac{2}{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3 \times \frac{\sqrt{6}}{3} = \sqrt{6}$

Another solution :

$3\sqrt{\frac{2}{3}} = \sqrt{3^2 \times \frac{2}{3}} = \sqrt{3 \times 2} = \sqrt{6}$

4. $\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Example 2 Simplify to the simplest form :

1 $\sqrt{45}-2\sqrt{20}+2\sqrt{5}$

2 $2\sqrt{18}+\sqrt{50}-42\sqrt{\frac{1}{2}}$

3 $2\sqrt{27}-3\sqrt{\frac{1}{3}}-\frac{6}{\sqrt{3}}$

Solution

1 $\sqrt{45}-2\sqrt{20}+2\sqrt{5}=\sqrt{9\times5}-2\sqrt{4\times5}+2\sqrt{5}$

$=\sqrt{9}\times\sqrt{5}-2\times\sqrt{4}\times\sqrt{5}+2\sqrt{5}$

$=3\sqrt{5}-2\times2\sqrt{5}+2\sqrt{5}$

$=3\sqrt{5}-4\sqrt{5}+2\sqrt{5}=\sqrt{5}$

2 $2\sqrt{18}+\sqrt{50}-42\sqrt{\frac{1}{2}}=2\sqrt{9\times2}+\sqrt{25\times2}-42\times\frac{\sqrt{1}}{\sqrt{2}}$

$=2\times\sqrt{9}\times\sqrt{2}+\sqrt{25}\times\sqrt{2}-42\times\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=2\times3\sqrt{2}+5\sqrt{2}-21\sqrt{2}=-10\sqrt{2}$

3 $2\sqrt{27}-3\sqrt{\frac{1}{3}}-\frac{6}{\sqrt{3}}=2\sqrt{9\times3}-3\times\frac{\sqrt{1}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=6\sqrt{3}-\sqrt{3}-\frac{6\sqrt{3}}{3}$

$=6\sqrt{3}-\sqrt{3}-2\sqrt{3}=3\sqrt{3}$

Example 3 Find the result of each of the following :

1 $2\sqrt{3}\left(\sqrt{6}+5\right)$ 2 $\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)$ 3 $\left(\sqrt{2}+\sqrt{6}\right)^2$

Solution

1 $2\sqrt{3}\left(\sqrt{6}+5\right)=2\sqrt{3}\times\sqrt{6}+2\sqrt{3}\times5$

$=2\sqrt{18}+10\sqrt{3}$

$=2\sqrt{9\times2}+10\sqrt{3}$

$=6\sqrt{2}+10\sqrt{3}$

2 $\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)=\left(3\sqrt{2}\right)^2-(5)^2$

$=3^2\times\left(\sqrt{2}\right)^2-(5)^2$

$=9\times2-25$

$=18-25=-7$

Remember that

$(a-b)(a+b)=a^2-b^2$

3 $\left(\sqrt{2}+\sqrt{6}\right)^2 = \left(\sqrt{2}\right)^2 + 2\times\sqrt{2}\times\sqrt{6} + \left(\sqrt{6}\right)^2$

$= 2 + 2\sqrt{12} + 6$

$= 8 + 2\sqrt{4\times 3} = 8 + 4\sqrt{3}$

Remember that

- $(a+b)^2 = a^2 + 2\,ab + b^2$
- $(a-b)^2 = a^2 - 2\,ab + b^2$

Example 4

If $a = \dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$, find the value of $a^2 + 2\sqrt{3}$

Solution

To facilitate the solution , we will make the denominator an integer by multiplying both the numerator and the denominator by $\sqrt{2}$

$$\therefore a = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}\times\sqrt{2}-\sqrt{2}\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{12}-2}{2}$$

$$= \frac{\sqrt{4\times 3}-2}{2} = \frac{2\sqrt{3}-2}{2} = \frac{2\left(\sqrt{3}-1\right)}{2} = \sqrt{3}-1$$

$$\therefore a^2 = \left(\sqrt{3}-1\right)^2 = \left(\sqrt{3}\right)^2 - 2\times\sqrt{3}\times 1 + 1 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$$

$$\therefore a^2 + 2\sqrt{3} = 4 - 2\sqrt{3} + 2\sqrt{3} = 4$$

Another method to simplify a :

$$\because a = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}} \qquad \therefore a = \frac{\sqrt{6}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{\frac{6}{2}} - 1 = \sqrt{3}-1$$

TRY by yourself 1

1 ***Simplify to the simplest form :***

1 $\sqrt{75} - 2\sqrt{27} + \sqrt{3}$ 2 $2\sqrt{50} - 3\sqrt{2} - 4\sqrt{\dfrac{1}{8}}$

2 ***Write each of the following such that the denominator is an integer :***

1 $\dfrac{5\sqrt{3}}{2\sqrt{5}}$ 2 $\dfrac{1+\sqrt{3}}{3\sqrt{3}}$

Answers *of try by yourself*

1 (1) 0 (2) $6\sqrt{2}$

2 (1) $\dfrac{\sqrt{15}}{2}$ (2) $\dfrac{3+\sqrt{3}}{9}$

LESSON 7

The two conjugate numbers

If a and b are two positive rational numbers

Then each of the two numbers $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$ is conjugate to the other one and we find that :

- **Their sum =** $\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)=2\sqrt{a}=$ twice the first term.

- **Their product =** $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\underbrace{\left(\sqrt{a}\right)^2}_{\text{the square of }1^{\text{st}}\text{ term}} - \underbrace{\left(\sqrt{b}\right)^2}_{\text{the square of }2^{\text{nd}}\text{ term}}=a-b$

For example: $\left(\sqrt{3}-\sqrt{2}\right)$ its conjugate is $\left(\sqrt{3}+\sqrt{2}\right)$, then we find that

- Their sum $=2\sqrt{3}$
- Their product $=3-2=1$

! Remark

The product of the two conjugate numbers is always a rational number.

Remark

If we have a real number whose denominator is written in the form $\left(\sqrt{a}+\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

Example 1

Choose the correct answer from those given :

1 The number $\frac{4}{\sqrt{7}-\sqrt{3}}$ in the simplest form is ....

(a) $\sqrt{7}-\sqrt{3}$ (b) $\sqrt{7}+\sqrt{3}$

(c) $4\sqrt{7}-4\sqrt{3}$ (d) $4\sqrt{7}+4\sqrt{3}$

2 The conjugate of $\frac{1}{\sqrt{3}-\sqrt{2}}$ is ....

(a) $\sqrt{3}-\sqrt{2}$ (b) $\sqrt{3}-2$ (c) $\sqrt{3}+\sqrt{2}$ (d) $\sqrt{3}+2$

3 The multiplicative inverse of $1-\sqrt{2}$ is ....

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$ (c) $-1-\sqrt{2}$ (d) $1+\sqrt{2}$

4 If $\frac{1}{x}=\sqrt{10}-3$, then $x=$ ....

(a) $\sqrt{10}+3$ (b) $\sqrt{10}-3$ (c) $3-\sqrt{10}$ (d) $-3-\sqrt{10}$

Solution

1 **(b) The reason :** Multiplying the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{7}+\sqrt{3}\right)$

$$\therefore \frac{4}{\sqrt{7}-\sqrt{3}}=\frac{4}{\sqrt{7}-\sqrt{3}}\times\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

$$=\frac{4\left(\sqrt{7}+\sqrt{3}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}=\frac{4\left(\sqrt{7}+\sqrt{3}\right)}{7-3}$$

$$=\sqrt{7}+\sqrt{3}$$

2 **(a) The reason :** Multiplying the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{3}+\sqrt{2}\right)$

$$\therefore \frac{1}{\sqrt{3}-\sqrt{2}}=\frac{1}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{\sqrt{3}+\sqrt{2}}{3-2}=\sqrt{3}+\sqrt{2}$$

$\therefore$ The conjugate of $\frac{1}{\sqrt{3}-\sqrt{2}}$ is $\sqrt{3}-\sqrt{2}$

3 **(c) The reason :** The multiplicative inverse of $1-\sqrt{2}$ is $\frac{1}{1-\sqrt{2}}$,

by multiplying the two terms of the number by the conjugate of the denominator which is $\left(1+\sqrt{2}\right)$

$$\therefore \frac{1}{1-\sqrt{2}} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1-2}$$

$$= \frac{1+\sqrt{2}}{-1} = -1-\sqrt{2}$$

4 **(a) The reason :** $\because \frac{1}{x} = \sqrt{10}-3$ $\qquad \therefore x = \frac{1}{\sqrt{10}-3}$

$$\therefore x = \frac{1}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3} = \frac{\sqrt{10}+3}{10-9} = \sqrt{10}+3$$

Example 2 If $x = \frac{4}{2-\sqrt{2}}$ and $y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$, write each of x and y such that its denominator is a rational number , then find $x+y$

Solution

$$\because x = \frac{4}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{4\left(2+\sqrt{2}\right)}{4-2} = \frac{4\left(2+\sqrt{2}\right)}{2}$$

$$= 2\left(2+\sqrt{2}\right) = 4+2\sqrt{2}$$

$$, y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{\left(3-2\sqrt{2}\right)^2}{9-8} = \frac{9-12\sqrt{2}+8}{1} = 17-12\sqrt{2}$$

$$\therefore x+y = 4+2\sqrt{2}+17-12\sqrt{2} = 21-10\sqrt{2}$$

TRY by yourself 1

Write each of the following such that the denominator is a rational number :

1 $\frac{12}{\sqrt{6}-\sqrt{2}}$ $\qquad$ **2** $\frac{\sqrt{8}}{3+2\sqrt{2}}$

Important remarks from direct product (multiplying by inspection)

- We know that : $(x-y)(x+y)=x^2-y^2$
- And we know also :

$(x+y)^2 = x^2+2xy+y^2$

Then :

- $x^2+xy+y^2=(x+y)^2-xy$
- $x^2+y^2=(x+y)^2-2xy$

$(x-y)^2 = x^2-2xy+y^2$

Then :

- $x^2-xy+y^2=(x-y)^2+xy$
- $x^2+y^2=(x-y)^2+2xy$

Example 3 If $x=\dfrac{2}{\sqrt{5}-\sqrt{3}}$ and $y=\sqrt{5}-\sqrt{3}$, prove that x and y are conjugate numbers , then find the value of each of :

1 $x^2+2xy+y^2$ **2** x^2+xy+y^2

Solution

$$\because x=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}=\sqrt{5}+\sqrt{3}$$

$, \because y=\sqrt{5}-\sqrt{3}$

$\therefore x$ and y are conjugate numbers.

1 $x^2+2xy+y^2=\left(\sqrt{5}+\sqrt{3}\right)^2+2\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)+\left(\sqrt{5}-\sqrt{3}\right)^2$

$$=\left(5+2\sqrt{15}+3\right)+2(5-3)+\left(5-2\sqrt{15}+3\right)$$

$$=8+2\sqrt{15}+4+8-2\sqrt{15}=20$$

Another solution using the previous remarks :

Since $x^2+2xy+y^2=(x+y)^2$

$$\therefore x^2+2xy+y^2=\left[\left(\sqrt{5}+\sqrt{3}\right)+\left(\sqrt{5}-\sqrt{3}\right)\right]^2$$

$$=\left(2\sqrt{5}\right)^2=4\times5=20$$

2 $x^2 + xy + y^2 = \left(\sqrt{5}+\sqrt{3}\right)^2 + \left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right) + \left(\sqrt{5}-\sqrt{3}\right)^2$

$$= \left(5+3+2\sqrt{15}\right) + (2) + \left(5+3-2\sqrt{15}\right) = 18$$

Another solution using the previous remarks :

$$x^2 + xy + y^2 = (x+y)^2 - xy = \left(\sqrt{5}+\sqrt{3}+\sqrt{5}-\sqrt{3}\right)^2 - \left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$$

$$= \left(2\sqrt{5}\right)^2 - 2 = 20 - 2 = 18$$

TRY by yourself 1

If $x = \dfrac{3}{2\sqrt{2}-\sqrt{5}}$ and $y = 2\sqrt{2}-\sqrt{5}$, find the value of the expression : $x^2 - y^2$

Answers of try by yourself

1 (1) $3\sqrt{6}+3\sqrt{2}$ (2) $-8+6\sqrt{2}$

2 $8\sqrt{10}$

LESSON **8**

Operations on the cube roots

If a and b are two real numbers , then :

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$

For example:

- $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$
- $\sqrt[3]{2} \times \sqrt[3]{-4} = \sqrt[3]{2 \times -4} = \sqrt[3]{-8} = -2$
- $\sqrt[3]{16} = \sqrt[3]{8 \times 2} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$
- $\sqrt[3]{-54} = \sqrt[3]{-27 \times 2} = \sqrt[3]{-27} \times \sqrt[3]{2} = -3\sqrt[3]{2}$

2 $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)

For example:

- $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\dfrac{32}{4}} = \sqrt[3]{8} = 2$
- $\dfrac{\sqrt[3]{54}}{\sqrt[3]{-2}} = \sqrt[3]{-\dfrac{54}{2}} = \sqrt[3]{-27} = -3$
- $\sqrt[3]{\dfrac{8}{125}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{125}} = \dfrac{2}{5}$
- $\sqrt[3]{-\dfrac{27}{64}} = \dfrac{\sqrt[3]{-27}}{\sqrt[3]{64}} = \dfrac{-3}{4}$

Example 1 Find the result of each of the following in its simplest form :

1 $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}}$

2 $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}}$

Solution

1 $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}} = \sqrt[3]{\frac{2}{3} \times \frac{4}{9}} = \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$

2 $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}} = \sqrt[3]{\frac{5}{4} \div \frac{2}{25}} = \sqrt[3]{\frac{5}{4} \times \frac{25}{2}} = \sqrt[3]{\frac{125}{8}} = \frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$

! Remarks

If a and b are two real numbers , then :

1 $\sqrt[3]{a^3 + b^3} \neq a + b$, $\sqrt[3]{a^3 - b^3} \neq a - b$

2 $\sqrt[3]{-a} = -\sqrt[3]{a}$

3 $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$

For example: • $3\sqrt[3]{\frac{1}{9}} = \sqrt[3]{27 \times \frac{1}{9}} = \sqrt[3]{3}$

• $8\sqrt[3]{\frac{1}{4}} = 4 \times 2\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{8 \times \frac{1}{4}} = 4\sqrt[3]{2}$

4 $\sqrt[3]{\frac{a}{b}} = \sqrt[3]{\frac{a}{b} \times \frac{b^2}{b^2}} = \sqrt[3]{\frac{a\,b^2}{b^3}} = \frac{1}{b}\sqrt[3]{a\,b^2}$ (Where $b \neq 0$)

For example: $\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} = \sqrt[3]{\frac{9}{27}} = \frac{1}{3}\sqrt[3]{9}$

Example 2 Put each of the following in its simplest form :

1 $\sqrt[3]{24} + \sqrt[3]{3} - \sqrt[3]{81}$

2 $\sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}}$

3 $\sqrt[3]{81} + \sqrt{12} - 2\sqrt[3]{3} - 2\sqrt{3}$

Solution

1 $\sqrt[3]{24}+\sqrt[3]{3}-\sqrt[3]{81}=\sqrt[3]{8\times 3}+\sqrt[3]{3}-\sqrt[3]{27\times 3}$

$=\sqrt[3]{8}\times\sqrt[3]{3}+\sqrt[3]{3}-\sqrt[3]{27}\times\sqrt[3]{3}$

$=2\sqrt[3]{3}+\sqrt[3]{3}-3\sqrt[3]{3}=\text{zero}$

2 $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{27\times 2}+6\sqrt[3]{8\times 2}-3\times 2\sqrt[3]{\frac{1}{4}}$

$=\sqrt[3]{27}\times\sqrt[3]{2}+6\times\sqrt[3]{8}\times\sqrt[3]{2}-3\times\sqrt[3]{8\times\frac{1}{4}}$

$=3\times\sqrt[3]{2}+6\times 2\times\sqrt[3]{2}-3\times\sqrt[3]{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

Another solution :

$\because \sqrt[3]{\frac{1}{4}}=\sqrt[3]{\frac{1}{4}\times\frac{16}{16}}=\sqrt[3]{\frac{16}{64}}=\frac{\sqrt[3]{16}}{\sqrt[3]{64}}=\frac{1}{4}\sqrt[3]{16}$

$=\frac{1}{4}\sqrt[3]{8\times 2}=\frac{1}{4}\times 2\sqrt[3]{2}=\frac{1}{2}\sqrt[3]{2}$

$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=3\sqrt[3]{2}+6\times 2\sqrt[3]{2}-6\times\frac{1}{2}\sqrt[3]{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

One more solution :

$\because \sqrt[3]{\frac{1}{4}}=\sqrt[3]{\frac{1}{4}\times\frac{2}{2}}=\sqrt[3]{\frac{2}{8}}=\frac{\sqrt[3]{2}}{\sqrt[3]{8}}=\frac{\sqrt[3]{2}}{2}$

$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=3\sqrt[3]{2}+12\sqrt[3]{2}-6\times\frac{\sqrt[3]{2}}{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

3 $\sqrt[3]{81}+\sqrt{12}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{27\times 3}+\sqrt{4\times 3}-2\sqrt[3]{3}-2\sqrt{3}$

$=\sqrt[3]{27}\times\sqrt[3]{3}+\sqrt{4}\times\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}$

$=3\sqrt[3]{3}+2\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{3}$

Example 3 Find in the simplest form : $2\sqrt[3]{4}\left(5\sqrt[3]{\frac{1}{2}}-\sqrt[3]{32}\right)$

Solution

$$2\sqrt[3]{4}\left(5\sqrt[3]{\frac{1}{2}}-\sqrt[3]{32}\right)=2\times 5\sqrt[3]{4\times\frac{1}{2}}-2\times\sqrt[3]{4\times 32}$$

$$=10\sqrt[3]{2}-2\times\sqrt[3]{128}=10\sqrt[3]{2}-2\times\sqrt[3]{64\times 2}$$

$$=10\sqrt[3]{2}-2\times 4\sqrt[3]{2}=10\sqrt[3]{2}-8\sqrt[3]{2}=2\sqrt[3]{2}$$

Example 4 If $x=\sqrt[3]{5}+2$ and $y=\sqrt[3]{5}-2$, find the value of $(x+y)^3-(x-y)^3$

Solution

$\because x+y=\sqrt[3]{5}+2+\sqrt[3]{5}-2=2\sqrt[3]{5}$

, $x-y=\sqrt[3]{5}+2-\left(\sqrt[3]{5}-2\right)=\sqrt[3]{5}+2-\sqrt[3]{5}+2=4$

$\therefore (x+y)^3-(x-y)^3=\left(2\sqrt[3]{5}\right)^3-(4)^3=2^3\times\left(\sqrt[3]{5}\right)^3-4^3$

$$=8\times 5-64=40-64=-24$$

TRY by yourself

Simplify each of the following to the simplest form :

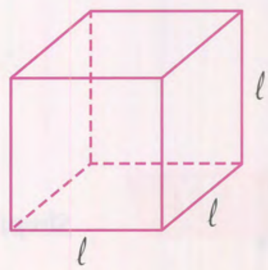
1. $5\sqrt[3]{2}-\sqrt[3]{16}+\sqrt[3]{-54}$
2. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

Answers of try by yourself

1. 0
2. $\frac{4}{3}\sqrt[3]{9}$

LESSON 9 Applications on the real numbers

The cube



It is a solid whose six faces are congruent squares.

i.e. All its edges are equal in length.

Assuming that the edge length of the cube = ℓ length unit , then :

1. The area of each face = ℓ^2 square unit.
2. Its lateral area = $4\ \ell^2$ square unit.
3. Its total area (the area of its 6 faces) = $6\ \ell^2$ square unit.
4. Its volume = ℓ^3 cube unit.

Example 1 **Choose the correct answer from those given :**

1. A cube of volume 64 cm^3. , then the sum of its edge lengths is ....
   (a) 16 cm. (b) 32 cm. (c) 48 cm. (d) 64 cm.

2. A cube of volume 125 cm^3. , then its total area = ...
   (a) 200 cm^2. (b) 150 cm^2. (c) 125 cm^2. (d) 25 cm^2.

3. A cube of volume 216 cm^3. , then its lateral area = ...
   (a) 36 cm^2. (b) 72 cm^2. (c) 144 cm^2. (d) 216 cm^2.

4 The lateral area of a cube is 4 cm^2, then its volume = ...

(a) 1 cm^3 (b) 2 cm^3 (c) 4 cm^3 (d) 16 cm^3

5 The total area of a cube is 294 cm^2, then its lateral area = ...

(a) 28 cm^2 (b) 49 cm^2 (c) 196 cm^2 (d) 343 cm^2

Solution

1 **(c) The reason :** $\because$ The volume of the cube $= \ell^3$ where ℓ is its edge length

$\therefore \ell^3 = 64$ $\therefore \ell = \sqrt[3]{64} = 4$ cm.

$\therefore$ The sum of the edge lengths $= 12\,\ell = 12 \times 4 = 48$ cm.

2 **(b) The reason :** $\because$ The volume of the cube $= \ell^3$ where ℓ is its edge length

$\therefore \ell^3 = 125$ $\therefore \ell = \sqrt[3]{125} = 5$ cm.

$\therefore$ The total area of the cube $= 6\,\ell^2 = 6 \times 5^2 = 150$ cm^2.

3 **(c) The reason :** $\because$ The volume of the cube $= \ell^3$ where ℓ is its edge length

$\therefore \ell^3 = 216$ $\therefore \ell = \sqrt[3]{216} = 6$ cm.

$\therefore$ The lateral area of the cube $= 4\,\ell^2 = 4 \times 6^2 = 144$ cm^2.

4 **(a) The reason :** $\because$ The lateral area of the cube $= 4\,\ell^2$ where ℓ is its edge length

$\therefore 4\,\ell^2 = 4$ $\therefore \ell^2 = 1$ $\therefore \ell = \sqrt{1} = 1$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 1^3 = 1$ cm^3.

5 **(c) The reason :** $\because$ The total area of the cube $= 6\,\ell^2$ where ℓ is its edge length

$\therefore 6\,\ell^2 = 294$ $\therefore \ell^2 = \frac{294}{6} = 49$

$\therefore$ The lateral area $= 4\,\ell^2 = 4 \times 49 = 196$ cm^2.

TRY by yourself 1

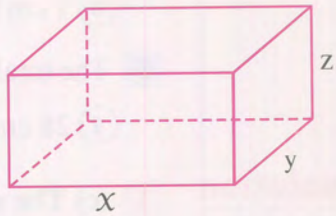
Complete the following table :

	Edge length of the cube	Area of one face	Lateral area	Total area	Volume
1	3 cm.	....	....	....	....
2	....	49 cm^2	....	....	....
3	....	....	144 cm^2	....	....
4	....	....	....	150 cm^2	....
5	....	....	....	....	64 cm^3

The cuboid

It is a solid that contains 6 faces , each of them is a rectangle and each two opposite faces are congruent.

Assuming that the lengths of the edges of the cuboid are x , y and z length unit , then :



1 Its lateral area = the perimeter of the base × height = $2(x + y) \times z$ square unit.

2 Its total area (the area of its six faces) = the lateral area + twice the area of the base

$$= 2(x + y) \times z + 2xy$$
$$= 2(xy + yz + zx) \text{ square unit.}$$

WATCH VIDEO

3 Its volume = the area of the base × the height

$$= x \times y \times z \text{ cube unit.}$$

Remarks

- The cuboid may contain two opposite faces , each of them is a square.
- The cube is a special case of the cuboid.

  i.e. The cube is a cuboid with edges having the same length.

Example 2 **The height of a cuboid is 4 cm. and its base is a square of side length 5 cm. Find :**

1 Its volume. 2 Its lateral area. 3 Its total area.

Solution

1 The volume of the cuboid = the area of the base × the height

$= 5 \times 5 \times 4 = 100 \text{ cm}^3.$

2 The lateral area of the cuboid = the perimeter of the base × the height

$= 4 \times 5 \times 4 = 80 \text{ cm}^2.$

3 The total area of the cuboid

= the lateral area + twice the area of the base = $80 + 2 \times 25 = 130 \text{ cm}^2.$

TRY *by yourself* **2**

The dimensions of a cuboid are 3 cm. , 4 cm. and 5 cm. Calculate its volume and its total area.

The circle

If M is a circle with radius length r , then :

1 The circumference of the circle = $2\pi r$ length unit.

2 The area of the circle = πr^2 square unit.

Example 3 **The area of a circle is 25π cm². Calculate its circumference in terms of π**

Solution

$\because$ The area of the circle = πr^2 $\qquad$ $\therefore \cancel{\pi} r^2 = 25 \cancel{\pi}$

$\therefore r^2 = 25$ $\qquad$ $\therefore r = \sqrt{25} = 5$ cm.

$\therefore$ The circumference of the circle = $2\pi r$

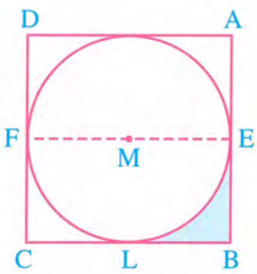
$= 2 \times 5 \times \pi = 10\pi$ cm.

Example 4 **In the opposite figure :**

A circle M is drawn inside a square (touching its sides).
If the area of the square = 196 cm², find :

1 The area of the shaded part.

2 The perimeter of the shaded part.



Solution

$\because$ The area of the square = 196 cm².

$\therefore$ The side length of the square = $\sqrt{196} = 14$ cm.

, $\because$ the side length of the square = 2 r

$\therefore 14 = 2r$ $\qquad$ $\therefore r = 7$ cm.

1 The area of the shaded part

= (the area of the square – the area of the circle) ÷ 4

$= \left(196 - \frac{22}{\cancel{7}} \times \cancel{7} \times 7\right) \div 4 = 42 \div 4 = 10.5$ cm².

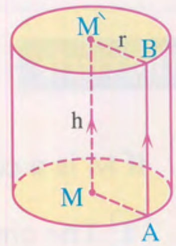
2 The perimeter of the shaded part

$= BE + BL + \frac{1}{4}$ circumference of the circle $= 7 + 7 + \left(\frac{1}{\cancel{4}_2} \times \cancel{2} \times \frac{22}{\cancel{7}} \times \cancel{7}\right)$

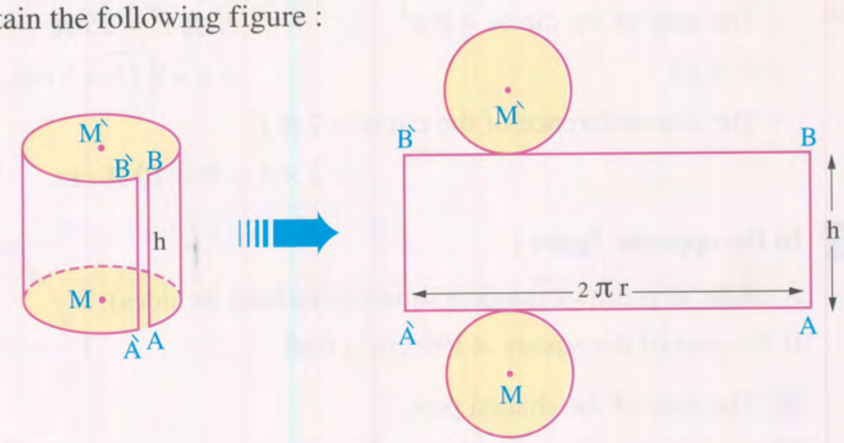
$= 14 + 11 = 25$ cm.

TRY by yourself 3 The circumference of a circle is 88 cm. Find its area. $\left(\pi = \frac{22}{7}\right)$

The right circular cylinder



- It is a solid having two parallel congruent bases , each of them is a circular-shaped surface while its lateral surface is a curved surface which is called cylindrical surface.
- The line segment $\overline{MM̀}$ drawn between the two centres of the two bases is perpendicular to each plane of the two bases and it is called the height of the cylinder.
- If we draw $\overline{AB}$ on the cylindrical surface such that A ∈ the circle M , B ∈ the circle M̀ , $\overline{AB}$ // $\overline{MM̀}$ and if we cut the lateral surface of the cylinder at $\overline{AB}$ and flattened it out, then we will obtain the following figure :



This figure consists of the surface of the rectangle ABB̀À and it is the same cylindrical surface of the cylinder in addition to the two surfaces of two circles which represent the two bases of the cylinder , then we find :

AB = the height of the cylinder.

AÀ = the circumference of the base of the cylinder.

∴ The lateral area of the cylinder = the area of the rectangle ABB̀À = AÀ × AB

= the circumference of the base of the cylinder × its height

and if we assume that the length of the radius of the base = r and its height = h , then :

1 The lateral area of the cylinder = $2 \pi r h$ square unit.

2 The total area of the cylinder = the lateral area of the cylinder + twice the area of the base = $2 \pi r h + 2 \pi r^2$ square unit.

WATCH VIDEO

3 The volume of the cylinder = the area of the base × height = $\pi r^2 h$ cube unit.

Example 5 **A right circular cylinder is of height 10 cm. and its volume is 1540 cm^3. Find its total area $\left(\pi = \frac{22}{7}\right)$**

Solution

$\because$ The volume of the cylinder = $\pi r^2 h$

$\therefore 1540 = \frac{22}{7} \times r^2 \times 10$

$\therefore 1540 = \frac{220}{7} r^2$

$\therefore r^2 = 1540 \times \frac{7}{220} = 49$

$\therefore r = \sqrt{49} = 7$ cm.

$\therefore$ The total area of the cylinder $= 2 \pi r h + 2 \pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 10 + 2 \times \frac{22}{7} \times 7^2$$

$$= 440 + 308 = 748 \text{ cm}^2.$$

TRY by yourself 4

A right circular cylinder is of volume 90π cm^3. and its height is 10 cm. Find the diameter length of its base.

The sphere

- It is a solid with a curved surface whose all points are equidistant from a fixed point inside the sphere.
- The equal distances are called the radius length of the sphere.
- The fixed point is called the centre of the sphere.
- If we cut the sphere by a plane passing through its centre , then the resulted section is a circle having the same centre of the sphere and its radius length is the same of the sphere. Assuming that the radius length of the sphere = r , then :

1. The area of the sphere = $4 \pi r^2$ square unit.
2. The volume of the sphere = $\frac{4}{3} \pi r^3$ cube unit.

WATCH VIDEO

Example 6 **The volume of a sphere = $\frac{500}{3} \pi$ cm³. Find the length of its diameter.**

Solution

$\because$ The volume of the sphere = $\frac{4}{3} \pi r^3$ $\qquad \therefore \frac{500}{3} \cancel{\pi} = \frac{4}{3} \cancel{\pi} r^3$

$\therefore r^3 = \frac{500}{3} \times \frac{3}{4} = 125$ $\qquad \therefore r = \sqrt[3]{125} = 5$ cm.

$\therefore$ The diameter length of the sphere = $2 \times 5 = 10$ cm.

Example 7 **A right circular cylinder is of height 6 cm. and its volume = $\frac{2}{3}$ the volume of a sphere whose radius length is 3 cm.**
Find the radius length of the base of the cylinder.

Solution Let the radius length of the sphere be r_1 cm. and the radius length of the base of the cylinder be r_2 cm.

$\because$ The volume of the sphere = $\frac{4}{3} \pi r_1^3 = \frac{4}{3} \pi (3)^3 = 36 \pi$ cm³.

$\because$ The volume of the cylinder = $\frac{2}{3}$ the volume of the sphere.

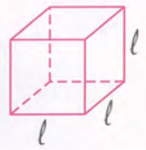
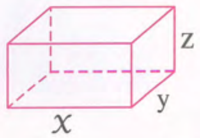
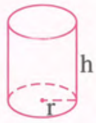
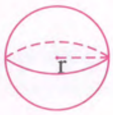
$\therefore \cancel{\pi} r_2^2 h = \frac{2}{3} \times 36 \cancel{\pi}$ $\qquad \therefore r_2^2 \times 6 = 24$

$\therefore r_2^2 = 4$ $\qquad \therefore r_2 = \sqrt{4} = 2$ cm.

$\therefore$ The radius length of the base of the cylinder = 2 cm.

TRY by yourself 5 The area of a sphere is 36π cm². Find its volume in terms of π

In the following, we will summarize the previous rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4\,\ell^2$	$6\,\ell^2$	ℓ^3
The cuboid		$2\,(x+y)\times z$	$2\,(x\,y+y\,z+z\,x)$	$x\,y\,z$
The cylinder		$2\,\pi\,r\,h$	$2\,\pi\,r\,h+2\,\pi\,r^2$ $=2\,\pi\,r\,(h+r)$	$\pi\,r^2\,h$
The sphere		–	$4\,\pi\,r^2$	$\frac{4}{3}\,\pi\,r^3$

Answers *of try by yourself*

1 (1) 9 cm², 36 cm², 54 cm², 27 cm³.

(2) 7 cm., 196 cm², 294 cm², 343 cm³.

(3) 6 cm., 36 cm², 216 cm², 216 cm³.

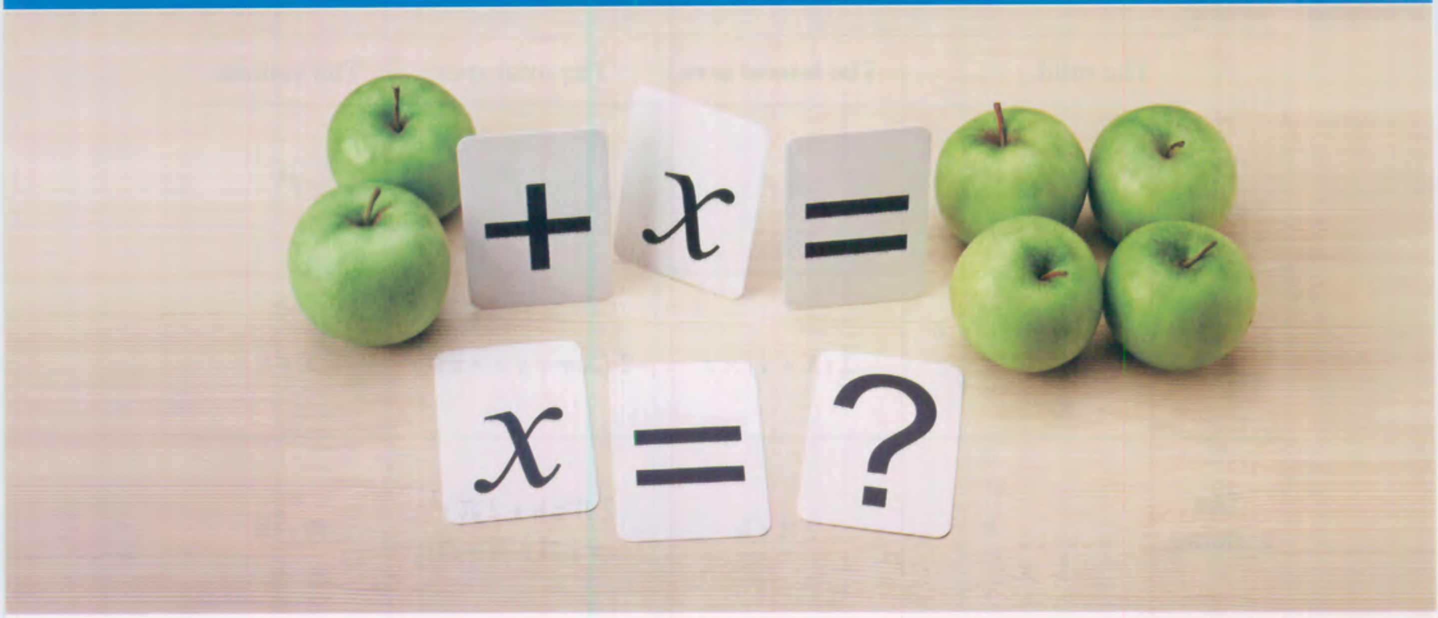
(4) 5 cm., 25 cm², 100 cm², 125 cm³.

(5) 4 cm., 16 cm², 64 cm², 96 cm².

2 The volume = 60 cm³, the total area = 94 cm².

3 616 cm². **4** 6 cm. **5** 36 π cm³.

LESSON 10 Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$



First Solving equations of the first degree in one unknown in $\mathbb{R}$

* Each of the equations :
  * $2x - 5 = 3$
  * $\sqrt{3}x - 1 = 8$
  * $\frac{1}{2}x - \sqrt{5} = 0$

is called an equation of the first degree in one variable (one unknown) which is x because the exponent of the variable x equals one.

* Solving the equation of the first degree in one variable means finding the real number which satisfies this equation.
* The following examples will show how to solve an equation of the first degree in one variable :

Example 1 **Find in $\mathbb{R}$ the S.S. of each of the following equations , then represent the solution on the number line :**

1. $3x + 2 = 1$
2. $\sqrt{3}x - 1 = 2$
3. $7x - \sqrt{7} = 6\sqrt{7}$
4. $x - \sqrt{5} = 1$

Solution

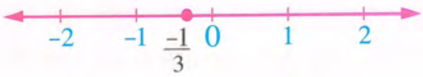
1 $\because 3x + 2 = 1$ (adding -2 to both sides)

$\therefore 3x + 2 - 2 = 1 - 2 \qquad \therefore 3x = -1$

(multiplying both sides by $\frac{1}{3}$ the multiplicative inverse of the coefficient of x)

$\therefore 3x \times \frac{1}{3} = -1 \times \frac{1}{3} \qquad \therefore x = -\frac{1}{3} \quad \therefore$ The S.S. $= \{-\frac{1}{3}\}$

- We can represent the number $-\frac{1}{3}$ on the number line as follows :



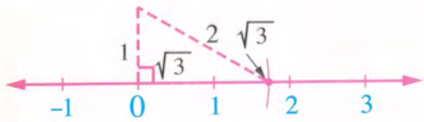
2 $\because \sqrt{3}\,x - 1 = 2 \qquad \therefore \sqrt{3}\,x = 2 + 1$

$\therefore \sqrt{3}\,x = 3 \qquad \therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\therefore x = \frac{3\sqrt{3}}{3} \qquad \therefore x = \sqrt{3}$

$\therefore$ The S.S. $= \{\sqrt{3}\}$

- We can represent the number $\sqrt{3}$ on the number line as follows :



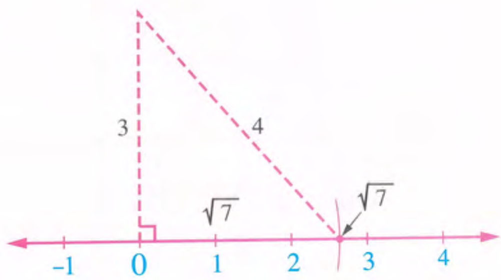
3 $\because 7x - \sqrt{7} = 6\sqrt{7} \qquad \therefore 7x = 6\sqrt{7} + \sqrt{7}$

$\therefore 7x = 7\sqrt{7} \qquad \therefore x = \frac{7\sqrt{7}}{7}$

$\therefore x = \sqrt{7}$

$\therefore$ The S.S. $= \{\sqrt{7}\}$

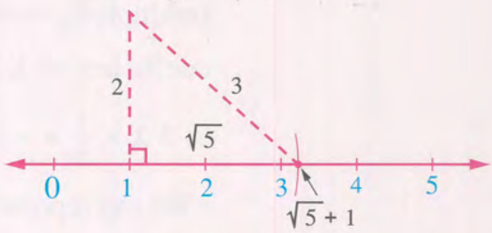
- We can represent the number $\sqrt{7}$ on the number line as follows :



4 $\because x - \sqrt{5} = 1$ $\therefore x = 1 + \sqrt{5}$

$\therefore$ The S.S. $= \{1 + \sqrt{5}\}$

- We can represent the number $\left(1 + \sqrt{5}\right)$ on the number line as follows :



TRY by yourself 1

Find in $\mathbb{R}$ the S.S. of each of the following equations , then represent the solution on the number line :

1 $2x + 5 = 4$ 2 $\sqrt{5}\,x - 1 = 4$ 3 $x - \sqrt{3} = 2$

Second Solving inequalities of the first degree in one unknown in $\mathbb{R}$

- Each of the inequalities :
  - $2x < 5$
  - $3x + 2 \le 1$
  - $5 + x > 2x - 1 \ge 3 + x$

  is called an inequality of the first degree in one unknown denoted by x

- Solving the inequality means finding all values of the unknown (x) which satisfy this inequality.

WATCH VIDEO

- The S.S. of the inequality in $\mathbb{R}$ will be written as an interval as will be shown later.

The methods of solving these inequalities in $\mathbb{R}$ depend on the properties of the inequality relation which will be summarized in the following :

Let a , b and c be three real numbers and assuming that $a < b$, then :

WATCH VIDEO

$a + c < b + c$ $\xrightarrow{\text{whether}}$ c is **positive** or **negative** (the addition property)

$ac < bc$ $\xrightarrow{\text{if}}$ c is **positive** (the property of multiplying by a positive real number)

$ac > bc$ $\xrightarrow{\text{if}}$ c is **negative** (the property of multiplying by a negative real number)

i.e. When we multiply (or divide) the two sides of an inequality by a negative number , we should change the symbol of the inequality.

Example 2 **Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line :**

WATCH VIDEO

1 $2x + 6 < 2$ **2** $5 - 4x \le -3$

Solution

1 $\because 2x + 6 < 2$ (adding the additive inverse of the number 6 (it is -6) to both sides)

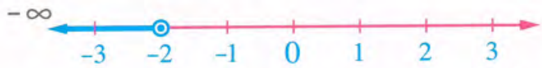
$\therefore 2x + 6 - 6 < 2 - 6$ $\therefore 2x < -4$

(multiplying both sides by the multiplicative inverse of the number 2 (it is $\frac{1}{2}$))

$\therefore 2x \times \frac{1}{2} < -4 \times \frac{1}{2}$ $\therefore x < -2$

$\therefore$ The S.S. is all the real numbers which are less than -2

i.e. The S.S. $=]-\infty , -2[$



2 $\because 5 - 4x \le -3$ (adding -5 to both sides)

$\therefore -4x \le -8$ (dividing both sides by -4) $\therefore x \ge 2$

(Notice the change in the symbol of the inequality because we divided by a negative number)

$\therefore$ The S.S. $= [2 , \infty[$

Example 3 **Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line :**

1 $-3 < 2x - 1 \le 5$ **2** $3 < 3 - 5x < 13$

Solution

1 $\because -3 < 2x - 1 \le 5$ (adding 1 to all sides)

$\therefore -2 < 2x \le 6$ (dividing all sides by 2)

$\therefore -1 < x \le 3$

$\therefore$ The S.S. $=]-1 , 3]$

2 $\because 3 < 3 - 5x < 13$ (subtracting 3 from all sides)

$\therefore 0 < -5x < 10$ (dividing all sides by -5)

$\therefore 0 > x > -2$

(Notice the change in the symbols of the inequality because we divided by a negative number).

$\therefore$ The S.S. $=]-2 , 0[$

Example 4 **Find in $\mathbb{R}$ the S.S. of each of the following inequalities :**

1 $X - 2 \ge 3X - 5$ **2** $X - 1 < 3X - 3 \le X + 5$

Solution

1 $\because X - 2 \ge 3X - 5$ (adding 2 to both sides)

$\therefore X \ge 3X - 3$ (adding $-3X$ to both sides)

$\therefore -2X \ge -3$ $\left(\text{multiplying both sides by } -\frac{1}{2}\right)$

$\therefore X \le \frac{3}{2}$ (Notice the change in the symbol of the inequality)

$\therefore$ The S.S. $=]-\infty , \frac{3}{2}]$

2 $\because X - 1 < 3X - 3 \le X + 5$ (adding 3 to all sides)

$\therefore X + 2 < 3X \le X + 8$ (adding $-X$ to all sides)

$\therefore 2 < 2X \le 8$ $\left(\text{multiplying by } \frac{1}{2}\right)$

$\therefore 1 < X \le 4$ $\therefore$ The S.S. $=]1 , 4]$

Another solution for number (2) :

We can divide this inequality into two inequalities as follows :

$X - 1 < 3X - 3 \longrightarrow (1)$ and $3X - 3 \le X + 5 \longrightarrow (2)$

Then the solution set of the origin inequality is the intersection set of the two sets of solutions of the two inequalities (1) and (2)

- **Finding the S.S. of the inequality (1) :**

$\because X - 1 < 3X - 3$ (adding 1 to both sides)

$\therefore X < 3X - 2$ (adding $-3X$ to both sides)

$\therefore -2X < -2$ $\left(\text{multiplying both sides by } -\frac{1}{2}\right)$

$\therefore X > 1$ $\therefore$ The S.S. $=]1 , \infty[$

- **Finding the S.S. of the inequality (2) :**

$\because 3x - 3 \le x + 5$ (adding 3 to both sides)

$\therefore 3x \le x + 8$ (adding $-x$ to both sides)

$\therefore 2x \le 8$ $\left(\text{multiplying both sides by } \frac{1}{2}\right)$

$\therefore x \le 4$ $\therefore$ The S.S. $=]-\infty , 4]$

- The S.S. of the origin inequality $=]1 , \infty[\cap]-\infty , 4] =]1 , 4]$

TRY by yourself 2

Find in $\mathbb{R}$ the S.S. of each of the following inequalities :

1. $3x - 1 > 8$
2. $2 - 2x \ge -6$
3. $-16 < 5x + 4 \le 9$
4. $2x + 1 > 4x - 3 > 2x - 11$

Answers *of try by yourself*

1 (1) $\{-\frac{1}{2}\}$ (2) $\{\sqrt{5}\}$ (3) $\{2 + \sqrt{3}\}$

2 (1) $]3 , \infty[$ (2) $]-\infty , 4]$ (3) $]-4 , 1]$ (4) $]-4 , 2[$

UNIT 2

Relation between Two Variables

Lessons of the unit :

1. Relation between two variables.
2. Slope of straight line.
3. Real life applications on the slope.

▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the relation between two variables of first degree.
- represent the relation between two variables of first degree graphically.
- recognize the slope of the straight line.
- find the slope of the straight line passing through two given points.
- recognize the slope of the straight line parallel to x-axis and the slope of the straight line parallel to y-axis.
- verify using the slope of the straight line that the three points are collinear or not.
- find the uniform velocity of a car by using the slope of the straight line.
- solve applications on the slope of the straight line.

LESSON **1**

Relation between two variables

The concept of the relation between two variables

- **Islam has 50 pounds. If Islam went to the amusement park , he would find two kinds of favourite games :**

The first kind

costs 5 pounds for playing one game.

The second kind

costs 10 pounds for playing one game.

- What are the different possibilities for playing the two kinds such that he spends all his money ?
- **To find all the possibilities :**
  - Assume that he will play x games of the first kind and y games of the second kind.
  - Then the cost of playing the first kind is $5x$ pounds and the cost of playing the second kind is $10y$ pounds.
  - In order to spend all his money , it should be : $5x + 10y = 50$
  - This is an algebraic relation between the two variables x and y and it is called an equation of the first degree in two variables.

$$5x + 10y = 50 \quad \div 5$$
$$x + 2y = 10$$
$$2y = 10 - x \quad \div 2$$
$$y = \frac{10 - x}{2}$$

- We can simplify the previous relation by dividing all terms by 5 to get an equivalent equation which is : $x + 2y = 10$

It can be written also in the form : $2y = 10 - x$

i.e. $y = \frac{10 - x}{2}$

For example:

- If Islam decided that he will not play the first kind.

  i.e. $x = 0$, then $y = \frac{10 - 0}{2} = 5$

  i.e. He can spend all his money by playing 5 games of the second kind.

  We express that by the ordered pair (0 , 5)

- If he decided to play one game of the first kind.

  i.e. $x = 1$, then $y = \frac{10 - 1}{2} = 4\frac{1}{2}$

  but in this case , he cannot play $4\frac{1}{2}$ games of the second kind because the number of games must be a natural number.

- If he decided to play two games of the first kind

  i.e. $x = 2$, then $y = \frac{10 - 2}{2} = 4$

  i.e. He can spend all his money by playing 2 games of the first kind and 4 games of the second kind . We express that by the ordered pair (2 , 4)

Thus we can know the different possibilities and put them in a table such as the following :

Number of games of the 1st kind (x)	0	2	4	6	8	10
Number of games of the 2nd kind (y)	5	4	3	2	1	0

! Remarks

- There is an infinite number of ordered pairs which satisfy the previous relation but some of them can't represent the possible numbers of each games because the number of games must be a natural number.
  - As we mentioned before $\left(1 , 4\frac{1}{2}\right)$ satisfies the relation but it is not possible to represent the number of games because $4\frac{1}{2} \notin \mathbb{N}$
  - Similarly (– 2 , 6) satisfies the relation but it is not to be used because $-2 \notin \mathbb{N}$
- To find all the possibilities , we write the equation : $x + 2y = 10$ putting y in one hand side as : $y = \frac{10 - x}{2}$

  We can also put x in one hand side as : $x = 10 - 2y$

  And the following example shows that.

Example 1 **What are the different possibilities for a person to pay L.E. 45 using two kinds of bills (banknotes) of L.E. 5 and L.E. 10 ?**

Solution Let the number of bills of L.E. 5 be X , then its value = 5 X pounds and the number of bills of L.E. 10 be y , then its value = 10 y pounds.

$\therefore 5X + 10y = 45$, dividing the two sides by 5

$\therefore X + 2y = 9$

Putting X in one hand side , then the equation will be in the form :

$X = 9 - 2y$

The following table shows all possibilities to pay the sum of money :

y	X	(X, y)	Number of bills of each kind
0	$9 - 2 \times 0 = 9$	(9 , 0)	9 bills of 5 pounds
1	$9 - 2 \times 1 = 7$	(7 , 1)	7 bills of 5 pounds and 1 bill of 10 pounds
2	$9 - 2 \times 2 = 5$	(5 , 2)	5 bills of 5 pounds and 2 bills of 10 pounds
3	$9 - 2 \times 3 = 3$	(3 , 3)	3 bills of 5 pounds and 3 bills of 10 pounds
4	$9 - 2 \times 4 = 1$	(1 , 4)	1 bill of 5 pounds and 4 bills of 10 pounds

Notice that :

If y = 5 , then $X = -1 \notin \mathbb{N}$, then y = 5 is impossible.

TRY by yourself 1 Find the different possibilities for a person to pay L.E. 65 of bills (banknotes) of L.E. 5 and L.E. 20

The linear relation

- The linear relation is a relation of the first degree between two variables X and y , it is in the form

$aX + by = c$, where a , b and c are real numbers , a and b are not both equal to zero

- There is an infinite number of ordered pairs which satisfy this relation.

- If we represent it graphically , the graph will be a straight line therefore it is called a linear relation , this will be shown later when we study the graphic representation of the linear relation.

Example 2

Find three ordered pairs satisfying each of the following relations :

1. $3x + y = 5$
2. $3x - 2y = 6$
3. $2x = 3$
4. $y = -2$

Solution

We can find these ordered pairs by setting a value for x and substituting in the relation to get its corresponding value of y or we do the converse :

1 • Set $x = 0$

$\therefore 3 \times 0 + y = 5$ $\qquad \therefore y = 5$

$\therefore (0 , 5)$ satisfies the relation.

• Set $x = 1$

$\therefore 3 \times 1 + y = 5$ $\qquad \therefore y = 5 - 3 = 2$

$\therefore (1 , 2)$ satisfies the relation.

• Set $x = -2$

$\therefore 3 \times (-2) + y = 5$ $\qquad \therefore y = 5 + 6 = 11$

$\therefore (-2 , 11)$ satisfies the relation.

2 By substituting directly as we did in 1 we can get the ordered pairs but we will present another method of solution by putting one of the two variables in one hand side alone.

$\because 3x - 2y = 6$ $\qquad \therefore -2y = 6 - 3x$ (multiply by (-1))

$\therefore 2y = 3x - 6$ $\qquad \therefore y = \frac{3x - 6}{2}$

• Set $x = 0$ $\qquad \therefore y = \frac{3 \times 0 - 6}{2} = \frac{-6}{2} = -3$

$\therefore (0 , -3)$ satisfies the relation.

• Set $x = 1$ $\qquad \therefore y = \frac{3 \times 1 - 6}{2} = -\frac{3}{2} = -1\frac{1}{2}$

$\therefore \left(1 , -1\frac{1}{2}\right)$ satisfies the relation.

• Set $x = 2$ $\qquad \therefore y = \frac{3 \times 2 - 6}{2} = 0$

$\therefore (2 , 0)$ satisfies the relation.

3 $\because 2x = 3$ $\therefore x = \frac{3}{2}$ $\therefore x = 1\frac{1}{2}$

This relation will be satisfied for all ordered pairs (x , y) where $x = 1\frac{1}{2}$

whatever the value of y such as $\left(1\frac{1}{2} , 0\right)$, $\left(1\frac{1}{2} , 1\right)$ and $\left(1\frac{1}{2} , 2\right)$

4 $y = -2$

This relation will be satisfied for all ordered pairs (x , y) , where $y = -2$, whatever the value of x such as $(0 , -2)$, $(1 , -2)$ and $(2 , -2)$

Example 3 Choose the correct answer from those given :

1 Which of the following ordered pairs satisfies the relation $= 2x - y = 1$?

(a) $(0 , 1)$ (b) $(5 , 3)$ (c) $(3 , 5)$ (d) $(-2 , 5)$

2 If $(2 , -3)$ satisfies the relation : $2x - y = c$, then $c =$ ....

(a) -7 (b) -1 (c) 1 (d) 7

3 If $(-2 , 1)$ satisfies the relation : $3x + b y = 1$, then $b =$ ....

(a) -7 (b) -5 (c) c (d) 7

4 If $(k , 2k)$ satisfies the relation : $5x - y = 6$, then $k =$ ....

(a) -18 (b) -2 (c) 2 (d) 18

5 If $(k , -2)$ satisfies the relation : $5x + 4y = 7$, then $k =$ ....

(a) -3 (b) $-\frac{1}{5}$ (c) $\frac{1}{5}$ (d) 3

Solution

1 **(c) The reason :** By substituting each ordered pair in the given relation , we find that $(3 , 5)$ satisfies the relation as follows : putting : $x = 3$, $y = 5$

$\therefore 2x - y = 2(3) - 5 = 6 - 5 = 1$

$\therefore (3 , 5)$ satisfies the relation.

2 **(d) The reason :** $\because (2 , -3)$ satisfies the relation : $2x - y = c$

$\therefore 2(2) - (-3) = c$

$\therefore 4 + 3 = c$

$\therefore c = 7$

3 **(d) The reason :** $\because$ $(-2, 1)$ satisfies the relation : $3X + by = 1$

$\therefore 3(-2) + b \times 1 = 1$ $\therefore -6 + b = 1$

$\therefore b = 1 + 6$ $\therefore b = 7$

4 **(c) The reason :** $\because$ $(k, 2k)$ satisfies the relation : $5X - y = 6$

$\therefore 5k - 2k = 6$ $\therefore 3k = 6$

$\therefore k = 2$

5 **(d) The reason :** $\because$ $(k, -2)$ satisfies the relation : $5X + 4y = 7$

$\therefore 5k + 4(-2) = 7$ $\therefore 5k - 8 = 7$

$\therefore 5k = 15$ $\therefore k = 3$

TRY by yourself 2

1 Find four ordered pairs satisfying the relation : $3X + y = 2$

2 If $(3k, 2k)$ satisfies the relation : $X - 3y = 9$, find the value of k

The graphic representation of the linear relation

WATCH VIDEO

- We mentioned that linear relation between two variables X and y is usually written in the form : $aX + by = c$, where a, b and c are real numbers, a and b are not both equal to zero.
This linear relation is represented graphically by a straight line (that is why it is called linear).
- To graph a linear relation, you need to graph at least two ordered pairs satisfying this relation. You can add a third ordered pair to check that the three points lie on the same straight line which is the graphic representation of the relation.

Example 4 **Represent the relation :** $2X - y = 3$ graphically

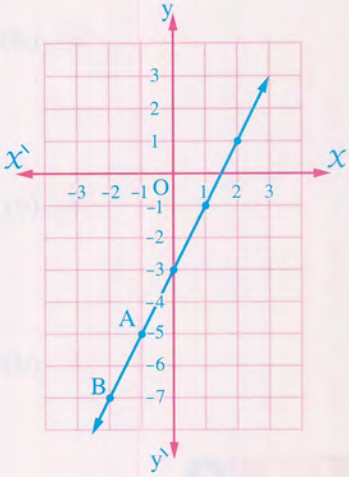
Solution To represent this relation graphically, we should determine three ordered pairs satisfying the relation : $2X - y = 3$, as follows :

- Set $X = 0$ $\therefore 2 \times 0 - y = 3$ $\therefore -y = 3$ $\therefore y = -3$
- Set $X = 1$ $\therefore 2 \times 1 - y = 3$ $\therefore -y = 1$ $\therefore y = -1$
- Set $X = 2$ $\therefore 2 \times 2 - y = 3$ $\therefore -y = -1$ $\therefore y = 1$

It is preferable to put the values of X and y in a table as the following :

X	0	1	2
y	-3	-1	1

Then we determine the points which represent these ordered pairs : (0 , – 3) , (1 , – 1) and (2 , 1) on orthogonal coordinates system , then we draw the straight line passing through these points , it will be the graphic representation of the relation : $2x - y = 3$



Remark

All the points of the straight line which represents the relation determine ordered pairs which satisfy the relation.

For example:

The point A determines the ordered pair (– 1 , – 5) which satisfies the relation when we put $x = -1$ we find that $2 \times (-1) - y = 3$ i.e. $y = -5$ and also the point B (– 2 , – 7)

TRY by yourself 3 ***Represent the relation :*** $y - 2x = -1$ graphically.

Special cases

We studied before the relation : a x + b y = c , where a , b are not both equal to zero and it is called a linear relation and it is represented graphically by a straight line and now we study the following cases :

Examples

1 If a = 0 , b ≠ 0

Then the relation becomes in the form :

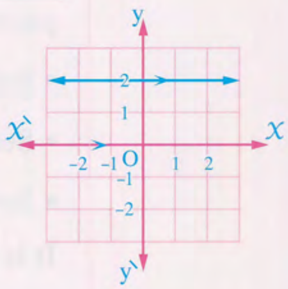
$$b\,y = c$$

and it is represented graphically by a straight line parallel to x-axis and intersects y-axis at the point $\left(0 , \frac{c}{b}\right)$

For example :

The relation : $2y = 4$

i.e. $y = 2$ is represented by a straight line parallel to x-axis and intersects y-axis at the point (0 , 2)



Notice that :

The relation : $y = 0$ is represented by x-axis

2 If b = 0 , a ≠ 0

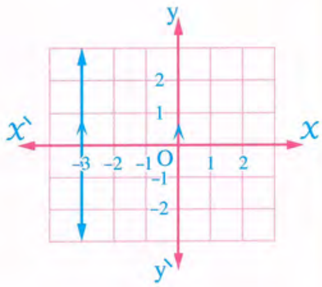
Then the relation becomes in the form :

$$a\,x = c$$

and it is represented graphically by a straight line parallel to y-axis and intersects x-axis at the point $\left(\frac{c}{a}, 0\right)$

For example :

The relation : $x = -3$ is represented by a straight line parallel to y-axis and intersects x-axis at the point $(-3, 0)$



Notice that :

The relation : $x = 0$ is represented by y-axis

3 If c = 0

Then the relation becomes :

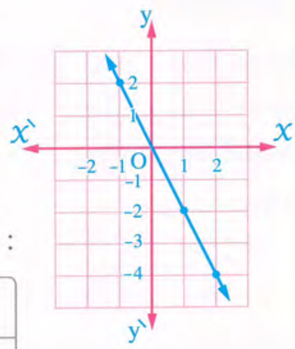
$$a\,x + b\,y = 0$$

and it is represented by a straight line passing through the origin point $(0, 0)$

For example :

The relation : $2\,x + y = 0$ is represented graphically by a straight line passing through the origin point as shown in the opposite graph :

x	1	− 1	2
y	− 2	2	− 4



Example 5 **Graph the straight line which represents the relation : $2\,x + 5\,y = 10$ and if this straight line intersects x-axis at the point A and y-axis at the point B , find the area of Δ OAB where O is the origin point.**

Solution

$\because 2\,x + 5\,y = 10$ $\therefore 2\,x = 10 - 5\,y$

$\therefore x = \dfrac{10 - 5\,y}{2}$

- Set $y = 0$ $\therefore x = \dfrac{10 - 5\,(0)}{2} = 5$

  $\therefore (5, 0)$ satisfies the relation.

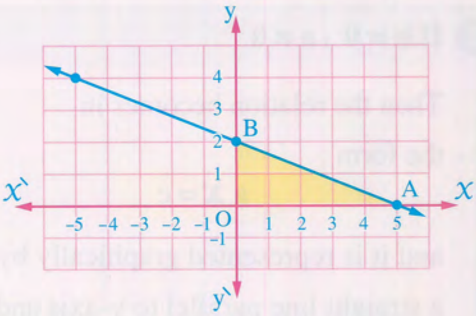
- Set $y = 2$ $\therefore x = \dfrac{10 - 5\,(2)}{2} = 0$

  $\therefore (0, 2)$ satisfies the relation.

- Set $y = 4$ $\therefore x = \dfrac{10 - 5\,(4)}{2} = -5$

  $\therefore (-5, 4)$ satisfies the relation

x	5	0	−5
y	0	2	4



$\because$ The straight line intersects x-axis at the point (5 , 0)

$\therefore$ OA = 5 length units.

, $\because$ the straight line intersects y-axis at the point (0 , 2)

$\therefore$ OB = 2 length units.

$\therefore$ The area of Δ OAB $= \frac{1}{2}$ OA $\times$ OB $= \frac{1}{2} \times 5 \times 2 = 5$ square units.

! Remark

In the previous example , we can get the points of intersection of the straight line representing the relation : $2x + 5y = 10$ and the coordinate axes without using the graph as the following :

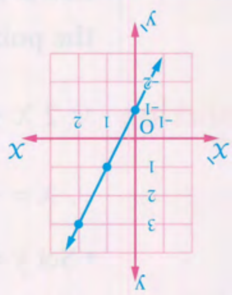
- Set $y = 0$ $\qquad \therefore 2x + 5 \times 0 = 10$
  $\therefore 2x = 10$ $\qquad \therefore x = 5$
  $\therefore$ The point of intersection with x-axis is (5 , 0)
- Set $x = 0$ $\qquad \therefore 2(0) + 5y = 10$
  $\therefore 5y = 10$ $\qquad \therefore y = 2$
  $\therefore$ The point of intersection with y-axis is (0 , 2)

Answers *of try by yourself*

1 13 bills of 5 pounds - 9 bills of 5 pounds and 1 bill of 20 pounds - 5 bills of 5 pounds and 2 bills of 20 pounds - 1 bill of 5 pounds and 3 bills of 20 pounds.

2 (1) (−1 , 5) , (0 , 2) , (1 , −1) , (2 , −4) "There are other solutions"
(2) −3

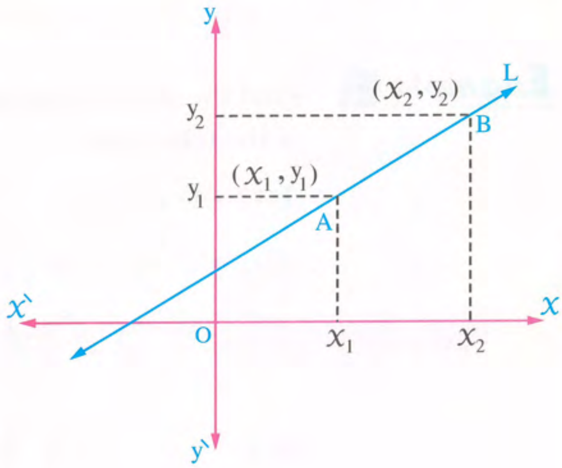
3



LESSON 2

Slope of straight line

If a point moves on a straight line L from the location A (x_1, y_1) to the location B (x_2, y_2), then :



– The change in the x-coordinates $= x_2 - x_1$

It is called (the horizontal change).

– The change in the y-coordinates $= y_2 - y_1$

It is called (the vertical change).

The ratio of the change in the y-coordinates to the change in the x-coordinates is called the slope of the straight line (S).

Definition

The slope of the straight line $= \dfrac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \dfrac{\text{the vertical change}}{\text{the horizontal change}}$

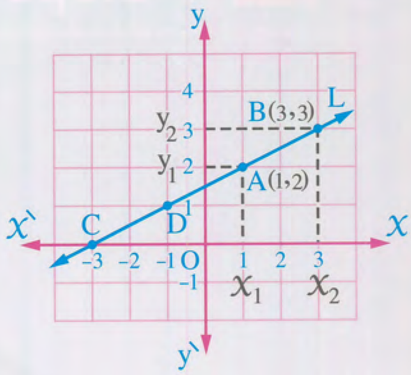
i.e. • $S = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

• S is undefined if $x_1 = x_2$

WATCH VIDEO

Example 1 **In the opposite figure :**

Find the slope of the straight line L



Solution We determine two points on the straight line such as A = (1 , 2) and B = (3 , 3)

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore S = \frac{3-2}{3-1} = \frac{1}{2}$

! Remark

In the previous example , notice that if we used another two points of the straight line to find its slope as the points C (– 3 , 0) and D (– 1 , 1) we find that :

$S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{-1-(-3)} = \frac{1}{2}$ (the same result)

i.e. The slope of the straight line is constant for any two selected points on it.

Example 2 **Find the slope of the straight line passing through each pair of points in the following :**

1 (2 , 4) , (4 , 5)

2 (1 , 3) , (4 , 2)

3 (– 2 , – 3) , (– 4 , 1)

4 (3 , 1) , (– 1 , 0)

Solution

1 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-4}{4-2} = \frac{1}{2}$

2 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{4-1} = \frac{-1}{3}$

3 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{-4-(-2)} = \frac{4}{-2} = -2$

4 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{-1-3} = \frac{-1}{-4} = \frac{1}{4}$

TRY by yourself 1

Find the slope of the straight line passing through each pair of points in the following :

1 (2 , 1) , (3 , 4)

2 (3 , – 5) , (– 4 , 2)

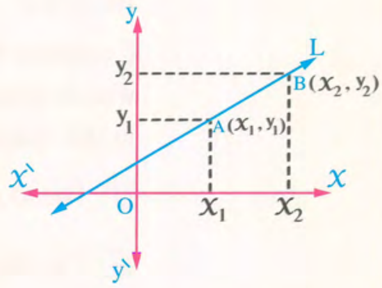
3 (– 3 , – 1) , (1 , 0)

4 (– 6 , 3) , (– 4 , 2)

! Remarks

- If a point moves on a straight line from the location A (x_1, y_1) to the location B (x_2, y_2), where $x_2 > x_1$, then

1 If $y_2 > y_1$

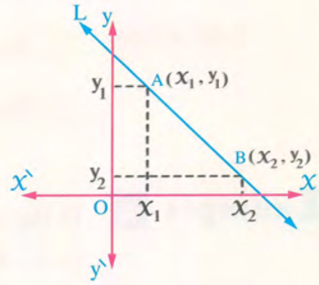


i.e. y increases as x increases, then the slope of the straight line is a positive number.

i.e. $S > 0$

---

2 If $y_2 < y_1$

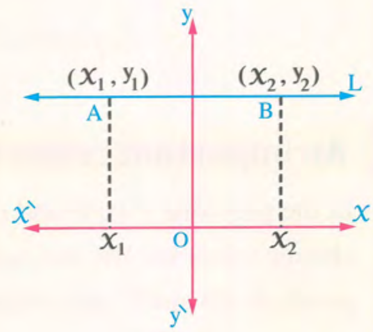


i.e. y decreases as x increases, then the slope of the straight line is a negative number.

i.e. $S < 0$

---

3 If $y_2 = y_1$



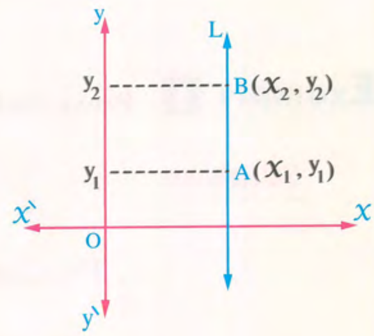
i.e. y is constant as x changes, then the slope of the straight line = zero

i.e. $S = 0$

i.e. The slope of the straight line parallel to x-axis = zero

---

4 If $x_2 = x_1$



, then the slope of the straight line is undefined because there is no change in the x-axis.

i.e. $x_2 - x_1 = 0$

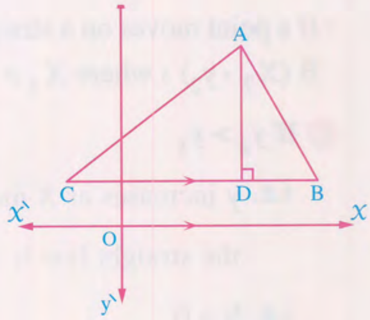
i.e. The slope of the straight line parallel to y-axis is undefined.

Example 3 **In the opposite figure :**

ABC is a triangle in which

$\overline{BC} \parallel \overleftrightarrow{XX'}$, $\overline{AD} \perp \overline{BC}$

Complete the following using one of the words (positive , negative , zero , undefined) in the spaces :



1 The slope of $\overleftrightarrow{AB}$ is ....

2 The slope of $\overleftrightarrow{BC}$ is ....

3 The slope of $\overleftrightarrow{AC}$ is ....

4 The slope of $\overleftrightarrow{AD}$ is ....

Solution

1 Negative

2 Zero

3 Positive

4 Undefined

Example 4 **If the slope of the straight line passing through the two points (– 3 , 4) and (1 , y) is 2 , find the value of y**

Solution $\because S = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\therefore 2 = \dfrac{y-4}{1-(-3)}$ $\therefore 2 = \dfrac{y-4}{4}$

$\therefore y - 4 = 2 \times 4$ $\therefore y - 4 = 8$ $\therefore y = 12$

! An important remark

In the previous , we found that the slope of the straight line is constant and it does not change whatever the two selected points on the line , therefore to prove that the three points A , B and C are collinear , then we find the slope of $\overleftrightarrow{AB}$ and the slope of $\overleftrightarrow{BC}$

If the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$, then A , B and C are collinear.

Example 5 **Prove that the points A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.**

Solution $\because S = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \dfrac{2-3}{4-2} = -\dfrac{1}{2}$, the slope of $\overleftrightarrow{BC} = \dfrac{0-2}{8-4} = \dfrac{-2}{4} = -\dfrac{1}{2}$

, $\because$ the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is common.

$\therefore$ The points A , B and C are collinear.

Example 6 **If the points A , B and C are collinear where A (3 , 2) , B (5 , – 1) and C (1 , k) , find the value of k**

Solution

$\because S = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \dfrac{-1-2}{5-3} = \dfrac{-3}{2}$

, the slope of $\overleftrightarrow{BC} = \dfrac{k-(-1)}{1-5} = \dfrac{k+1}{-4}$

, $\because$ A , B and C are collinear , the slope of the straight line is constant for any two points on it.

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \dfrac{-3}{2} = \dfrac{k+1}{-4}$

$\therefore 2(k+1) = -3 \times (-4)$

$\therefore 2k + 2 = 12$

$\therefore 2k = 10$ $\qquad \therefore k = 5$

TRY by yourself 2

1 If the slope of the straight line passing through the two points (3 , – 1) , (7 , a) is $\frac{3}{4}$, find the value of a

2 ***Prove that :*** C (– 1 , 2) $\in \overleftrightarrow{AB}$, where A (1 , 3) and B (3 , 4)

Answers *of try by yourself*

1 | 1 | 3 | 2 | – 1 | 3 | $\frac{1}{4}$ | 4 | $-\frac{1}{2}$

2 | 1 | 2 | 2 | Prove by yourself [**Hint :** Prove that the slope of $\overleftrightarrow{AC}$ = the slope of $\overleftrightarrow{AB}$].

LESSON 3

Real life applications on the slope

- We studied before that if there is a linear relation between two variables x and y , then :

The slope of the straight line which represents this relation $= \dfrac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}}$

i.e. The slope of the straight line (S) expresses the rate of change of y with respect to x

- In our life , there are many applications which we need to know the rate of change in dealing with them.

For example:

1 If the opposite graph represents the motion of a car , then :

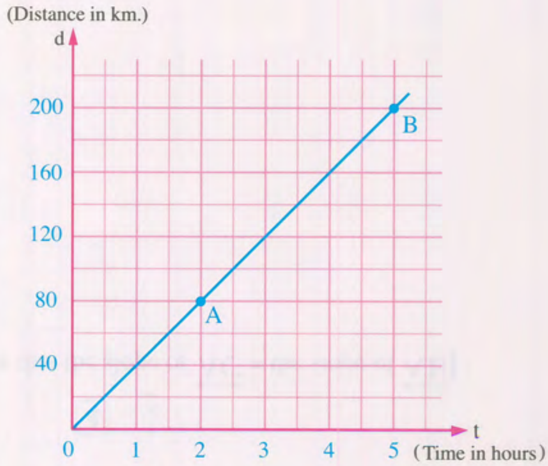
The uniform velocity of the car (v)

= the rate of change of the distance (d) with respect to the time (t)

i.e. The uniform velocity of the car (v)

= the slope of the straight line (S)

and by selecting two points on the straight line as A (2 , 80) and B (5 , 200)



$$\therefore\ v = \frac{d_2 - d_1}{t_2 - t_1} = \frac{200 - 80}{5 - 2} = \frac{120}{3} = 40 \text{ km./hr.}$$

2 If the opposite graph represents the change in the capital of a company (y) within the time (t) , then :

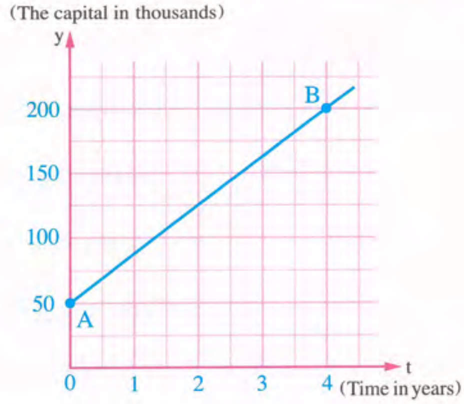
The rate of change in the capital of the company

= the slope of the straight line $\overleftrightarrow{AB}$

$\therefore$ The rate of change of the capital of the company

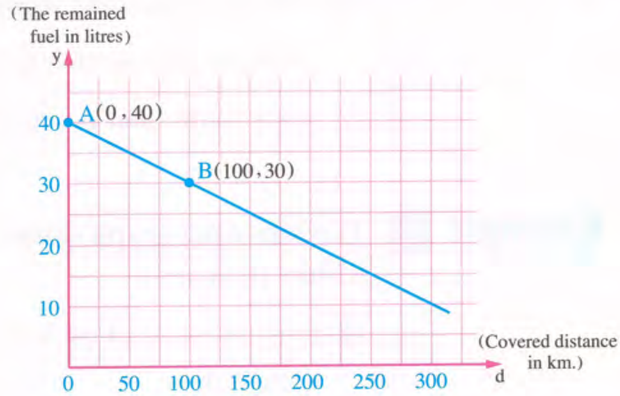
$$= \frac{y_2 - y_1}{t_2 - t_1} = \frac{200 - 50}{4 - 0}$$

$$= \frac{150}{4} = 37.5 \text{ thousand pounds / year.}$$



i.e. The capital of the company increases in the rate = $37.5 \times 1000 = 37500$ pounds/year.

3 A person filled the tank of his car whose capacity is 40 litres with fuel. After he covered a distance 100 km. , he found that the remained fuel in the tank = 30 litres. The opposite figure shows the relation between the covered distance in km. (d) and the amount of the remained fuel in the tank in litres (y) , then :



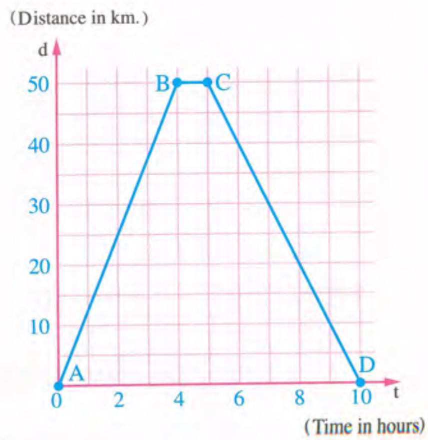
The rate of consumption of fuel = the slope of $\overleftrightarrow{AB}$

i.e. The rate of consumption of fuel $= \frac{y_2 - y_1}{d_2 - d_1} = \frac{30 - 40}{100 - 0} = \frac{-10}{100} = -\frac{1}{10}$ litre/km.

(The negative sign denotes the amount of fuel decreases in the rate of one litre for each 10 km.)

Example 1

Waleed rode his bicycle from Cairo to Benha , then he returned back to Cairo. The opposite graph represents the bicycle motion during going and returning back :



1 Find his velocity in going trip.

2 Find his velocity in returning back trip.

3 Find the average velocity during all trips.

4 What do you say about the horizontal line segment in the graph ?

Solution

1. Taking the two points A (0 , 0) and B (4 , 50)

$\therefore$ v (during going trip) $= \frac{50-0}{4-0} = 12.5$ km./hr.

2. Taking the two points C (5 , 50) and D (10 , 0)

$\therefore$ v (during returning back trip) $= \frac{0-50}{10-5} = \frac{-50}{5} = -10$ km./hr.

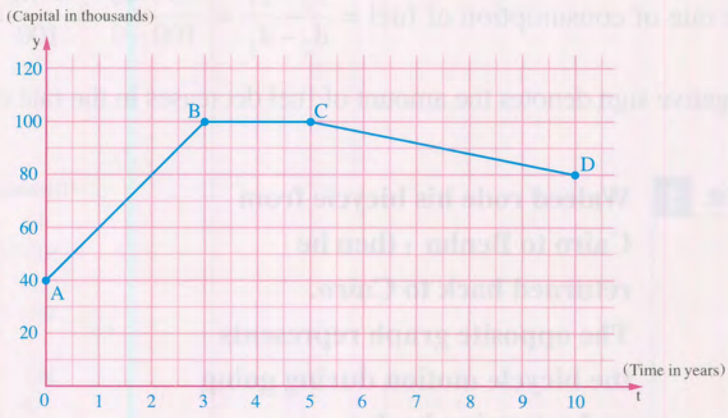
(The negative sign means that Waleed moved in the opposite direction of his first motion returning back to Cairo with velocity 10 km./hr.)

3. The average velocity $= \frac{\text{the total distance}}{\text{the total time}} = \frac{100}{10} = 10$ km./hr.

4. The horizontal line segment in the graph shows that Waleed stopped for an hour after he covered a distance equal to 50 km. , then he returned back to the start point.

Example 2 **The following graph shows the change of the capital of a company within 10 years :**

1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$ What is the meaning of each of them ?
2. Calculate the capital of the company at the beginning.



Solution $\because$ A (0 , 40) , B (3 , 100) , C (5 , 100) and D (10 , 80)

1. • The slope of $\overleftrightarrow{AB} = \frac{100-40}{3-0} = \frac{60}{3} = 20$

It expresses the increase in the capital of the company within the first three years from the beginning in the rate of 20000 pounds/year.

• The slope of $\overleftrightarrow{BC} = \frac{100 - 100}{5 - 3} = \frac{0}{2} = 0$

It expresses that the capital of the company is still constant without increasing or decreasing within the fourth and the fifth years from the beginning.

• The slope of $\overleftrightarrow{CD} = \frac{80 - 100}{10 - 5} = \frac{-20}{5} = -4$

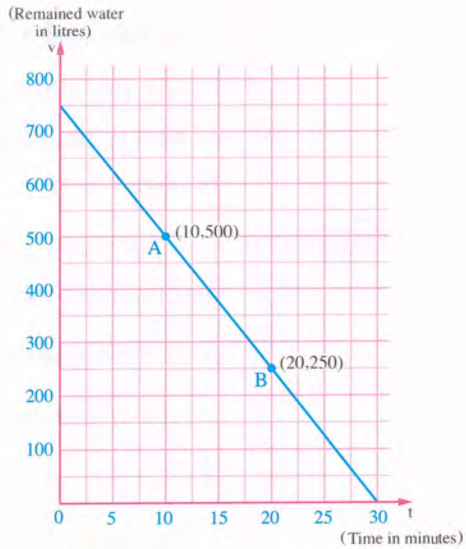
It expresses the decrease in the capital of the company within the last five years in the rate of 4000 pounds / year.

2 $\because$ A (0 , 40)

$\therefore$ The capital of the company in the beginning = 40000 pounds.

Example 3

A tank of water is filled with water completely.
A tap is opened below the tank to empty it. The opposite graph represents the relation between the time (t) in minutes and the amount of water remained in the tank (v) in litres :



1 What is the greatest capacity of the tank ?

2 What is the time needed to empty the tank ?

3 What is the amount remained in the tank after 20 minutes ?

4 What is the rate of emptying the tank ?

Solution

1 From the graph , we find that $\overleftrightarrow{AB}$ intersects the axis which represents the amount of remained water (v) at the point (0 , 750)

$\therefore$ The greatest capacity of the tank = 750 litres.

2 From the graph , we find that $\overleftrightarrow{AB}$ intersects the axis which represents the time (t) at the point (30 , 0)

$\therefore$ The needed time for emptying the tank is 30 minutes.

3 $\because$ The point $(20 , 250) \in \overleftrightarrow{AB}$

$\therefore$ After 20 minutes , the remained amount of water in the tank is 250 litres.

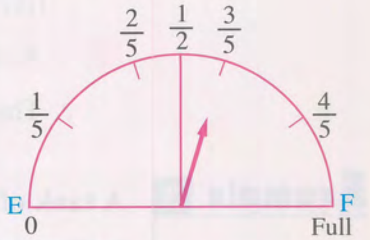
4 The rate of emptying the tank = the slope of $\overleftrightarrow{AB}$

$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{250 - 500}{20 - 10} = \frac{-250}{10} = -25$$

$\therefore$ The tank is emptied by the rate 25 litres/minute.

Example 4

Hossam filled the tank of his car with fuel given that its capacity is 50 litres. After Hossam covered a distance 200 km. , he noticed that fuel meter shows that the tank has fuel = $\frac{3}{5}$ its capacity.



Graph the relation between the distance covered by the car and the amount of fuel in the tank and calculate the distance covered by the car till the tank becomes empty.

Solution

Let the covered distance = d (km.)
and the remained amount of fuel = y (litres)

$\therefore$ In the beginning , the distance = 0 km.

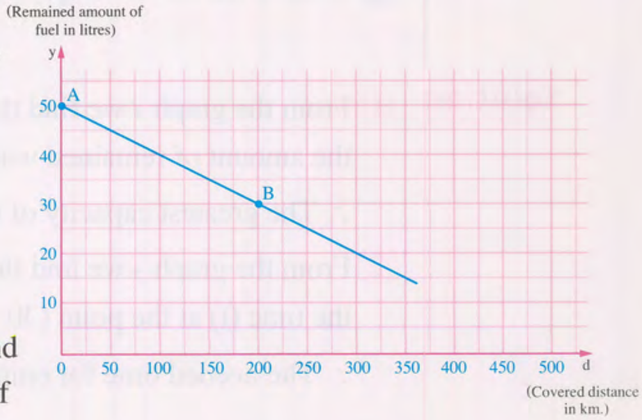
i.e. d = 0 and the amount of fuel in the tank = 50 litres.

i.e. y = 50

$\therefore$ The point A (0 , 50) expresses the amount of fuel in the tank in the beginning of motion.

$\because \frac{3}{5}$ the capacity of the tank $= \frac{3}{5} \times 50 = 30$ litres.

$\therefore$ The point B (200 , 30) expresses the amount of fuel in the tank after a covered distance 200 km. from the beginning.



$\therefore \overleftrightarrow{AB}$ represents the relation between the covered distance (d) and the remained amount of fuel in the tank (y)

$\therefore$ The rate of decrease of fuel = the slope of $\overleftrightarrow{AB}$

$$= \frac{y_2 - y_1}{d_2 - d_1} = \frac{30 - 50}{200 - 0} = \frac{-20}{200} = -\frac{1}{10} \text{ litre/km.}$$

i.e. The amount of fuel in the tank decreases with rate of one litre per 10 km.

$\therefore$ The covered distance from beginning the motion till the tank becomes empty

$$= \frac{\text{the amount of fuel in the beginning}}{\text{rate of decrease of fuel}} = \frac{50}{\frac{1}{10}} = 50 \times 10 = 500 \text{ km.}$$

! Remark

We can find the covered distance from the beginning till the tank becomes empty from the graph by finding the point of intersection of $\overleftrightarrow{AB}$ with the axis which represents the distance d which is (500 , 0)

i.e. The covered distance by the car when the tank becomes empty = 500 km.

TRY by yourself

The opposite graph represents the motion of a car measured from a fixed point A :

1. Determine the uniform velocity of the car.
2. Calculate the covered distance after two hours from the beginning of the motion.

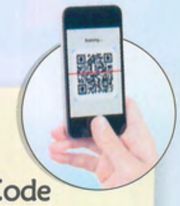
Answers of try by yourself

1. 40 km. / hour
2. 80 km.

UNIT 3 Statistics

Lessons of the unit :

1. Collecting and organizing data.
2. The ascending and descending cumulative frequency tables and their graphical representation.
3. Mean.
4. Median.
5. Mode.



▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- organize data in frequency tables with sets.
- form each of the ascending and descending cumulative frequency tables.
- graph each of the ascending and descending cumulative frequency tables.
- find the mean of a set of data organized in a frequency table with sets.
- find the median of a frequency distribution with sets.
- calculate the mode from a frequency table with sets.

LESSON 1

Collecting and organizing data

In the last year , you knew how to organize data and put them in a simple frequency table , but when summarizing large masses of data , it is useful to distribute them into sets , and determine the number of individuals belonging to each set.

The table consisting of sets and their corresponding frequencies is called "frequency table with sets"

The following example shows how to organize data into a frequency table with sets.

Example **In the following table , these are the marks of 54 students in one of the classes in grade two preparatory in a school , which they took in an exam in mathematics where the full mark is 60**

WATCH VIDEO

42	54	36	46	34	45	51	40	48
48	40	47	25	48	45	36	56	44
38	47	30	37.5	40	20	42	28	50
47	55	27	45	30	42	51	43	46
29	43	59	35	44.5	32	24	39	54
41	36	45	39	42	58	35	50	45

The required is forming the frequency table with sets.

Solution 1 Determine the range

(it is the difference between the greatest mark and the smallest mark)

$\because$ The smallest mark is 20 and the greatest mark is 59

$\therefore$ The range = 59 – 20 = 39

2 Divide these data into a suitable number of sets of marks , say 10 disjoint sets , the length of each of them is 4 , then you obtain the following sets :

- **The first set :**
  The students who obtain 20 marks till less than 24 marks , which is written as (20 –)

- **The second set :**
  The students who obtain 24 marks till less than 28 marks , it is written as (24 –)

- **The third set :**
  The students who obtain 28 marks till less than 32 marks , it is written as (28 –) and so on till you reach the tenth set.

- **The tenth set :**
  The students who obtain 56 marks till less than 60 , it is written as (56 –)

3 **Form the tally table as follows :**

Sets	Tallies	Frequency
20 –	/	1
24 –	///	3
28 –	////	4
32 –	////	4
36 –	~~////~~ //	7
40 –	~~////~~ ~~////~~	10
44 –	~~////~~ ~~////~~ //	12
48 –	~~////~~ //	7
52 –	///	3
56 –	///	3
Total		54

(The tally table)

4 Omit the tallies column from the table to get the final form of the frequency table with sets. It can be written vertically or horizontally.

The following is the horizontal form of the frequency table :

Sets	20-	24-	28-	32-	36-	40-	44-	48-	52-	56-	Total
Frequency	1	3	4	4	7	10	12	7	3	3	54

From the previous table , we deduce that :

- The set that has the greatest frequency is 44 –
- The set that has the least frequency is 20 –

TRY by yourself

The following is the weights of 50 persons :

52	35	40	57	43	40	36	49	43	58
47	48	51	30	59	36	45	41	44	37
42	54	38	55	42	47	46	34	53	44
47	32	41	62	50	39	58	46	43	49
40	41	64	44	54	45	38	40	48	41

Form the frequency table with sets.

Answers *of try by yourself*

Sets	30 –	35 –	40 –	45 –	50 –	55 –	60 –
Frequency	3	7	16	11	9	5	2

LESSON 2

The ascending and descending cumulative frequency tables and their graphical representation

Prelude

- In the previous lesson , you learnt how to form a frequency table with sets and how to get some information from it as the following table which represents the distribution of weekly wages of 50 workers in one factory :

Sets of wages	54 -	58 -	62 -	66 -	70 -	Total
No. of workers (Frequency)	5	12	22	7	4	50

From this table , you can know the number of workers (the frequency) in each set.

For example:

– The number of workers whose wages lie between 58 and less than 62 pounds is 12 workers.

– The number of workers whose wages lie between 66 and less than 70 pounds is 7 workers.

- **But some other information cannot be obtained directly from this table such as :**
  – The number of workers who obtain wages less than 62 pounds.
  – The number of workers who obtain wages equal to 58 pounds or more.

- **In order to be able to know such information , you need to study how to form another type of tables called cumulative frequency tables (ascending and descending) and this what will be shown in the following examples :**

Example 1

The following frequency table shows the weekly wages in pounds of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the ascending cumulative frequency table and represent it graphically , then find :

1. The number of workers whose weekly wages are less than 60 pounds.
2. The percentage of the number of workers whose weekly wages are less than 60 pounds.

Solution

- Form the ascending cumulative frequency table as follows :

Sets of wages	54 -	58 -	62 -	66 -	70 -
Number of workers (Frequency)	5	12	22	7	4

Less than $54 = 0$

Less than $58 = 5 + 0 = 5$

Less than $62 = 5 + 12 = 17$

Less than $66 = 5 + 12 + 22 = 39$

Less than $70 = 5 + 12 + 22 + 7 = 46$

Less than $74 = 5 + 12 + 22 + 7 + 4 = 50$

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

The ascending cumulative frequency table.

Notice that :

The ascending cumulative frequency begins with zero and ends at the total frequency.

To represent the ascending cumulative frequency table graphically , do as follows :

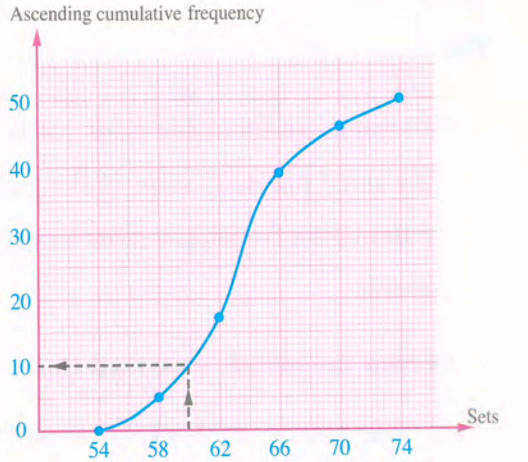
1. Specialize the horizontal axis for sets and the vertical axis for the ascending cumulative frequency.
2. Choose a suitable scale to represent data on the vertical axis so that it contains the ascending cumulative frequency easily.

3 Represent the ascending cumulative frequency of each set , then draw the graph (the curve) such that it passes through the points which we located as shown in the opposite figure.

- **From the graph , we find that :**

1 The number of workers whose weekly wages are less than 60 pounds = 10 workers.

2 The percentage of the number of workers whose weekly wages are less than 60 pounds $= \frac{10}{50} \times 100\%$
$= 20\%$



The ascending cumulative frequency curve

Example 2

The following frequency table shows the weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the descending cumulative frequency table and represent it graphically , then find :

1 The number of workers whose weekly wages are 60 pounds or more.

2 The percentage of the number of workers whose weekly wages are 60 pounds or more.

Solution

- Form the descending cumulative frequency table as follows :

Sets of wages	54 -	58 -	62 -	66 -	70 -
Number of workers (Frequency)	5	12	22	7	4

54 and more = 5 + 12 + 22 + 7 + 4 = 50 → 54 and more
58 and more = 12 + 22 + 7 + 4 = 45 → 58 and more
62 and more = 22 + 7 + 4 = 33 → 62 and more
66 and more = 7 + 4 = 11 → 66 and more
70 and more = 4 → 70 and more
74 and more = 0 → 74 and more

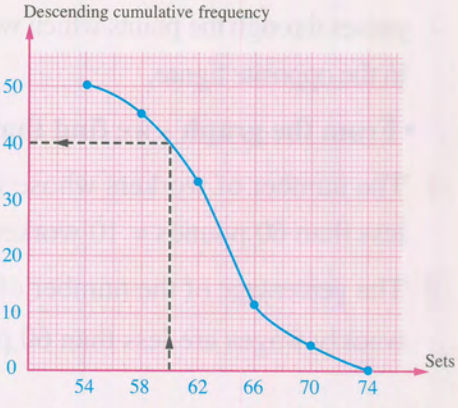
The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

The descending cumulative frequency table

Notice that :

The descending cumulative frequency begins with the total frequency and ends with zero.

- To represent this table graphically , follow the same previous steps in the ascending cumulative frequency table to get the opposite graph.



The descending cumulative frequency curve

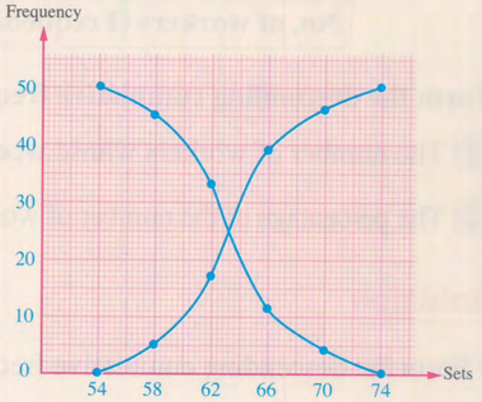
- **From the graph , we find that :**

1 The number of workers whose weekly wages are 60 pounds or more = 40 workers.

2 The percentage of those workers $= \frac{40}{50} \times 100\% = 80\%$

! Remark

You can graph the two curves of the ascending and descending cumulative frequency of a frequency distribution in one sketch as shown in the opposite graph.



TRY *by yourself*

The following table shows the frequency distribution of marks of 40 students in math exam :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	4	8	12	10	6	**40**

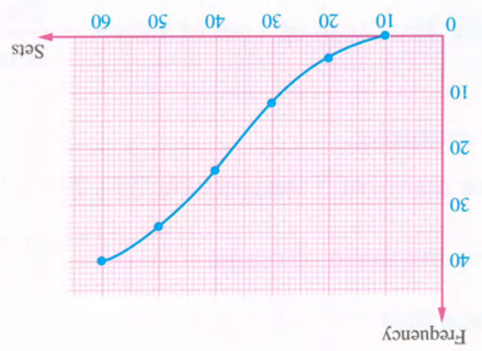
Graph each of :

1 The ascending cumulative frequency curve.

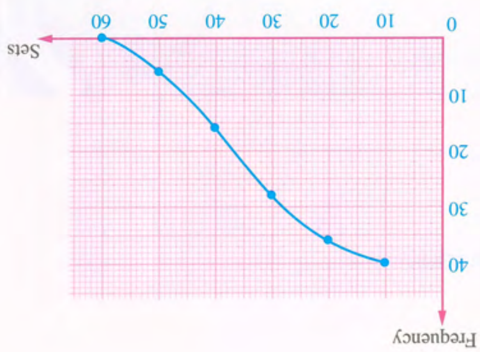
2 The descending cumulative frequency curve.

Answers *of try by yourself*

1



2



LESSON 3 Mean

You studied last year some of the measures of central tendency of a set of values which are the mean , the median and the mode.

Now you will study how you can find these three measures of a set of data organized in a frequency table with sets.

WATCH VIDEO

Remember that

To calculate the mean of a set of values , do as follows :

1 Find the sum of these values.

2 Divide this sum by the number of these values

i.e. The mean of a set of values $= \dfrac{\text{The sum of values}}{\text{Number of values}}$

For example:

If the marks of 5 students are 25 , 23 , 21 , 22 , 24

, then the mean of marks $= \dfrac{25 + 23 + 21 + 22 + 24}{5} = 23$ marks.

Notice that : $23 \times 5 = 115$

, the sum of marks of the 5 students $= 25 + 23 + 21 + 22 + 24 = 115$

i.e. The mean is the value which is given to each item of a set , then the sum of these new values is the same sum of the original values.

Finding the mean of data from the frequency table with sets

Example **The following table shows the distribution of the marks of 50 students in mathematics :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Solution

1 **Determine the centres of sets according to the rule :**

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

, then the centre of the first set $= \frac{10 + 20}{2} = 15$

, the centre of the second set $= \frac{20 + 30}{2} = 25$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \frac{50 + 60}{2} = 55$

2 **Form the vertical table :**

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

3 The mean $= \frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34$ marks.

TRY *by yourself*

The following table shows the daily wages in pounds of 50 workers in a factory :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Find the mean of the wage of the worker in pounds.

Answers of try by yourself

31 Pounds.

LESSON 4 Median

Remember that

The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it.

- To find the median of a set of values , do as follows :

Arrange the values ascendingly or descendingly

then

If the values number is odd

Then :

The median is the value lying in the middle exactly.

For example:

- **If the values are :**

42 , 23 , 17 , 30 , 20

- **We arrange them ascendingly as follows**

17 , 20 , $\boxed{23}$, 30 , 42

The median = 23

If the values number is even

Then :

$$\text{The median} = \frac{\text{The sum of the two values lying in the middle}}{2}$$

For example:

- **If the values are :**

27 , 13 , 23 , 24 , 13 , 21

- **We arrange them ascendingly as follows**

13 , 13 , $\boxed{21 , 23}$, 24 , 27

$$\text{The median} = \frac{21 + 23}{2} = 22$$

Finding the median of a frequency distribution with sets graphically

To find the median of a frequency distribution with sets graphically , do the following steps :

1. Form the ascending or the descending cumulative frequency table , then draw the cumulative frequency curve of it.
2. Find the order of the median = $\frac{\text{The total of frequency}}{2}$
3. Determine the point which represents the order of the median on the vertical axis, from this point , draw a horizontal straight line to intersect the curve at a point , then from this point , draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.

The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

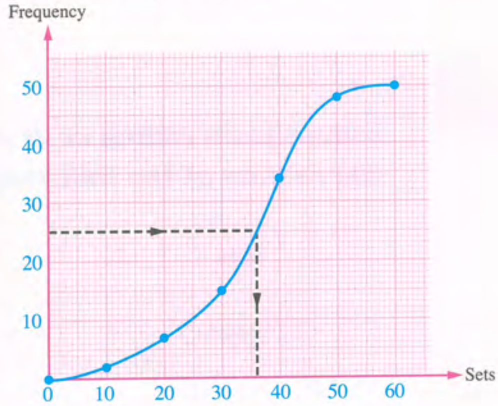
Example **The following table shows the frequency distribution of marks of 50 students in math exam :**

Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the students.

Solution * **First : Using the ascending cumulative frequency curve :**

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

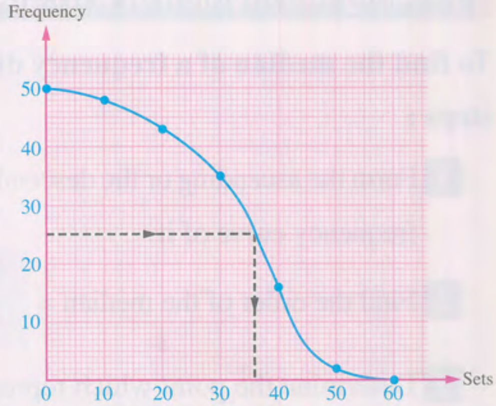


$\because$ The order of the median = $\frac{50}{2} = 25$

$\therefore$ From the graph , the median = 36 approximately

* Second : Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



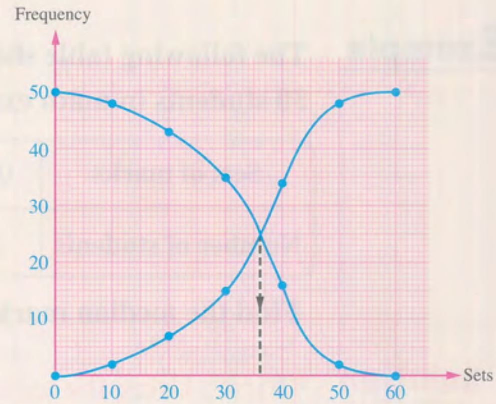
$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the graph , the median = 36 approximately

! Remark

You can find the median by more accurate method , this by drawing the two curves (the ascending and descending cumulative frequency curves) together in one graph to intersect at one point.

From this point , draw a vertical straight line to meet the horizontal axis at a point which represents the median as shown in the opposite graph to get the median = 36 approximately.



TRY *by yourself*

Using the ascending or descending cumulative frequency curve , find the median of the following frequency distribution :

Sets	4 –	8 –	12 –	16 –	20 –	Total
Frequency	2	4	8	6	4	24

Answers *of try by yourself*

15 approximately.

EL-MOASSER

By a group of supervisors

EXERCISES

2nd PREP.

FIRST TERM

Maths

GPS

AL TALABA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
e-mail: info@elmoasserbooks.com
15014 www.elmoasserbooks.com

Contents

A research project on each unit

First Algebra and Statistics

UNIT 1 Real Numbers

Exercises of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers $\grave{\mathbb{Q}}$
3. The set of real numbers $\mathbb{R}$
   - Ordering numbers in $\mathbb{R}$
4. Intervals.
5. Operations on the real numbers.
6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

A research project on unit one

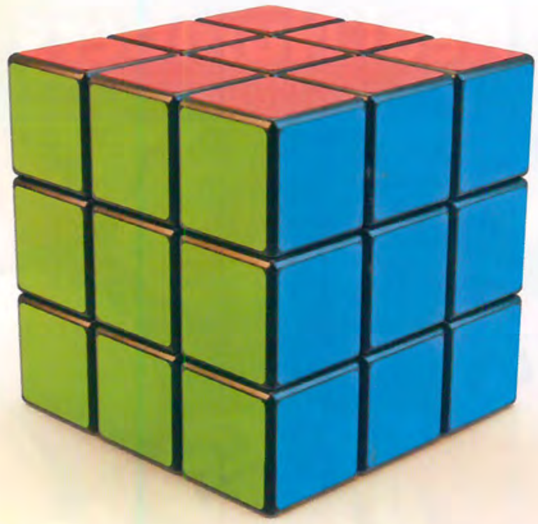
Scan the QR code to solve an interactive test on each lesson

EXERCISE 1 ?

The cube root of a rational number

Interactive test

From the school book



Remember Understand Apply Problem Solving

1 Complete the following table :

Number a	8	125	-27	......	$3\frac{3}{8}$	$-\frac{8}{125}$	......	......
$\sqrt[3]{a}$	......	......	......	-10	......	......	6	-4

2 Complete :

1. $\sqrt[3]{216} =$ ....
2. $\sqrt[3]{-343} =$ ....
3. $\sqrt[3]{0.001} =$ ....
4. $\sqrt[3]{-\frac{8}{27}} =$ ....
5. $\sqrt[3]{8} + \sqrt[3]{-8} =$ ....
6. $\sqrt[3]{27} - \sqrt[3]{64} =$ ....
7. $\sqrt[3]{27} - \sqrt[3]{-27} =$ ....
8. $\sqrt{9} + \sqrt[3]{-8} =$ ....
9. $-\sqrt[3]{-1} - \sqrt{1} =$ ....
10. $\dfrac{-\sqrt[3]{64}}{\sqrt{64}} =$ ....
11. $\sqrt[3]{a^3} =$ ....
12. $\sqrt[3]{-27a^6} =$ ....
13. $\sqrt[3]{\text{..........}} = 4$
14. $\sqrt{16} = \sqrt[3]{\text{..........}}$
15. $|\sqrt[3]{-125}| = \sqrt{\text{..........}}$
16. $\sqrt[3]{64 + \text{..........}} = 5$

3 **Choose the correct answer from those given :**

1 $\sqrt[3]{(-8)^2} = \dots\dots\dots\dots$

(a) 2 (b) -2 (c) 4 (d) -4

2 $\sqrt[3]{\left(\frac{1}{8}\right)^2} = \dots\dots\dots\dots$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

3 $\sqrt[3]{-64} + \sqrt{16} = \dots\dots\dots\dots$

(a) zero (b) 8 (c) -8 (d) ± 8

4 $\sqrt{25} - \sqrt[3]{-125} = \dots\dots\dots\dots$

(a) 10 (b) zero (c) 5 (d) ± 5

5 $\sqrt{(-2)^2} + \sqrt[3]{(-2)^3} = \dots\dots\dots\dots$

(a) -4 (b) 8 (c) 4 (d) zero

6 $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25} = \dots\dots\dots\dots$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) -2

7 $\sqrt[3]{0.001 \times \frac{1}{8}} = \dots\dots\dots\dots$

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20

8 $\sqrt[3]{1000} \times \sqrt[3]{-0.008} = \dots\dots\dots\dots$

(a) $\frac{1}{2}$ (b) 10 (c) 2 (d) -2

9 $\sqrt[3]{-27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0.125} = \dots\dots\dots\dots$

(a) 1 (b) zero (c) -1 (d) $\frac{11}{2}$

10 If $-\sqrt{25} = \sqrt[3]{y}$, then $y = \dots\dots\dots\dots$

(a) 5 (b) -5 (c) 125 (d) -125

11 If $x^3 = 64$, then $\sqrt{x} =$ ....

(a) 4 (b) -4 (c) 2 (d) -2

12 If $x^3 = 27$, then $x^2 =$ ....

(a) 3 (b) 6 (c) 9 (d) 81

13 $\sqrt[3]{x^6} = \sqrt{\dots\dots\dots}$

(a) x^3 (b) x^2 (c) x (d) x^4

14 If $\frac{x}{3} = \frac{9}{x^2}$, then $x =$ ....

(a) 1 (b) 3 (c) 9 (d) 27

4 Find the value of x in each of the following :

1 $\sqrt[3]{x} = 5$
2 $\sqrt[3]{x} = -\frac{1}{4}$
3 $\sqrt[3]{x} = -\sqrt{4}$
4 $\sqrt[3]{x} - 3 = -1$
5 $x^3 = -8$
6 $x^3 = 64$
7 $x^3 + 5 = 32$
8 $2x^3 = 54$
9 $\frac{1}{5}x^3 = -200$

5 Find the S.S. of each of the following equations in $\mathbb{Q}$:

1 $x^3 + 27 = 0$
2 $8x^3 + 7 = 8$
3 $x^3 + 16 = \frac{3}{8}$
4 $2x^3 - 5 = x^3 + 3$
5 $(x+3)^3 = 343$
6 $(3x+1)^3 = -8$
7 $(2x+1)^3 - 7 = 20$
8 $(5x-2)^3 + 10 = 18$

6 Find each of the following :

1 $\sqrt[3]{2\frac{1}{4} \div \frac{2}{3}}$
2 $-\sqrt[3]{2^9 \times 3^6}$
3 $\sqrt{\sqrt[3]{729}}$
4 $\sqrt[3]{\sqrt[3]{512}}$
5 $\sqrt{27\sqrt[3]{27}}$

Applications

7 A cube of volume 27 cm^3. Find the area of one face. « 9 cm^2. »

8 Find the total area of a cube whose volume is 216 cm^3. « 216 cm^2. »

9 If the half of the cube of a number equals 32 , find this number. « 4 »

10 🕮 Find the inner edge length of a cube vessel with capacity of one litre. « 10 cm. »

11 🕮 Find the diameter length of a sphere whose volume is $\frac{1372}{81}\pi$ cube unit. « $\frac{14}{3}$ length unit »

12 Find the length of the diameter of a sphere whose volume is 113.04 cm^3. ($\pi = 3.14$) « 6 cm. »

For excellent pupils

13 **Find the S.S. of each of the following equations in $\mathbb{Q}$:**

1. $(x^2+6)^3 = 1000$
2. $(x^3-14)^2 = 169$
3. $\sqrt[3]{(x-1)^2} = \sqrt[3]{25}$
4. $\sqrt[3]{(x-2)(x^2-4x+4)} = 3$

14 If $\sqrt[3]{\sqrt{x}+19} = 3$, find the value of $\sqrt[3]{x}$ « 4 »

15 A man was asked about the age of his father and the age of each of his three sons.
His answer was as follows :
My age is half the age of my father. The age of my eldest son is the square root of the age of my father and the age of my middle son is the cube root of the age of my father and the age of my youngest daughter is the quotient of the age of my eldest son by the age of my middle son. Given that the age of my eldest son is twice the age of my middle son.
What is the age of each of his father and his three sons ? « 64 , 8 , 4 , 2 »

EXERCISE **2**

The set of irrational numbers $\overline{\mathbb{Q}}$

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 **In each of the following , show which of them is a rational number and which of them is an irrational number :**

1 -5	2 $2\frac{2}{3}$	3 2.06	4 2.3×10^5
5 $-\sqrt{36}$	6 $\sqrt[3]{36}$	7 $\sqrt{7}$	8 zero
9 $\lvert -5 \rvert$	10 $\sqrt[3]{-\frac{64}{81}}$	11 $\sqrt{\frac{25}{16}}$	12 $\sqrt{\frac{1}{3}}$
13 $\sqrt[3]{3\frac{3}{8}}$	14 $\sqrt[3]{0.343}$	15 $\frac{\pi}{2}$	16 $(-5)^{\text{zero}}$
17 $\frac{\text{zero}}{3}$	18 $\frac{\sqrt{9}}{\sqrt{4}}$	19 $\sqrt{9}+\sqrt{16}$	20 $\sqrt{4}-\sqrt{11}$

2 **Find an approximated value for each of the following numbers :**

1 $\sqrt{11}$ "to the nearest hundredth".

2 $\sqrt[3]{7}$ "to the nearest tenth".

3 $\sqrt[3]{-9}$ "to the nearest tenth".

3 **Find two consecutive integers for each of the following numbers to be included between them :**

1 $\sqrt{5}$ 2 $\sqrt{12}$ 3 $\sqrt[3]{10}$ 4 $\sqrt[3]{-20}$

4 **If x is an integer , find the value of x in each of the following cases :**

1 $x < \sqrt{2} < x + 1$ « 1 »

2 $x < \sqrt{80} < x + 1$ « 8 »

3 $x < \sqrt[3]{5} < x + 1$ « 1 »

4 $x < \sqrt[3]{50} < x + 1$ « 3 »

5 $x < \sqrt[3]{-100} < x + 1$ « – 5 »

6 $x < |-\sqrt{35}| < x + 1$ « 5 »

5 **Find an approximated value for each of the following numbers , then check your answer using the calculator :**

1 $\sqrt{20}$ 2 $\sqrt[3]{17}$ 3 $\sqrt{5} + 1$ 4 $\sqrt[3]{9} - 1$

6 **Choose the correct answer from the given ones :**

1 The irrational number in the following numbers is ....

(a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$

2 If $x = \sqrt{2}$, $y = 2$, then which of the following does not represent a rational number ?

(a) $x^2 + y$ (b) $x + y^2$ (c) $\sqrt{x^2 y}$ (d) $\sqrt{2}\, x y$

3 The irrational number located between 2 and 3 is ....

(a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

4 The irrational number located between –2 and –1 is ....

(a) – 3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

5 $\sqrt{10} \approx$ ....

(a) 2.99 (b) 3.71 (c) 3 (d) – 3.2

6 The nearest integer to $\sqrt[3]{25}$ is ....

(a) 5 (b) 3 (c) 2 (d) 12.5

7 If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then $n =$ ....

(a) 25 (b) 5 (c) – 5 (d) 24

8 The side length of a square whose area is 6 cm^2 is ....

(a) a natural number. (b) an integer.

(c) a rational number. (d) an irrational number.

9 The area of a square whose side length is $\sqrt{3}$ cm. is ………… cm².

(a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

10 The square whose area is 10 cm², its side length is ………… cm.

(a) 5 (b) – 5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$

11 The S.S. of the equation : $(x-\sqrt{5})(x+\sqrt{3})=0$ in $\mathbb{Q}'$ is …………

(a) $\{\sqrt{5}\}$ (b) $\{-\sqrt{3}\}$ (c) $\{-\sqrt{5},\sqrt{3}\}$ (d) $\{\sqrt{5},-\sqrt{3}\}$

7 Find the value of x in each of the following cases and determine whether $x \in \mathbb{Q}$ or $x \in \mathbb{Q}'$:

1 $5x^2 = 10$ « $\pm\sqrt{2}$ »

2 $4x^2 = 9$ « $\pm\frac{3}{2}$ »

3 $x^3 = 125$ « 5 »

4 $3x^3 = 27$ « $\sqrt[3]{9}$ »

5 $0.1x^2 = 10$ « ±10 »

6 $0.001x^3 = -8$ « – 20 »

7 $(x-1)^2 = 4$ « 3 or – 1 »

8 $(x-5)^3 = 1$ « 6 »

8 Find in $\mathbb{Q}'$ the S.S. of each of the following equations :

1 $x^2 = 13$

2 $x^3 = 16$

3 $\frac{2}{5}x^2 = \frac{25}{2}$

4 $\frac{5}{4}x^3 = -2$

5 $125x^3 - 7 = 20$

6 $\frac{1}{4}x^2 + 2 = 66$

7 $(x^3+5)(x^2-3) = $ zero

8 $(x+\sqrt{7})(x^3-6) = $ zero

9 Prove that :

1 $\sqrt{2}$ is included between 1.4 and 1.5

2 $\sqrt{11}$ is included between 3.31 and 3.32

3 $\sqrt[3]{2}$ is included between 1.2 and 1.3

4 $\sqrt[3]{15}$ is included between 2.4 and 2.5

5 $\sqrt[3]{-17}$ is included between – 2.6 and – 2.5

6 $\sqrt{3} + 1$ is included between 2.7 and 2.8

10 **Determine the point that represents each of the following numbers on the number line :**

1 $\sqrt{3}$ 2 $-\sqrt{11}$ 3 $\sqrt{10}$ 4 $\sqrt{5}+1$ 5 $2-\sqrt{7}$

11 Draw the number line and label point A which represents $\sqrt{2}$

- Label point B which represents $1+\sqrt{2}$
- Label point C which represents $1-\sqrt{2}$

12 Draw the right-angled triangle ABC at B where AB = 1 cm. and BC = 3 cm. , then use the figure to determine the points that represent the following numbers on the number line :

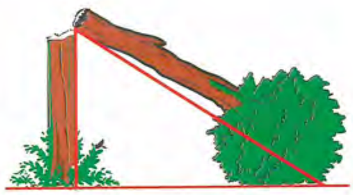
1 $\sqrt{10}$ 2 $-\sqrt{10}$ 3 $2+\sqrt{10}$ 4 $3-\sqrt{10}$

13 Calculate the side length and the diagonal length of a square whose area equals 10 cm^2.

« $\sqrt{10}$ cm. , $\sqrt{20}$ cm. »

Life Application

14 A tree is 3 metres long. Its upper part was broken because of the wind and it made an angle with the surface of the ground. If the length of the left part of the tree is 1 metre , find the distance between the base of the tree and the point of touching of its top with the ground.



« $\sqrt{3}$ metres »

For excellent pupils

15 Without using the calculator , prove that $\sqrt{3}+\sqrt{2}$ is included between 3 and 4

EXERCISE 3

The set of real numbers $\mathbb{R}$ and ordering numbers in $\mathbb{R}$

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 Complete the following table by placing (✓) in the suitable place as shown in the first case :

The number	Natural	Integer	Rational	Irrational	Real
-5	✗	✓	✓	✗	✓
$\sqrt{2}$					
$1\frac{1}{2}$					
$\sqrt[3]{9}$					
$\|-2\|$					
$-\sqrt{4}$					
$\frac{5}{2}$					
0.3					
$\sqrt{-1}$					

2 If $x \in \mathbb{R}$, state whether x is positive or negative or anything else in each of the following cases :

1. $x > 0$
2. $x < 0$
3. $x > |-4|$
4. $|-5| < x < 7$
5. $-2 < x < 0$
6. $|-1| < x < |-7|$

3 **Put the suitable sign (> , < or =) :**

1. $\sqrt{5}$ 2
2. $\sqrt{7}$ 2.6
3. $\sqrt[3]{24}$ 3
4. $\sqrt[3]{-24}$ .... -2
5. $3-\sqrt{5}$ .... $\sqrt[3]{-1}$
6. $\sqrt[3]{8}$ .... $\sqrt{4}$
7. $1+\sqrt{3}$ .... $\sqrt{5}$
8. $\sqrt[3]{3}-1$ 0.2
9. $\sqrt{2}-1$ .... $1-\sqrt{2}$

4 **Choose the correct answer from those given :**

1. $\mathbb{R}=$ ....
   (a) $\mathbb{Q}\cup\grave{\mathbb{Q}}$ (b) $\mathbb{Z}_+\cup\mathbb{Z}_-$ (c) $\mathbb{R}_+\cup\mathbb{R}_-$ (d) $\mathbb{N}\cup\mathbb{R}_-$
2. $\mathbb{Q}\cap\grave{\mathbb{Q}}=$ ....
   (a) $\mathbb{Q}$ (b) $\grave{\mathbb{Q}}$ (c) $\mathbb{R}$ (d) $\varnothing$
3. $\mathbb{Q}\cup\grave{\mathbb{Q}}=$ ....
   (a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\grave{\mathbb{Q}}$
4. $\mathbb{R}_+\cap\mathbb{R}_-=$ ....
   (a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}_-$
5. $\mathbb{R}_+\cup\mathbb{R}_-=$ ....
   (a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}^*$
6. $\mathbb{R}-\grave{\mathbb{Q}}=$ ....
   (a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{Q}$ (d) $\{0\}$
7. $\mathbb{R}-\mathbb{Q}=$ ....
   (a) $\grave{\mathbb{Q}}$ (b) $\mathbb{R}$ (c) $\varnothing$ (d) $\{0\}$
8. $\mathbb{R}_+\cap\{-1, 0, 1\}=$ ....
   (a) $\{0, 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$
9. $\{x : x\in\mathbb{R}, x<0\}=$ ....
   (a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$ (c) $\mathbb{R}^*$ (d) $\mathbb{R}$
10. If x is a negative real number , then which of the following numbers is positive ?
    (a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$
11. If $\frac{1}{a}$ and $\frac{a}{\sqrt{5}}$ are two real numbers included between 0 and 1 , then a = ...
    (a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2
12. If $x\in\mathbb{R}_+$, $y\in\mathbb{R}_+$ and $x^2>y^2$, then ....
    (a) $x>y$ (b) $x<y$ (c) $x=y$ (d) $x\le y$

13 $\sqrt{(2-\pi)^2}$ .... $(2-\pi)$ (where π is the ratio between the circumference of the circle and its diameter length)

(a) = (b) < (c) > (d) ≤

14 The S.S. of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....

(a) $\{-1\}$ (b) $\{1, -1\}$ (c) $\{1\}$ (d) $\varnothing$

5 Arrange the following numbers ascendingly :

1 $\sqrt{8}$, $-\sqrt{3}$, $\sqrt{15}$, $\sqrt{5}$, $-\sqrt{7}$ and $-\sqrt{11}$

2 $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 0.6 and $\sqrt[3]{-1}$

6 Arrange the following numbers descendingly :

1 $\sqrt{62}$, 8 , $-\sqrt{50}$ and $\sqrt{70}$

2 $\sqrt{6}$, 9 , $-\sqrt{10}$, $-\sqrt{7}$, $-\sqrt{50}$ and $\sqrt{101}$

7 Write three positive irrational numbers less than 2

8 Write three negative irrational numbers greater than $-\sqrt{6}$

9 Write four irrational numbers included between 15 and 17

10 Prove that $\sqrt{3}$ is between 1.7 and 1.8 , then represent $\sqrt{3}$, 1.7 and 1.8 on the number line.

11 Solve the following equations to the nearest hundredth given $x \in \mathbb{R}$:

1 $x^2 - 6 = 0$

2 $\frac{3}{4}x^2 = 24$

3 $\frac{1}{2}x^2 - 5 = 0$

4 $5x^3 + 3 = 2$

5 $\frac{3}{4}x^2 + 2 = -11$

6 $\frac{2}{x^3} + 5 = 21$ $(x \neq 0)$

7 $(x^2 - 9)(x^3 - 5) = 0$

8 $(2x^3 - 5)(x^2 + 1) = 0$

Geometric Applications

12 Find the side length of a square whose area is 5 cm^2. Is the side length a rational number ? «$\sqrt{5}$ cm.»

13 Find the edge length of a cube whose volume is 1.728 cm^3. Is the edge length a rational number ? «$\frac{6}{5}$ cm.»

14 A cube whose total area is 13.5 cm^2. Find its edge length. Is the edge length a rational number ? «1.5 cm.»

15 A square is of side length 6 cm. Find its diagonal length. «$\sqrt{72}$ cm.»

16 A square is of area 32 cm^2. Find its side length and its diagonal length. «$\sqrt{32}$ cm. , 8 cm.»

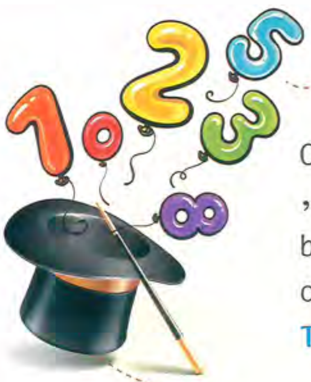
17 An isosceles right-angled triangle , the length of one side of its right-angle = 5 cm. Find the length of its hypotenuse. «$\sqrt{50}$ cm.»

18 A rectangle with dimensions 5 cm. and 7 cm. Find the length of its diagonal. And if its area equals the area of a square , then find the side length of the square and its diagonal length. «$\sqrt{74}$ cm. , $\sqrt{35}$ cm. , $\sqrt{70}$ cm.»

For excellent pupils

19 Without using the calculator , prove that : $\sqrt[3]{3} > \sqrt{2}$

20 Two real numbers , the sum of their squares is 7 and the greater number is 2
Find the other number. «$\sqrt{3}$ or $-\sqrt{3}$»



Wonders of numbers

Choose a number from 1 to 9 , multiply it by 3 , add 3 to the product , and multiply the result by 3 once again "use calculator" Find the sum of the digits of the product.

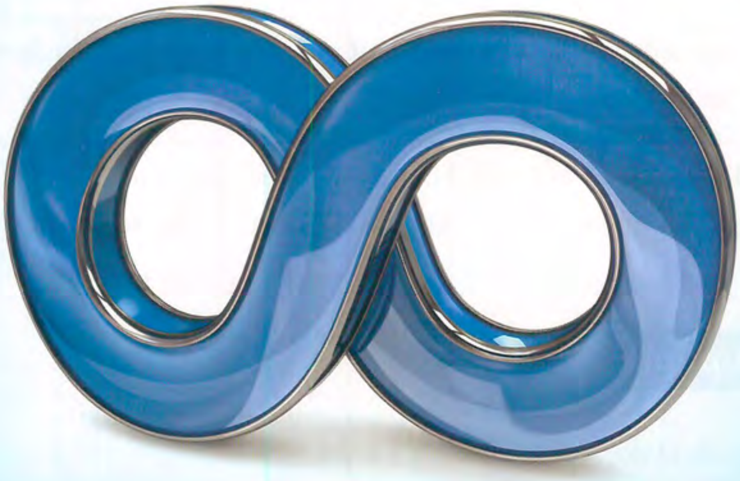
The answer is always 9.

EXERCISE 4 ?

Intervals

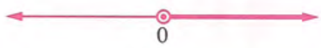
Interactive test

From the school book



Remember Understand Apply Problem Solving

1 **Complete the following table :**

The interval	Expression by description method	Its representation on the number line
1 [– 1 , 2]	$\{x : -1 \leq x \leq 2 , x \in \mathbb{R}\}$	
2 [1 , 3[	....	....
3	$\{x : 0 < x \leq 3 , x \in \mathbb{R}\}$	....
4	....	
5]– ∞ , 1]	....	....
6	....	
7	$\{x : x < 4 , x \in \mathbb{R}\}$	....
8 [– 2 , ∞[	....	....

2 **Choose the correct answer from the given ones :**

1 $\mathbb{R} =$ ....

(a) $\mathbb{R}_+ \cap \mathbb{R}_-$ (b) $\mathbb{R}_+ \cup \mathbb{R}_-$ (c)]– ∞ , ∞[(d) $\mathbb{Q} \cap \grave{\mathbb{Q}}$

2 $\mathbb{R}_+ =$ ....

(a)]0 , ∞[(b)]– ∞ , 0[(c) [0 , ∞[(d)]– ∞ , 0]

3 $\mathbb{R}_- = \dots\dots$

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

4 The set of non-negative real numbers = ……….

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

5 The set of non-positive real numbers = ……….

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

3 Complete each of the following using one of the symbols $\in$ or $\notin$:

1 $3 \dots\dots [3 , 5]$

2 $-2 \dots\dots]-2 , 1]$

3 $0 \dots\dots [-1 , 4[$

4 $|-3| \dots\dots [2 , \infty[$

5 $\sqrt{9} \dots\dots]-3 , \infty[$

6 $\sqrt[3]{-1} \dots\dots]-\infty , 1[$

7 $1.3 \times 10^{-5} \dots\dots \mathbb{R}_+$

8 $\sqrt{2} \dots\dots [2 , 5]$

9 $5 \dots\dots]\sqrt{5} , \sqrt{23}[$

10 $\sqrt[3]{-125} \dots\dots]-\sqrt{25} , \sqrt{25}]$

4 If $X = [2 , 5[$ and $Y = [-1 , 3[$, find using the number line :

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $Y - X$

5 $\grave{X}$

6 $\grave{Y}$

5 If $X =]-\infty , 3]$ and $Y = [-4 , \infty[$, find using the number line :

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $Y - X$

5 $\grave{X}$

6 $\grave{Y}$

6 If $X = [-1 , 4]$, $Y = [3 , \infty[$ and $Z = \{3 , 4\}$, find the following using the number line :

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $X - Z$

5 $Y \cap Z$

6 $Y - X$

7 $\grave{X}$

8 $\grave{Y}$

7 Find using the number line :

1 $[-1 , 4] \cap [2 , 5]$

2 $[-1 , 4] \cup [2 , 5]$

3 $]-2 , 3] \cap]0 , 1[$

4 $]-2 , 3] \cup]0 , 1[$

5 $[2 , 6] - [-1 , 3[$

6 $[-1 , 3[- [2 , 6]$

7 $[-3 , 0[\cup]0 , 2]$

8 $[-3 , 0] \cap]0 , 2]$

9 $[1 , 2] - [-2 , 4]$

10 $[-2 , 4] - [1 , 2]$

11 $[-1 , 4] \cap [5 , 7[$

12 $[-1 , 5] -]-1 , 5[$

8 **Find using the number line :**

1. $[-1, \infty[\cup [-3, 4]$
2. $[2, \infty[\cap]-2, 3[$
3. $]-\infty, 3] \cap [-4, \infty[$
4. $[2, \infty[\cup]-\infty, 3]$
5. $]-\infty, 3] - [-1, \infty[$
6. $]-\infty, -3] - [-3, 1]$
7. $]-\infty, 2] -]-\infty, 0]$
8. $]-\infty, 3[\cup]4, \infty[$

9 **Complete the following :**

1. $[3, 5] \cup \{3, 5\} =$ ....
2. $]3, 5[\cup \{3, 5\} =$ ....
3. $[3, 5] \cap \{3, 5\} =$ ....
4. $]3, 5[\cap \{3, 5\} =$ ....
5. $[3, 5] - \{3, 5\} =$ ....
6. $]3, 5[- \{3, 5\} =$ ....
7. $\{3, 5\} - [3, 5] =$ ....
8. $\{3, 5\} -]3, 5[=$ ....
9. $]3, 5[\cup \{3\} =$ ....
10. $[3, 5] - \{5\} =$ ....
11. $]2, 5[\cap \{-2, 3, 4\} =$ ....
12. $]-3, 5] \cup \{-2, 3, 4\} =$ ....

10 **Complete the following :**

1. $]1, 7[\cup]3, 5[=$ ....
2. $]-3, 2] - [0, 2] =$ ....
3. $[3, 4[\cup]3, 4] =$ ....
4. $]2, 5] \cap [2, 5[=$ ....
5. $[3, 5] - [3, 5[=$ ....
6. $[3, 7] - [4, 7] =$ ....
7. $[2, 7] -]2, 7[=$ ....
8. $[-2, 4] \cap [4, 6] =$ ....
9. If $X \cap [2, 7] = [3, 4[$, then $X =$ ....
10. If x is a positive real number , then $x > x^2$ when $x \in]$ .... , ... $[$

11 **Choose the correct answer from the given ones :**

1. 📖 $[-3, 4] - \{-3, 5\} =$ ....
   (a) $]-3, 4[$ (b) $]-3, 4]$ (c) $]-3, 5[$ (d) $[-3, 5[$
2. If $x \in [-3, \infty[$, then ....
   (a) $x < -3$ (b) $x \leq -3$ (c) $x > -3$ (d) $x \geq -3$

3 If $X = \{x : x \in \mathbb{R} , 2 < x \leq 5\}$, then $[3 , 4]$ X

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

4 $\{3\} \cap [3 , 6] =$ ....

(a) $\varnothing$ (b) $\{3\}$ (c) $]3 , 6]$ (d) $\{6\}$

5 $\{8 , 9 , 10\} -]8 , 10[=$ ....

(a) $\varnothing$ (b) $\{8 , 10\}$ (c) $\{9\}$ (d) $\mathbb{N}$

6 The sum of all real numbers in $[-75 , 75]$ is ....

(a) -75 (b) 75 (c) 150 (d) zero

12 Complete the following :

1 $\mathbb{R} \cap [-3 , 3] =$ ....

2 $\mathbb{R} \cup]-1 , 4] =$ ....

3 $\mathbb{R} - [-1 , \infty[=$ ....

4 $\mathbb{R}_- - [-3 , 1] =$ ....

5 $]-2 , 5] - \mathbb{R}_+ =$ ....

6 $[-2 , 2] - \mathbb{R}_- =$ ....

7 $]-3 , 2] \cap \mathbb{Z}_+ =$ ....

8 $\mathbb{N} \cap [-5 , 2[=$ ....

9 $\mathbb{Z} \cap [-1 , 3[=$ ....

10 $\mathbb{R}_+ \cap [0 , 5] =$ ....

11 $\mathbb{R}_- \cap [-3 , 2] =$ ....

13 Choose from column (B) the suitable interval which represents the figure in column (A) :

(A)	(B)
1 	$\mathbb{R} -]-3 , 1]$
2 	$\mathbb{R} - [-3 , 1[$
3 	$\mathbb{R} -]-3 , 1[$
4 	$[-3 , 3[- \{1\}$
5 	$[-3 , 1[$
	$]-3 , 1[$

Life Application

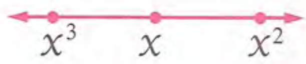
14 Two kinds of food , the first kind needs to be kept in a temperature between – 3 and 4 degrees and the other kind needs to be kept in a temperature between 2 and 10 degrees.

What is the temperature needed to keep the two kinds altogether at the same place ?

For excellent pupils

15 **Choose the correct answer from the given ones :**

1. **In the opposite figure :**



If x is a real number , then $x \in$ ....

(a) $\mathbb{R}_-$ (b) $\mathbb{R}_+$ (c) $]-\infty , -1]$ (d) $]-\infty , -1[$

2. If $x \in [-3 , 4]$, then $x^2 \in$ ....

(a) $[9 , 16]$ (b) $[0 , 9]$ (c) $[0 , 16]$ (d) $[-9 , 0]$

3. If $x \in [-5 , 4]$, then $x^2 \in$ ....

(a) $[0 , 16]$ (b) $[16 , 25]$ (c) $[0 , 25]$ (d) $[-5 , 0]$

4. If $x \in [1 , 16]$, then $-\sqrt{x} \in$ ....

(a) $[1 , 4]$ (b) $[-1 , 4]$ (c) $[-4 , -1]$ (d) $[-4 , 0]$

5. If $X \subset \mathbb{R}$, $[2 , 5] - X =]2 , 5[$, then $X =$ ....

(a) $[2 , 5]$ (b) $\{2 , 5\}$ (c) $[2 , 5[$ (d) $]2 , 5]$

6. If $X \subset \mathbb{R}$, $]4 , 7] \cup X = [1 , 7]$, then $X =$ ....

(a) $[1 , 3[$ (b) $[1 , 3]$ (c) $[1 , 4[$ (d) $[1 , 5]$

7. If $M \subset \mathbb{R}$, $M \cap [3 , 8[= [3 , 8[$, then $M =$ ....

(a) $]3 , 8[$ (b) $]3 , 8]$ (c) $[3 , 9]$ (d) $[3 , 7]$

8. If $]-\infty , k[\cap [-2 , 5] = [-2 , 3[$, then $k =$ ....

(a) -2 (b) 5 (c) 3 (d) zero

9. If $[-1 , x] \cap [y , 5] = [2 , 3]$, then $x^y =$ ....

(a) 8 (b) $\frac{1}{5}$ (c) 9 (d) -1

16 If $X \cap Y = [4 , 7]$, $X \cup Y = [3 , 7]$ and $X \subset Y$, **find :** X , Y and $Y - X$

EXERCISE 5

Operations on the real numbers

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 **Find each of the following in the simplest form :**

1. $\sqrt{3} + 2\sqrt{3}$
2. $2\sqrt{5} - 3\sqrt{5} + \sqrt{5}$
3. $5\sqrt[3]{7} - 8\sqrt[3]{7} + 2\sqrt[3]{7}$
4. $4\sqrt{5} - 2\sqrt{5} + 5\sqrt{5} - \sqrt{5}$

2 **Find each of the following in the simplest form :**

1. $\sqrt{5} - \sqrt{3} + 2\sqrt{5} + \sqrt{3}$
2. 📖 $2\sqrt{3} + 5 + \sqrt{3} - 6$
3. 📖 $2\sqrt{7} - 3\sqrt{2} + \sqrt{7} + 5\sqrt{7}$
4. $2\sqrt{2} - 3\sqrt[3]{2} + 5\sqrt{2} + \sqrt[3]{2}$
5. $\frac{1}{4}\sqrt{2} + \frac{2}{7}\sqrt{5} + \frac{3}{4}\sqrt{2} - \frac{2}{7}\sqrt{5}$
6. $8\sqrt{\frac{1}{4}} + 2\sqrt[3]{3} - \sqrt[3]{64} - 5\sqrt[3]{3}$

3 **Find the result of each of the following :**

1. $\sqrt{3} \times \sqrt{3}$
2. $-2\sqrt{5} \times 3\sqrt{5}$
3. $2 \times 3\sqrt{2}$
4. $\frac{1}{3}\sqrt{3} \times \sqrt{3}$
5. $(\sqrt[3]{5})^3 \times 3\sqrt{3}$
6. $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \div \frac{20\sqrt{3}}{5\sqrt{7}}$

4 **Find the result of each of the following in the simplest form :**

1. $2\left(\sqrt{2} + \sqrt{5}\right)$
2. 📖 $\sqrt{2}\left(5 + \sqrt{2}\right)$
3. 📖 $\sqrt{7}\left(\sqrt{7} + 2\right)$
4. 📖 $-\sqrt{3}\left(-5 - \sqrt{3}\right)$
5. $-2\sqrt{5}\left(3 - \sqrt{5}\right)$
6. $\sqrt{7}\left(\frac{2}{\sqrt{7}} - \sqrt{7} + 3\right)$
7. $-3\left(8 + 2\sqrt{3}\right) + 6\sqrt{3}$
8. 📖 $\sqrt{5}\left(3 - \sqrt{5}\right) - 2\left(1 + \sqrt{5}\right)$

5 **Find the result of each of the following operations :**

1. $(\sqrt{2}+1)(\sqrt{2}-1)$
2. $(4-3\sqrt{2})(4+3\sqrt{2})$
3. $(\sqrt{5}-1)^2$
4. $(2\sqrt{3}+4)^2$
5. $(\sqrt{3}+2)(\sqrt{3}-1)$
6. $(5-\sqrt{3})^2-28$

6 **Make the denominator in each of the following an integer :**

1. $\frac{3}{\sqrt{3}}$
2. $\frac{10}{\sqrt{5}}$
3. $-\frac{6}{\sqrt{3}}$
4. $\frac{8}{\sqrt{6}}$
5. $\frac{2}{3\sqrt{2}}$
6. $\frac{6}{2\sqrt{3}}$
7. $\frac{25}{2\sqrt{10}}$
8. $\frac{\sqrt{2}+3}{\sqrt{2}}$
9. $\frac{\sqrt{5}-15}{2\sqrt{5}}$

7 **Choose the correct answer from those given :**

1. $\sqrt{7}+\sqrt{7}=$ ....
   (a) 7 (b) 14 (c) $2\sqrt{7}$ (d) $\sqrt{14}$

2. $\sqrt{3}+(-\sqrt{3})=$ ....
   (a) $2\sqrt{3}$ (b) $2\sqrt{6}$ (c) $\sqrt{6}$ (d) zero

3. $2\sqrt{3}+3\sqrt{3}=$ ....
   (a) $5\sqrt{6}$ (b) $5\sqrt{3}$ (c) $6\sqrt{3}$ (d) $5\sqrt[3]{3}$

4. $5+7\sqrt{2}-4+\sqrt{2}=$ ....
   (a) 15 (b) $1+7\sqrt{2}$ (c) $1+8\sqrt{2}$ (d) $1+6\sqrt{2}$

5. $-2\sqrt{3}\times\sqrt{3}=$ ....
   (a) -6 (b) $-2\sqrt{3}$ (c) $2\sqrt{3}$ (d) 6

6. $(2\sqrt[3]{5})^3=$ ....
   (a) 10 (b) 20 (c) $4\sqrt[3]{5}$ (d) 40

7. The additive inverse of the number $\frac{6}{\sqrt{2}}$ is ....
   (a) $-2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $-3\sqrt{2}$ (d) $3\sqrt{2}$

8. The additive inverse of the number $(\sqrt{2}-\sqrt{5})$ is ....
   (a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$

9 The multiplicative inverse of the number $\sqrt{5}$ is ....

(a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$

10 The multiplicative inverse of the number $\frac{\sqrt{2}}{6}$ is ....

(a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{2}}{2}$

11 $\left(\sqrt{5}+3\sqrt{5}\right)\div\sqrt{5}=$ ....

(a) $3\sqrt{5}$ (b) 3 (c) 5 (d) 4

12 If $x=\sqrt{2}+10$, $y=\sqrt{2}-10$, then $(x+y)^2=$ ....

(a) 4 (b) 6 (c) 8 (d) $4\sqrt{2}$

8 Complete the following :

1 The multiplicative neutral in $\mathbb{R}$ is .... and the additive neutral in $\mathbb{R}$ is ....

2 The additive inverse of the number $1-\sqrt{2}$ is ....

3 The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\cdots\cdots}{6}$

4 The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\cdots\cdots}{\sqrt{3}}$

5 $7+\sqrt{3}=5+(\cdots\cdots+\cdots\cdots)$

6 If $a=\frac{\sqrt{2}}{\sqrt{3}}$, $b=\frac{\sqrt{3}}{\sqrt{2}}$, then $\frac{a}{b}=$ ....

7 $\left(\sqrt{3}-2\right)^2=7-$ ....

8 If $\sqrt{x}=\sqrt{2}+1$, then $x=$ ....

9 If $x^2=\left(2\sqrt{3}-\sqrt{7}\right)\left(2\sqrt{3}+\sqrt{7}\right)$, then $x=$ ....

10 If $x^2-y^2=16$, $x-y=\sqrt{2}$, then $x+y=$ ....

11 If the side length of a square is ℓ cm. and its area is 15 cm^2 , then the area of the square of side length 2ℓ cm. is ....

12 If $a\in\mathbb{R}$ and $b\in\mathbb{R}$, then $a-b$ means the sum of the number a and .... of the number b

13 If $a\in\mathbb{N}$, $b\in\mathbb{Q}$ and $c\in\mathbb{R}$, then $a+b+c\in$ ....

9 **If $x = \sqrt{5} - 2$ and $y = \sqrt{5} + 2$, find the value of each of the following :**

1. $x + y$
2. $x - y$
3. $x y$
4. $x^2 - y^2$
5. $x^2 + 2 x y + y^2$
6. $x^2 - 2 x y + y^2$

---

10 If $x = \sqrt[3]{2\sqrt{2}}$ **, find the value of :** $(x + \sqrt{2})^2$ « 8 »

---

11 If $\frac{a}{2\sqrt{2} + 2} = \frac{b}{2\sqrt{2} - 2} = 1$ **Prove that :** $a \times b = a - b$

---

12 **If $x = \sqrt{15} + 2$ and $y = 4 - \sqrt[3]{25}$, estimate the value of each of the following :**

1. x , y
2. $x \times y$
3. $x + y$

Check the reasonability of each value using your calculator.

Geometric Application

13 A rectangle is of dimensions $(6 + \sqrt{5})$ cm. and $(6 - \sqrt{5})$ cm.

Calculate its perimeter and its area. « 24 cm. , 31 cm.^{2} »

For excellent pupils

14 If $a - b = 2\sqrt{3}$ **, find the value of :** $a (a - b)^3 + b (b - a)^3$ « 144 »

---

15 If the multiplicative inverse of the number $\sqrt{a} - 1$ is $\frac{\sqrt{a} + 1}{4}$, find the numerical value of a « 5 »

---

16 If $x = 2 y = 4 z = \sqrt{2}$ **, find the value of :** $x^2 + 2 y^2 + 4 z^2$ « $3\frac{1}{2}$ »

---

17 If the number y is the additive inverse of x and $\frac{1}{2} (y - x) = 1 - \sqrt{2}$

Prove that : $x y - 2\sqrt{2} = -3$

Wonders of numbers

- $1 \times 1 = 1$
- $11 \times 11 = 121$
- $111 \times 111 = 12321$
- $1111 \times 1111 = 1234321$

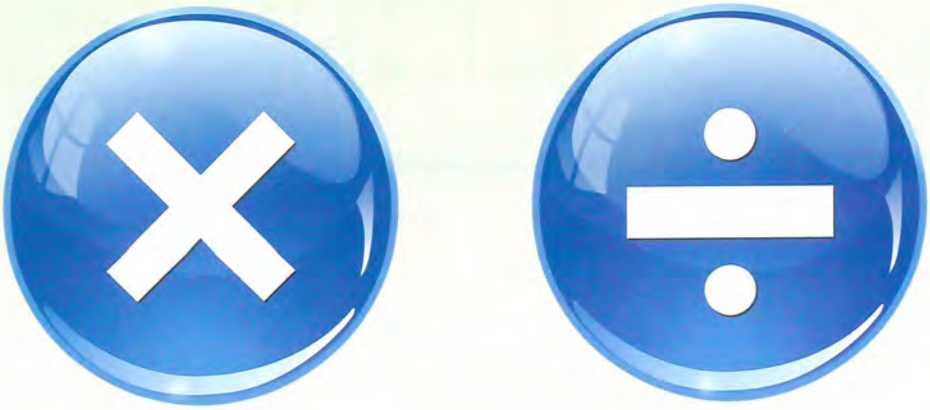
What happens when you multiply 11111 × 11111 ?

EXERCISE **6**

Operations on the square roots

Interactive test

From the school book



● Remember ● Understand ● Apply ● Problem Solving

1 Put each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :

1. $\sqrt{12}$
2. $\sqrt{28}$
3. $2\sqrt{72}$
4. $\frac{2}{5}\sqrt{1000}$
5. $2\sqrt{\frac{1}{2}}$
6. $6\sqrt{\frac{2}{3}}$

2 Simplify each of the following to the simplest form :

1. $\sqrt{50}+\sqrt{8}$ « $7\sqrt{2}$ »
2. $\sqrt{20}-\sqrt{45}$ « $-\sqrt{5}$ »
3. $3\sqrt{2}+\sqrt{8}-\sqrt{18}$ « $2\sqrt{2}$ »
4. $\sqrt{98}-\sqrt{128}-\sqrt{18}+4\sqrt{2}$ « zero »
5. $2\sqrt{18}+\sqrt{50}+\frac{1}{3}\sqrt{162}$ « $14\sqrt{2}$ »
6. $\sqrt{98}+\sqrt{50}-\frac{1}{2}\sqrt{200}-\sqrt{2}$ « $6\sqrt{2}$ »
7. $\sqrt{27}+5\sqrt{18}-\sqrt{300}$ « $15\sqrt{2}-7\sqrt{3}$ »

3 Put each of the following in the simplest form :

1. $2\sqrt{5}+4\sqrt{20}-5\sqrt{\frac{1}{5}}$ « $9\sqrt{5}$ »
2. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}$ « $\sqrt{2}$ »
3. $2\sqrt{5}+6\sqrt{\frac{1}{3}}-\sqrt{12}-5\sqrt{\frac{1}{5}}$ « $\sqrt{5}$ »
4. $\sqrt{3}+\frac{3}{\sqrt{3}}-\sqrt{2}\times\sqrt{6}$ « zero »
5. $\sqrt{18}-\frac{\sqrt{12}}{\sqrt{6}}$ « $2\sqrt{2}$ »
6. $\sqrt{(-5)^2}+\sqrt{18}-\frac{6}{\sqrt{2}}$ « 5 »

4 **Simplify each of the following to the simplest form :**

1. $2\sqrt{3} \times 5\sqrt{2}$ « $10\sqrt{6}$ »
2. $2\sqrt{18} \times 3\sqrt{2}$ « 36 »
3. $\sqrt{5} \times 2\sqrt{10}$ « $10\sqrt{2}$ »
4. $\sqrt{\frac{2}{7}} \times \sqrt{\frac{7}{2}}$ « 1 »
5. $\frac{3\sqrt{15}}{\sqrt{5}}$ « $3\sqrt{3}$ »
6. $12\sqrt{\frac{2}{3}} \times \sqrt{54}$ « 72 »

5 **Simplify each of the following to the simplest form :**

1. $\sqrt{6}\left(\sqrt{3} - \sqrt{2}\right)$
2. $5\sqrt{2}\left(2\sqrt{2} + \sqrt{12}\right)$
3. $\left(3\sqrt{5} - \sqrt{7}\right)\left(3\sqrt{5} + \sqrt{7}\right)$
4. $\left(\sqrt{3} - \sqrt{2}\right)^2$
5. $\left(\sqrt{3} + \sqrt{5}\right)^2 - \sqrt{60}$
6. $\sqrt{18} - \frac{12}{\sqrt{6}} + \sqrt{2}\left(2\sqrt{3} - 3\right)$

6 **Write each of the following such that the denominator is an integer :**

1. $\frac{\sqrt{3}}{\sqrt{2}}$
2. $\sqrt{\frac{5}{3}}$
3. $\frac{5\sqrt{3}}{\sqrt{5}}$
4. $\frac{4\sqrt{3} - \sqrt{2}}{2\sqrt{3}}$

7 **Choose the correct answer from those given :**

1. $\frac{\sqrt{63}}{\sqrt{7}} = \dots\dots\dots\dots$

(a) 3 (b) $\sqrt{3}$ (c) 9 (d) ± 3

2. $\sqrt{8} - \sqrt{2} = \dots\dots\dots\dots$

(a) $\sqrt{6}$ (b) $\sqrt{2}$ (c) 2 (d) 1

3. $\left(\sqrt{8} + \sqrt{2}\right)^2 = \dots\dots\dots\dots$

(a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

4. $\left(\sqrt{7} - \sqrt{5}\right)\left(\sqrt{7} + \sqrt{5}\right) = \dots\dots\dots\dots$

(a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$

5. $\sqrt{5} + \sqrt{5} = \dots\dots\dots\dots$

(a) $\sqrt{10}$ (b) $\sqrt{20}$ (c) 5 (d) 10

6. $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} = \dots\dots\dots\dots$

(a) 1 (b) $\sqrt{\frac{1}{4}}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

7 $\frac{\sqrt{27}}{\sqrt{3}} \div \frac{\sqrt{72}}{\sqrt{2}} = \dots\dots$

(a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 4

8 The multiplicative inverse of the number $\sqrt{50}$ is ....

(a) $\frac{\sqrt{2}}{10}$ (b) $\frac{-\sqrt{2}}{10}$ (c) $-5\sqrt{2}$ (d) $5\sqrt{2}$

9 If $x = \frac{\sqrt{6}}{\sqrt{2}}$, then $x^{-1} = \dots\dots$

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $2\sqrt{3}$

10 If $x = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{28} + \sqrt{12}$, then $x = \dots\dots$

(a) y (b) $\frac{1}{2}$ y (c) 2 y (d) y^2

8 Complete the following :

1 $\frac{3\sqrt{2}}{2\sqrt{18}} = \dots\dots$

2 $\sqrt{3} \times \sqrt{6} = 3 \times \dots\dots$

3 $\frac{1}{2}\sqrt{48} = 2 \times \dots\dots$

4 $\sqrt{3\frac{3}{8}} = \frac{3}{2}\sqrt{\dots\dots}$

5 If $2\sqrt{27} - 2\sqrt{48} = x\sqrt{3}$, then $x = \dots\dots$

6 $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ... (in the same pattern).

7 If $x^2 = \frac{8}{9}$, then x in the simplest form = ...

8 If $x^2 = 5$, then $(x + \sqrt{5})^2 = \dots\dots$ or ....

9 Find the value of each of $x + y$, $x \times y$ in each of the following cases :

1 $x = 3 + \sqrt{5}$, $y = 1 - \sqrt{5}$ « 4 , $-2 - 2\sqrt{5}$ »

2 $x = \sqrt{3} - \sqrt{2}$, $y = \sqrt{3} + \sqrt{2}$ « $2\sqrt{3}$, 1 »

3 $x = 5 - 3\sqrt{2}$, $y = 5 - 3\sqrt{2}$ « $10 - 6\sqrt{2}$, $43 - 30\sqrt{2}$ »

10 If $x = \frac{\sqrt{2}}{\sqrt{3}}$ and $y = \frac{\sqrt{3}}{\sqrt{2}}$, find the value of : $6(x + y)$ « $5\sqrt{6}$ »

11 If $x = \frac{10}{\sqrt{5}}$, $y = \sqrt{45} + \sqrt{2}$ and $z = \sqrt{8} + \sqrt{5}$

, find in the simplest form the value of the expression $(x - y + z)^2$ « 2 »

12 If $x = 2\sqrt{5} + \sqrt{2}$, $y = 2\sqrt{5} - \sqrt{2}$

, find the value of the expression : $x^2 + 2xy + y^2$ « 80 »

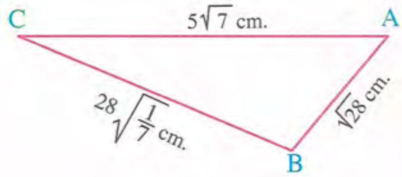
---

13 If $x = \sqrt{7} + \frac{1}{2}\sqrt{12}$ and $y = \frac{1}{3}\sqrt{63} - \sqrt{3}$, prove that : $x^2 y^2 = 16$

Geometric Applications

14 **In the opposite figure :**

Find the perimeter of Δ ABC in the simplest form.

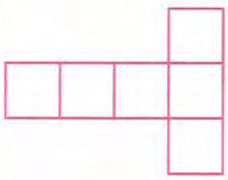


« $11\sqrt{7}$ cm. »

---

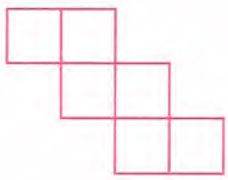
15 Each of the following figures consists of squares equal in area. Find the perimeter of each figure in the simplest form if its area is known :

1



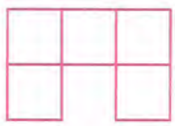
Its area = 300 cm^2.

2



Its area = 72 cm^2.

3



Its area = 40 cm^2.

« $70\sqrt{2}$ cm. , $28\sqrt{3}$ cm. , $24\sqrt{2}$ cm. »

For excellent pupils

16 If $a^x = 6$ and $a^{-y} = \sqrt{3}$, find the value of : a^{x+y} « $2\sqrt{3}$ »

---

17 **Simplify each of the following to the simplest form :**

1 $\dfrac{(\sqrt{5})^3 \times (\sqrt{5})^5}{(\sqrt{10})^6}$ « $\frac{5}{8}$ »

2 $\dfrac{2\sqrt{2} \times (\sqrt{6})^{-3}}{(\sqrt{3})^{-3}}$ « 1 »

---

18 If $\sqrt{27} + 2\sqrt{\frac{1}{2}} + \sqrt{18} + \sqrt{12} - \sqrt{50} = x\sqrt{2} + y\sqrt{3}$

, find the value of each of x and y where x and y are two rational numbers. « – 1 , 5 »

EXERCISE 7

The two conjugate numbers

Interactive test

From the school book

● Remember ● Understand ○ Apply Problem Solving

1 **Write the conjugate number of each of the following numbers :**

[1] $\sqrt{5}+\sqrt{3}$ [2] $5-2\sqrt{7}$ [3] $\sqrt{5}+\frac{2}{\sqrt{2}}$

2 **Make the denominator of each of the following a rational number :**

[1] $\frac{5}{\sqrt{7}-\sqrt{2}}$ [2] $\frac{\sqrt{3}}{2-\sqrt{3}}$ [3] $\frac{\sqrt{7}+3}{\sqrt{7}-3}$

3 If $x=\frac{2}{\sqrt{7}-\sqrt{5}}$ and $y=\sqrt{7}-\sqrt{5}$, find the value of : $(x+y)^2$ « 28 »

4 📖 If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$ and $y=\frac{4}{\sqrt{7}+\sqrt{3}}$, find the value of : $x^2 y^2$ « 16 »

5 If $x=\sqrt{5}+\sqrt{3}$, prove that : $\frac{4}{x}+2x=4\sqrt{5}$

6 📖 If $a=\sqrt{3}+\sqrt{2}$ and $b=\frac{1}{\sqrt{3}+\sqrt{2}}$, find the value of : a^2-b^2 in its simplest form. « $4\sqrt{6}$ »

7 📖 If $x=\sqrt{5}-\sqrt{3}$ and $y=\frac{2}{\sqrt{5}-\sqrt{3}}$, find the value of : $x^2+2xy+y^2$ « 20 »

8 📖 If $x=\sqrt{5}-\sqrt{2}$ and $y=\frac{3}{\sqrt{5}-\sqrt{2}}$, prove that x and y are conjugate numbers , then find the value of : $x^2-2xy+y^2$ « 8 »

9 **If $x = 3 + \sqrt{5}$ and $y = \frac{4}{3 + \sqrt{5}}$, prove that x and y are conjugate numbers , then find :**

1 Their product. 2 $x^2 + y^2$ « 4 , 28 »

10 If $x = \frac{2}{\sqrt{5} - \sqrt{3}}$ and $y = \frac{2}{\sqrt{5} + \sqrt{3}}$, find the value of : $x^2 - xy + y^2$ « 14 »

11 If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, find the value of : $\frac{x + y}{xy - 1}$ in its simplest form. « $\sqrt{5}$ »

12 If $a = \frac{4}{\sqrt{7} - \sqrt{3}}$ and $b = \frac{4}{\sqrt{7} + \sqrt{3}}$, find the value of : $\frac{a - b}{ab}$ « $\frac{\sqrt{3}}{2}$ »

13 If $x = 2\sqrt{2} - \sqrt{3}$ and $y = \frac{5}{\sqrt{8} - \sqrt{3}}$

, prove that x and y are conjugate numbers and calculate : $\frac{x + y}{xy}$ « $\frac{4\sqrt{2}}{5}$ »

14 If $x = \frac{5\sqrt{2} + 3\sqrt{5}}{\sqrt{5}}$ and $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$

, find the value of each of : 1 $x^2 + y^2$ 2 xy « 38 , 1 »

, then prove that : $x^2 + y^2 = 38\,xy$

15 If $x = \frac{1}{2 + \sqrt{3}}$ and $y = \frac{12}{\sqrt{3}}$, find the value of : $x^2 + y$ « 7 »

16 If $x = \frac{1}{\sqrt{3} - \sqrt{2}}$ and y is the multiplicative inverse of x

, find y , then prove that : $(x + y)^2 = 12$ « $\sqrt{3} - \sqrt{2}$ »

17 If $x = \sqrt{13} + \sqrt{6}$, $xy = 1$, find the value of : $x^2 - 49\,y^2$ « $4\sqrt{78}$ »

18 If $x = \frac{4}{\sqrt{7} - \sqrt{3}}$ and $y^{-1} = \frac{1}{\sqrt{7} - \sqrt{3}}$ (Remember that $y^{-1} = \frac{1}{y}$)

, prove that x and y are conjugate numbers , then find the value of : $x^2 y^2$ « 16 »

19 If $x = \sqrt{7} + \sqrt{5}$ and $y = \frac{2}{x}$, find the value of : $\frac{x + y}{x y}$ in its simplest form. «$\sqrt{7}$»

20 If $x = \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - \sqrt{5}}$, prove that : $x + \frac{1}{x} = 22$

21 **Complete the following :**

1. $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) = \cdots\cdots$
2. If $x = 3 + \sqrt{2}$, then its conjugate is $\cdots\cdots$ and the product of multiplying x by its conjugate is $\cdots\cdots$
3. The conjugate number of the number $\frac{2}{\sqrt{5} - \sqrt{3}}$ is $\cdots\cdots$
4. The conjugate number of the number $1 + \frac{7}{\sqrt{7}}$ in the simplest form is $\cdots\cdots$
5. The multiplicative inverse for $(\sqrt{3} + \sqrt{2})$ in its simplest form is $\cdots\cdots$
6. If $x = 2 + \sqrt{5}$ and y is the conjugate number of x , then $(x - y)^2 = \cdots\cdots$
7. If $\frac{x}{5 - \sqrt{5}} = 5 + \sqrt{5}$, then the value of x in its simplest form is $\cdots\cdots$
8. If $\frac{1}{x} = \sqrt{5} - 2$, then the value of x in its simplest form is $\cdots\cdots$
9. If $x = \sqrt{3} + 2$, $y = \sqrt{3} - 2$, then $(x y , x + y) = \cdots\cdots$
10. $(\sqrt{2} + \sqrt{3})^{-9} (\sqrt{2} - \sqrt{3})^{-9} = \cdots\cdots$

22 **In each of the following , if a and b are two integers , find the value of each of them :**

1. $\frac{11}{2\sqrt{5} + 3} = a\sqrt{5} + b$ «2 , −3»
2. $\frac{3}{2\sqrt{2} - \sqrt{5}} = a\sqrt{2} + b\sqrt{5}$ «2 , 1»
3. $\frac{7}{\sqrt{8} + 1} = a + b\sqrt{2}$ «−1 , 2»

23 **Simplify each of the following :**

1. $\dfrac{4}{\sqrt{5}+\sqrt{3}}+\dfrac{4}{\sqrt{5}-\sqrt{3}}$ « $4\sqrt{5}$ »

2. $\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ « $-4\sqrt{30}$ »

3. $\sqrt{75}-\sqrt[3]{125}+\dfrac{10}{\sqrt{3}-1}$ « $10\sqrt{3}$ »

For excellent pupils

24 If $x=\sqrt{4+\sqrt{7}}$, $y=\sqrt{4-\sqrt{7}}$
, find in the simplest form : $(x+y)^2$ « 14 »

---

25 If $x=\sqrt{5}+1$ and $y=\sqrt{5}-1$, find the value of : $x\,y^{-1}+y\,x^{-1}$ « 3 »

---

26 If $\dfrac{x}{y}=\sqrt{3}-\sqrt{2}$, find the value of $\dfrac{3x^2+3y^2}{xy}$ « $6\sqrt{3}$ »

---

27 If $x=\sqrt{7}+\sqrt{6}$ and $y=\sqrt{7}-\sqrt{6}$, find the value of : $\dfrac{x^8y^9-y}{(x+y)^5}$ « zero »

EXERCISE **8**

Operations on the cube roots

Interactive test

From the school book

Remember   Understand   Apply   Problem Solving

1 Put each of the following in the form $a\sqrt[3]{b}$ where a and b are two integers , b is the least possible positive value :

1) $\sqrt[3]{16}$
2) $\sqrt[3]{-54}$
3) $2\sqrt[3]{250}$
4) $\frac{2}{3}\sqrt[3]{-135}$
5) $3\sqrt[3]{\frac{1}{3}}$
6) $-10\sqrt[3]{\frac{2}{5}}$

2 Find the result of each of the following in its simplest form :

1) $\sqrt[3]{2}\times\sqrt[3]{32}$
2) $\frac{\sqrt[3]{72}}{\sqrt[3]{9}}$
3) $\frac{4\sqrt[3]{-54}}{2\sqrt[3]{-2}}$
4) (From the school book) $\frac{1}{2}\sqrt[3]{10}\times 6\sqrt[3]{100}$
5) (From the school book) $\sqrt[3]{\frac{2}{5}}\times\sqrt[3]{\frac{4}{25}}$
6) (From the school book) $\sqrt[3]{\frac{3}{4}}\div\sqrt[3]{\frac{2}{9}}$

3 Find the result of each of the following in its simplest form :

1) $\sqrt[3]{16}-\sqrt[3]{2}$ « $\sqrt[3]{2}$ »
2) (From the school book) $\sqrt[3]{125}-\sqrt[3]{24}$ « $5-2\sqrt[3]{3}$ »
3) $\sqrt[3]{81}+\sqrt[3]{-24}$ « $\sqrt[3]{3}$ »
4) $\sqrt[3]{54}+\sqrt[3]{16}-\sqrt[3]{250}$ « zero »
5) $2\sqrt[3]{54}-5\sqrt[3]{2}+\sqrt[3]{16}$ « $3\sqrt[3]{2}$ »
6) $\sqrt[3]{16}-\frac{1}{3}\sqrt[3]{54}+\sqrt[3]{-2}$ « zero »
7) $\sqrt[3]{16}+\sqrt[3]{10}\times\sqrt[3]{25}$ « $7\sqrt[3]{2}$ »
8) (From the school book) $\sqrt[3]{24}-6\sqrt[3]{13\frac{8}{9}}$ « $-8\sqrt[3]{3}$ »

4 **Prove that :**

1. $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=\text{zero}$
2. $\sqrt[3]{54}\times\sqrt[3]{16}\div\left(\sqrt[3]{4}\times 6\right)=1$

5 **Simplify each of the following to the simplest form :**

1. $\sqrt[3]{81}+\sqrt[3]{-24}-3\sqrt[3]{\frac{1}{9}}$ « zero »
2. $\sqrt[3]{54}+8\sqrt[3]{-\frac{1}{4}}+5\sqrt[3]{16}$ « $9\sqrt[3]{2}$ »
3. $\sqrt[3]{108}-2\sqrt[3]{4}-\sqrt[3]{\frac{1}{2}}$ « $\frac{1}{2}\sqrt[3]{4}$ »
4. $\sqrt[3]{3}-\sqrt[3]{4}\times\sqrt[3]{6}+3\sqrt[3]{\frac{1}{9}}$ « zero »

6 **Simplify each of the following to its simplest form :**

1. $\frac{7}{3}\sqrt{18}+\sqrt[3]{54}-7\sqrt{2}+\sqrt[3]{16}$ « $5\sqrt[3]{2}$ »
2. $\sqrt{27}+\frac{1}{3}\sqrt[3]{27}-9\sqrt{\frac{1}{3}}-1$ « zero »
3. $\sqrt[3]{-16}+\frac{14}{\sqrt{7}}-\sqrt{28}+\sqrt[3]{54}$ « $\sqrt[3]{2}$ »
4. $\sqrt{18}+\sqrt[3]{54}-\frac{\sqrt{216}}{\sqrt{12}}-\sqrt[3]{16}$ « $\sqrt[3]{2}$ »
5. $5\sqrt{2}-\frac{1}{2}\sqrt{200}+\left(\sqrt[3]{5}\times\sqrt[3]{25}\right)$ « 5 »

7 **Simplify the following to its simplest form :**

$2\sqrt[3]{16}\left(3\sqrt[3]{4}+5\sqrt[3]{32}-2\sqrt[3]{\frac{1}{2}}\right)$ « 96 »

8 **Choose the correct answer from those given :**

1. $\sqrt[3]{54}+\sqrt[3]{-2}=$ ....

(a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$

2. $\sqrt[3]{-64}+\sqrt{16}=$ ....

(a) zero (b) 8 (c) – 8 (d) ± 8

3. $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}=$ ....

(a) 8 (b) – 2 (c) 2 (d) $2\sqrt[3]{2}$

4 $\sqrt[3]{2}+\sqrt[3]{2}=$ ....

(a) $\sqrt[3]{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[3]{8}$ (d) $\sqrt[3]{16}$

5 $\sqrt[3]{\frac{2}{9}}=$ ....

(a) $\frac{\sqrt[3]{6}}{3}$ (b) $\sqrt[3]{\frac{1}{6}}$ (c) $\sqrt[3]{6}$ (d) $\sqrt[3]{2}$

9 Complete the following :

1 $\sqrt[3]{\frac{2}{3}}\times\sqrt[3]{-12}=$ ....

2 $\sqrt[3]{3}\times\sqrt[3]{9}=\sqrt{\text{..........}}$

3 $\sqrt[3]{54}-\sqrt[3]{-16}=\sqrt[3]{\text{..........}}$

4 $\frac{1}{2}\sqrt[3]{56}-\sqrt[3]{\frac{7}{27}}=$ ....

5 If $x=2$, $y=\sqrt[3]{-16}$, then $\left(\frac{x}{y}\right)^3=$ ....

6 $\frac{\sqrt[3]{250}-\sqrt[3]{16}}{\sqrt[3]{54}}=$ ....

10 If $a=\sqrt[3]{5}+1$, $b=\sqrt[3]{5}-1$, find the value of each of the following :

1 $(a-b)^5$

2 $(a+b)^3$ « 32 , 40 »

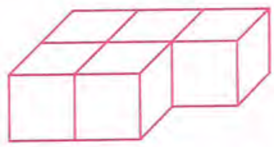
11 If $x=3+\sqrt[3]{6}$, $y=3-\sqrt[3]{6}$, find the value of : $\left(\frac{x-y}{x+y}\right)^3$ « $\frac{2}{9}$ »

12 Find the result of the following in its simplest form :

$\sqrt[3]{32}+4\sqrt[3]{\frac{1}{2}}-\left(2\sqrt[3]{-2}\right)^2+\left(\sqrt{2}\right)^{\text{zero}}-\left(\frac{2}{\sqrt{2}}\right)^2$ « – 1 »

Life Application

13 The opposite figure represents a number of cubic boxes ,
the volume of each one is 24 dm^3.
Find the area of the ground for putting the boxes.



« $20\sqrt[3]{9}$ dm^2 »

For excellent pupils

14 If $x=\sqrt[3]{2}+1$, $y=\sqrt[3]{2}-1$, prove that : $x^2+y^2=2\sqrt[3]{4}+2$

15 Make the denominator of $\frac{2}{\sqrt[3]{2}}$ a rational number.

EXERCISE 9 ?

Applications on the real numbers

Interactive test

From the school book

Remember Understand Apply Problem Solving

The cube

1 **Complete the following :**

1 If the edge length of a cube is 5 cm. , then its volume = ... cm^3.

2 The edge length of a cube is 4 cm. , then its total area = ... cm^2.

3 The lateral area of a cube whose edge length is ℓ cm. is .... cm^2.

4 The cube whose volume is ℓ^3 cm^3. , its total area = ... cm^2.

5 The cube whose edge length is 2ℓ cm. , then its volume = ... cm^3.

2 **A cube whose lateral area is 36 cm^2. Find :**

1 Its total area. 2 Its volume. « 54 cm^2. , 27 cm^3. »

3 **The perimeter of one face of a cube is 12 cm. Find :**

1 Its volume. 2 Its lateral area. « 27 cm^3. , 36 cm^2. »

4 **The sum of lengths of all edges of a cube is 60 cm. Find :**

1 Its volume. 2 Its total area. « 125 cm^3. , 150 cm^2. »

5 **Choose the correct answer from those given :**

1 The volume of a cube is 1 cm^3. , then the sum of its edge lengths = ... cm.

(a) 1 (b) 6 (c) 8 (d) 12

2 The volume of a cube is 64 cm^3, then its lateral area = ... cm^2.

(a) 4 (b) 8 (c) 64 (d) 96

3 A cube of volume 27 cm^3, then its total area = ... cm^2.

(a) 9 (b) 27 (c) 36 (d) 54

4 If the total area of a cube is 96 cm^2, then the area of one face = ... cm^2.

(a) 16 (b) 64 (c) 24 (d) 48

5 A cube of total area 150 cm^2, then its lateral area = ... cm^2.

(a) 25 (b) 100 (c) 125 (d) 150

6 If the area of the six faces of a cube = 54 cm^2, then its volume = ... cm^3.

(a) 54 (b) 44 (c) 72 (d) 27

7 If the volume of a cube = 64 cm^3, then the length of a diagonal of one face = ... cm.

(a) 16 (b) $4\sqrt{2}$ (c) 32 (d) 64

8 The volume of a cube is 5 cm^3. If the edge length became twice the first , then its volume = ... cm^3.

(a) 10 (b) 20 (c) 30 (d) 40

9 The edge length of a cube whose volume is $2\sqrt{2}$ cm^3 is .... cm.

(a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

The cuboid

6 The dimensions of the base of a cuboid are 9 cm. and 10 cm. and its height is 5 cm. Find :

1 Its volume. 2 Its lateral area. 3 Its total area.

« 450 cm^3, 190 cm^2, 370 cm^2 »

7 The dimensions of a cuboid are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. and $\sqrt{6}$ cm. Find its volume. « 6 cm^3 »

8 The dimensions of the base of a cuboid are $\sqrt{3}$ cm. and $\left(\sqrt{3}-1\right)$ cm. and its height equals $\left(3+\sqrt{3}\right)$ cm. Calculate its volume. « 6 cm^3 »

9 The lateral area of a cuboid is 480 cm^2 and its base is in the shape of a square whose side length is 10 cm. Calculate its height. « 12 cm. »

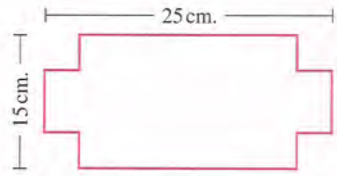
10 Find the total area of a cuboid whose volume is 720 cm^3 and its height is 5 cm. with a squared-shape base. « 528 cm^2 »

11 **Which is more in size :**

A cube whose total area is 294 cm^2 or a cuboid with dimensions $7\sqrt{2}$ cm. , $5\sqrt{2}$ cm. and 5 cm. ?

12 **In the opposite figure :**

A rectangular piece of cardboard has a length of 25 cm. and a width of 15 cm. A square whose side length = 4 cm. was cut from each of its four corners , then the projected parts were folded to form a basin in the shape of a cuboid.



Find the volume and the total area of that cuboid. « 476 cm^3 , 311 cm^2 »

The circle

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

13 A circle is of radius length 10.5 cm. Find each of its circumference and its area.

« 66 cm. , 346.5 cm^2 »

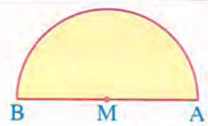
14 The area of a circle is 154 cm^2. Find its circumference and its diameter length.

« 44 cm. , 14 cm. »

15 A circle whose area is 64π cm^2. Find the length of its radius , then find its circumference approximating it to the nearest integer. ($\pi = 3.14$) « 8 cm. , 50 cm. »

16 **In the opposite figure :**

$\overline{AB}$ is a diameter of the semicircle. If the area of this region is 12.32 cm^2 , find the perimeter of the figure.

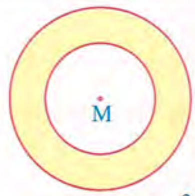


« 14.4 cm. »

17 **In the opposite figure :**

These are two concentric circles at M and their radii lengths are 3 cm. and 5 cm.

Find the area of the shaded part in terms of π



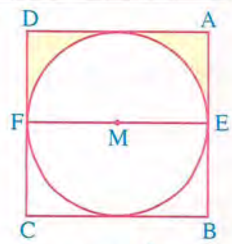
« 16π cm^2 »

18 **In the opposite figure :**

The circle M is inside the square ABCD

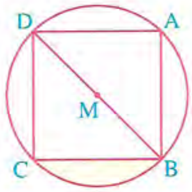
If the area of the shaded part = $10\frac{5}{7}$ cm^2 ,

find the perimeter of this part.



« $35\frac{5}{7}$ cm. »

19 **In the opposite figure :**



The square ABCD is inside the circle M

If the radius length of the circle M is 7 cm. ,

find the area of the shaded part and its perimeter. « 14 cm^2 , $(11 + 7\sqrt{2})$ cm. »

The right circular cylinder

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

20 A right circular cylinder , the radius length of its base is 14 cm. and its height is 20 cm.

Find the volume and the total area of the cylinder. « 12320 cm^3 , 2992 cm^2 »

21 Find the lateral area for a right circular cylinder of volume 924 cm^3 and of a height 6 cm.

« 264 cm^2 »

22 Find the total area of a right circular cylinder of volume 7536 cm^3 and its height is 24 cm.

(π = 3.14) « 2135.2 cm^2 »

23 **Which is more in volume :**

A right circular cylinder with base radius length 7 cm. and its height = 10 cm.

or a cube whose edge length is equal to 11 cm. ?

24 **Complete the following :**

1. A right circular cylinder whose base radius length is r cm. and its height = h cm. , then its lateral area = ... cm^2 and its volume = ... cm^3
2. A right circular cylinder with volume 40 π cm^3 and its height = 10 cm. , then its base radius length = ...
3. A right circular cylinder with volume 500 π cm^3 and its base radius length = 5 cm. , then its height = ...
4. A right circular cylinder with volume πr^3 cm^3 , then its height = ...
5. If the lateral area of a right circular cylinder is $2 \pi r^2$ cm^2 , then its height = ...

25 The circumference of the base of a right circular cylinder is 44 cm. and its height = 25 cm.

Find its volume. « 3850 cm^3 »

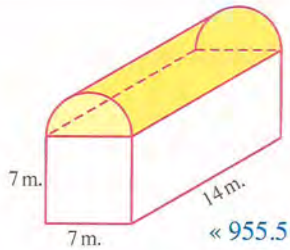
26 The lateral area of a right circular cylinder is 52 cm^2 and the length of the diameter of its base is 8 cm. Find its volume. « 104 cm^3 »

27 A right circular cylinder of volume 36 π cm^3 and height 4 cm. , the radius length of its base equals the edge length of a cube.
Find : The total area of the cube. « 54 cm^2 »

28 Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 72 π cm^3 « $2\sqrt[3]{9}$ cm. »

29 **In the opposite figure :**

A cuboid-shaped water tank with dimensions 7 m. , 7 m. and 14 m. and the upper part of it is in the form of half of a right circular cylinder.
Calculate the volume of the tank in m^3



« 955.5 m^3 »

30 A piece of paper has a shape of a rectangle ABCD in which AB = 10 cm. and BC = 44 cm. It was folded to form a right circular cylinder such that $\overline{AB}$ is coincident to $\overline{DC}$
Find the volume of the resulted cylinder. « 1540 cm^3 »

The sphere

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

31 Find the volume and the surface area of a sphere if the length of its diameter is 4.2 cm.
« 38.808 cm^3 , 55.44 cm^2 »

32 The volume of a sphere is 4188 cm^3 Find its radius length. (π = 3.141) « 10 cm. »

33 The volume of a sphere is 562.5 π cm^3
Find its surface area in terms of π « 225 π cm^2 »

34 **Choose the correct answer from those given :**

1 The volume of the sphere = ...

(a) $4 \pi r^2$ (b) $\frac{4}{3} \pi r^3$ (c) $\frac{3}{4} \pi r^3$ (d) $\frac{4}{3} \pi r^2$

2 The sphere whose radius length is $\sqrt[3]{3}$ cm. , its volume = ... cm^3

(a) 4π (b) $4\sqrt{3}\pi$ (c) $\frac{4}{3}\pi$ (d) $\frac{9}{4}\pi$

3 The volume of the sphere whose diameter length is 6 cm. equals .... cm^3

(a) 288 (b) 12π (c) 36π (d) 288π

4 If the volume of a sphere = $\frac{9}{16}\pi$ cm^3, then its radius length = ... cm.

(a) 3 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

5 If the surface area of a sphere is 9π cm^2, then its diameter length = ... cm.

(a) 9 (b) 3 (c) 1.5 (d) 6

6 If three quarters of the volume of a sphere equals 8π cm^3, then the length of its radius equals .... cm.

(a) 64 (b) 8 (c) 4 (d) 2

7 If the radius length of a sphere is r cm., then which of the following represents the ratio between the area of the sphere and its volume?

(a) $\frac{4}{r}$ (b) $\frac{3}{r}$ (c) $\frac{r}{4}$ (d) $\frac{r}{\pi}$

35 Find the radius length of a sphere if its volume equals the volume of a right circular cylinder whose height is 18 cm. and its base radius length is 4 cm. « 6 cm. »

36 Find the volume of a sphere if its radius length equals the radius length of a right circular cylinder with volume 7536 cm^3 and height 24 cm. ($\pi = 3.14$) « $4186\frac{2}{3}$ cm^3 »

37 A lead cuboid is of dimensions 77 cm., 24 cm. and 21 cm. It was melted to make a sphere. Find the radius length of that sphere. « 21 cm. »

38 A metallic sphere, with diameter length 6 cm. has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height. « 4 cm. »

39 A sphere with volume 36π cm^3 is placed inside a cube. If the sphere touches the six faces of the cube, **find :**

1 The radius length of the sphere.

2 The volume of the cube. « 3 cm., 216 cm^3 »

40 A metallic sphere is of radius length 16.8 cm. It is melted and it is converted to 8 small spheres which are equal in volume. Find the radius length of each small sphere. « 8.4 cm. »

41 A right circular cylinder has a height of 20 cm. Find its base radius length if its volume equals $\frac{4}{9}$ of the volume of a sphere with a diameter length of 30 cm. « 10 cm. »

For excellent pupils

42 A cuboid has a square-shaped base whose height = 3 cm. If the sum of lengths of its edges is 52 cm., find its volume. « 75 cm^3 »

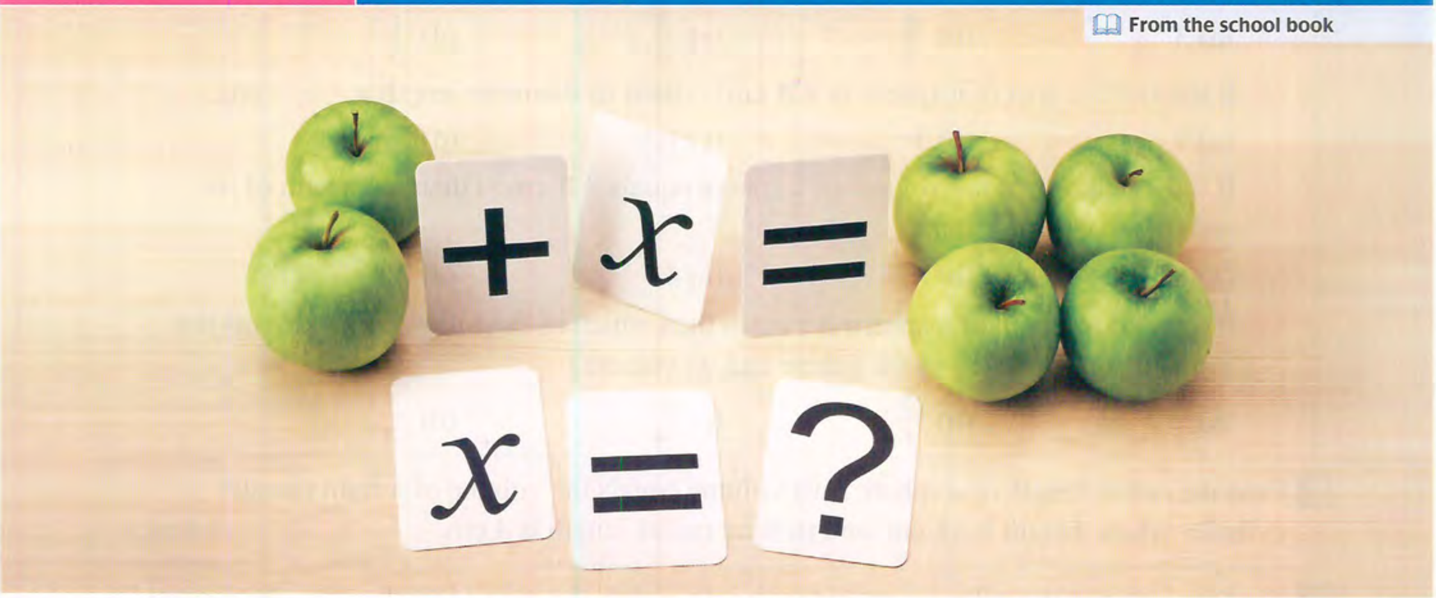
43 A hollow metal sphere is with internal radius length 2.1 cm. and external radius length 3.5 cm. Find its mass approximated to the nearest gram taking into consideration that the mass of a cubic centimetre of such a metal is 20 gm. « 2817 gm. »

EXERCISE

10 ? Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

Interactive test

📖 From the school book

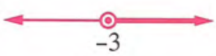


● Remember ● Understand ● Apply ● Problem Solving

1 **Find the solution set for each of the following equations in $\mathbb{R}$, then represent the solution on the number line :**

1 📖 $x + 5 = 0$	2 📖 $5x + 6 = 1$	3 📖 $2x + 4 = 3$
4 📖 $2x - 3 = 4$	5 $4x - 1 = \vert -2 \vert$	6 $\sqrt{5}\,x - 1 = 4$
7 $x - 1 = \sqrt{3}$	8 $2 - \sqrt{6}\,x = \vert -8 \vert$	9 📖 $x + 2\sqrt{3} = 3$

2 **Choose the correct answer from those given :**

1 The figure  represents the solution set of the inequality ·········· in $\mathbb{R}$

(a) $x > -3$ (b) $x \geq -3$ (c) $x < -3$ (d) $x \leq -3$

2 The figure  represents the solution set of the inequality ·········· in $\mathbb{R}$

(a) $-6 < x < 6$ (b) $-6 \leq x < 6$ (c) $-6 < x \leq 6$ (d) $-6 \leq x \leq 6$

3 If $x \in]3 , \infty[$, then ··········

(a) $x < 3$ (b) $x \leq 3$ (c) $x > 3$ (d) $x \geq 3$

4 The S.S. of the inequality : $x > 7$ in $\mathbb{R}$ is ··········

(a) $]-7 , \infty[$ (b) $[7 , \infty[$ (c) $]-\infty , 7[$ (d) $]7 , \infty[$

5 The S.S. of the inequality : $-1 < x \leq 5$ in $\mathbb{R}$ is ....

(a) $]-1 , 5]$ (b) $[-1 , 5]$ (c) $\{-1 , 5\}$ (d) $[-1 , 5[$

6 The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is ....

(a) $\{-3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$

3 Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $2x > 6$
2 $-7x \geq -14$
3 $x + 3 \leq 5$
4 $5 - x > 3$
5 $2x + 5 \geq 3$
6 $1 - 5x < 6$
7 $\frac{1}{2}x + 1 \leq 2$
8 $3 - 2x \leq 7$

4 Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $3 < x + 2 \leq 6$
2 $-5 < x + 3 < 9$
3 $-3 \leq -x < 3$
4 $1 < 5 - x \leq 3$
5 $\sqrt[3]{-8} \leq x + 1 \leq \sqrt{9}$
6 $5 < 3 - x \leq 3^2$
7 $-8 \leq 3x + 1 \leq 4$
8 $|-3| < 2x - 1 < 5$
9 $-3 < \frac{1}{2}x - 2 \leq$ zero
10 $0 \leq \frac{-2x + 6}{3} < 4$

5 Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $3x < 2x + 4$
2 $7x - 9 \geq 4x$
3 $5x - 3 < 2x + 9$
4 $7x - 12 \geq 5x - 8$
5 $x - 1 \leq 3 - x$
6 $1 - x \geq -2x - 3$

6 Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :

1 $x + 3 \geq 2x \geq x - 2$
2 $-x < x < 4 - x$
3 $4x \leq 5x + 2 < 4x + 3$
4 $x - 1 < 3x - 1 \leq x + 1$
5 $2 + 2x \leq 3x + 3 < 5 + 2x$
6 $\frac{3x - 4}{6} < x + 1 < \frac{x + 3}{2}$

7 Complete the following :

1 If $x - 3 \geq 0$, then x ....
2 If $5x < 15$, then x ....
3 If $1 - x > 4$, then x ....
4 If $-2x \leq 3$, then x ....

5 If $\sqrt{2}\,x \le 4$, then x ....

6 The S.S. of the inequality : $4 < 2x < 8$ in $\mathbb{R}$ is ....

7 The S.S. of the inequality : $-5 \le -x < 2$ in $\mathbb{R}$ is ....

8 The S.S. of the inequality : $2 - x < 0$ in $\mathbb{R}$ is ....

9 If $-3 < x < 3$ where $x \in \mathbb{R}$, then $2x \in]-6$, ... [

8 Choose the correct answer from those given :

1 The S.S. of the inequality : $x + 3 < 3$ in $\mathbb{R}$ is ....

(a) $]-\infty , 0[$ (b) $]-\infty , 0]$ (c) $[0 , \infty[$ (d) $]0 , \infty[$

2 The S.S. of the inequality : $1 > x - 5 > -1$ in $\mathbb{R}$ is ....

(a) $[4 , 6]$ (b) $]4 , 6[$ (c) $]4 , 6]$ (d) $[4 , 6[$

3 If $x > 5$, then $-x$ ....

(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

4 If $-2 < x < 2$, then $2x + 3$ belongs to ....

(a) $[-1 , 7]$ (b) $]-1 , 5[$ (c) $]-1 , 7[$ (d) $]-4 , 6[$

5 The number 5 belongs to the S.S. of the inequality ....

(a) $x > 5$ (b) $x < 5$ (c) $-x \ge -5$ (d) $-x \ge 5$

Life Application

9 A lift for carrying goods can carry 2200 kg. as a maximum weight. If we have 60 boxes of cans and the weight of one box is 45 kg. , what is the maximum number of boxes can the lift carry in one time without carrying any person ? « 48 boxes »

For excellent pupils

10 Prove that $\sqrt{3}$ belongs to the S.S. of the inequality : $0 < 4 - 2x < 6$ in $\mathbb{R}$

11 If $[4 , 7]$ is the S.S. of the inequality : $a \le x - 3 \le b$, find the value of each of a and b « 1 , 4 »

12 If $[m , m + n]$ is the S.S. of the inequality : $\frac{1}{5} \le \frac{2x + 1}{5} \le 1$, find the value of n « 2 »

13 If $5 \le \frac{2x}{3} + 1 \le 7$, find the smallest value of the expression : $x - 2$ « 4 »

14 Find in $\mathbb{R}$ the S.S. of the inequality : $\frac{x}{\sqrt{3} - \sqrt{5}} \ge \sqrt{3} + \sqrt{5}$

A Research Project

On Unit One

Project aims :

- Performing arithmetic operations on real numbers.
- Finding volumes as applications on operations on real numbers.
- Associating mathematics with science.

Do a research project on the following topic :

"The planet Earth where we live on is one of the solar system planets, which consists of eight planets rotating around the sun".

Discuss the following points using available resources :

1. State the names of the solar system planets.
2. Find the radius length of each planet in the solar system and calculate its volume.
3. Arrange the solar system planets in a descending order according to their volumes.
4. Find out the weight of a body on the surface of the Earth in the simplest form if its weight on the surface of the moon is $30\sqrt{18}$ kg.

UNIT 2

Relation between Two Variables

Exercises of the unit :

Scan the QR code to solve an interactive test on each lesson

EXERCISE 11

Relation between two variables

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 Complete the following ordered pairs which satisfy the relation : $y = 3x - 1$

$(5 , ...)$, $(2 , ...)$, $(0 , ...)$, $(-3 , ...)$

2 Show which of the following ordered pairs satisfies the relation : $y - 4x = 7$

1. $(1 , 2)$
2. $(3 , -5)$
3. $(-1 , 3)$

3 Find four ordered pairs satisfying each of the following relations :

1. $2x - y = 5$
2. $y = \frac{1}{2}x + 5$
3. $y = 2$
4. $2x = 5$

4 Using the linear relations , complete the following tables :

1. $4x - y = -1$

x	0	1	2	3
y	....	....	....	....

2. $y = 5x + 15$

x	-4	-3	-2
y	....	....	....

3. $a - b = 4$

a	1	....	....
b	....	0	-1

4. $a - 3b = 5$

a	2	....	-1
b	....	0	....

5 **If $y - 2x = 1$, find :**

1. y at $x = 3$
2. y at $x = -5$
3. x at $y = 1$
4. x at $y = -1$

6 If $(3 , 6)$ satisfies the relation : $y = kx$, find the value of k « 2 »

7 If $(3 , 1)$ satisfies the relation : $y - 3x = a$, find the value of a « – 8 »

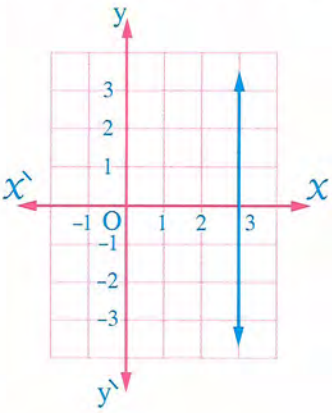
8 Find the value of b , where $(-3 , 2)$ satisfies the relation : $3x + by = 1$ « 5 »

9 If $(3 , a)$ satisfies the relation : $y - 2x = 4$, find the value of a « 10 »

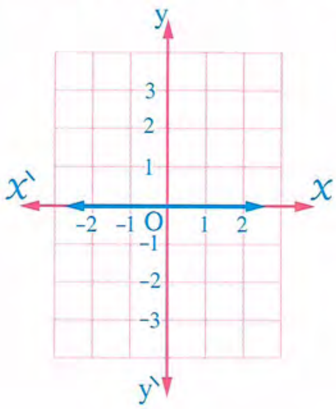
10 Find the value of k , where $(k , 2k)$ satisfies the relation : $x + y = 15$ « 5 »

11 **Find the relation that is represented by the line in each figure below :**

1.



2.



12 **Represent graphically each of the following relations :**

1. $x + y = 2$
2. $x - y = 3$
3. $x + 2y = 3$
4. $y - 3x = 1$
5. $y = -2x$
6. $y - 2x = -1$
7. $2x = 5$
8. $y + 1 = 0$

13 Graph the relation : $2x + 3y = 6$, if the straight line representing this relation intersects the x-axis at the point A and the y-axis at the point B , find the area of the triangle OAB where O is the origin point. « 3 square units »

14 If the straight line which represents the relation : $2x - y = a$ intersects the x-axis at the point $(3 , b)$, find a and b «6 , 0»

15 **Choose the correct answer from those given :**

1 Which of the following ordered pairs satisfies the relation : $2x + y = 5$?

(a) $(-1 , 3)$ (b) $(1 , 3)$ (c) $(3 , 1)$ (d) $(2 , 2)$

2 $(3 , 2)$ does not satisfy the relation ....

(a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$

3 The relation : $5x = 7y$ is represented by a straight line passes through the point ....

(a) $(5 , 7)$ (b) $(0 , 0)$ (c) $(5 , 0)$ (d) $(0 , 7)$

4 The point $(3 , 5)$ lies on the straight line which represents the relation ....

(a) $y = 3x - 5$ (b) $2x - y = 1$ (c) $3x + y = 1$ (d) $y = 3x - 1$

5 If $(2 , -5)$ satisfies the relation : $3x - y + c = 0$, then $c =$ ....

(a) 1 (b) -1 (c) 11 (d) -11

6 If $(-1 , 5)$ satisfies the relation : $3x + ky = 7$, then $k =$ ....

(a) 2 (b) -2 (c) 1 (d) 10

7 Which of the following relations is represented by a straight line parallel to the y-axis ?

(a) $y = -5$ (b) $x = -5$ (c) $x = y$ (d) $x + y = 0$

8 Which of the following relations is represented by a straight line parallel to the x-axis ?

(a) $2y = 6$ (b) $2x = 6$ (c) $x = -y$ (d) $x - y = 0$

9 Which of the following relations is represented by a straight line passes through the origin point ?

(a) $y = 5$ (b) $x = -3$ (c) $y = x + 2$ (d) $y = 3x$

10 The relation : $3x + 8y = 24$ is represented by a straight line intersecting the y-axis at the point ....

(a) $(0 , 8)$ (b) $(8 , 0)$ (c) $(0 , 3)$ (d) $(3 , 0)$

11 The relation : $2x + 7y = 14$ is represented by a straight line intersecting the x-axis at the point ....

(a) $(2 , 0)$ (b) $(0 , 2)$ (c) $(7 , 0)$ (d) $(0 , 7)$

12 The opposite table represents the relation between x and y , which of the following expresses this relation ?

x	1	2	3	4
y	− 2	− 5	− 8	− 11

(a) $x + y = -1$ (b) $x - y = 3$ (c) $3x + y = 1$ (d) $y = -x - 3$

13 The opposite table shows the relation between x and y , which is ....

x	1	2	3	4	5
y	1	3	5	7	9

(a) $y = x + 4$ (b) $y = x + 1$
(c) $y = 2x - 1$ (d) $y = 3x - 2$

14 The relation which expresses the two ordered pairs (2 , 1) and (4 , 3) together is ....

(a) $y = \frac{1}{2}x$ (b) $y = 2x - 5$ (c) $y = x - 1$ (d) $y = 3x + 3$

16 Two even natural numbers , twice the first plus the second equals 12

Find the different possibilities of the two numbers.

Geometric Application

17 The perimeter of a rectangle is 14 cm. What are the different possibilities of the length and the width given that each of them belongs to $\mathbb{Z}_+$?

Life Applications

18 Essam has 10 bills of L.E. 5 and other bills of L.E. 20 He bought some goods from a shopping centre for L.E. 65 Determine the different possibilities to pay this amount of money. Find the relation and graph it.

19 The selling price of a computer table is L.E. 100 and its chair is L.E. 50 If the store sells in one week with L.E. 500 , what are the represented expectations to the number of sold computer tables and chairs ? Represent the relation graphically.

For excellent pupils

20 The perimeter of an isosceles triangle is 19 cm. What are the different possible lengths of its sides given that its sides lengths $\in \mathbb{Z}_+$?

Notice that : The sum of the lengths of any two sides of the triangle is greater than the length of the third side.

EXERCISE **12?**

Slope of straight line

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 **Classify the slope of the straight line in each of the following figures showing whether it is (positive – negative – zero – undefined) :**

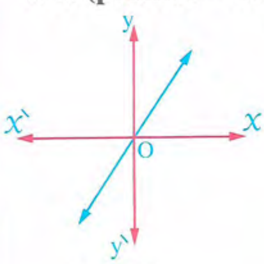


Fig. (1)

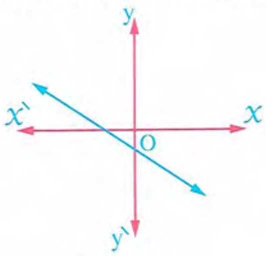


Fig. (2)

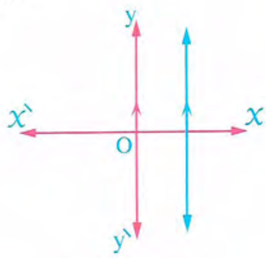


Fig. (3)

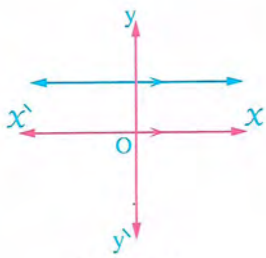
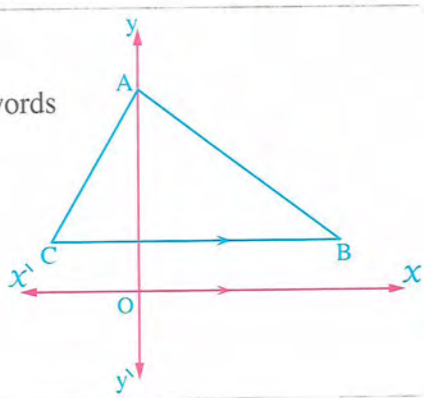


Fig. (4)

2 **In the opposite figure :**

ABC is a triangle. Complete by using one of the following words (positive , negative , zero , undefined)



1. The slope of $\overleftrightarrow{AB}$ is ....
2. The slope of $\overleftrightarrow{BC}$ is ....
3. The slope of $\overleftrightarrow{AO}$ is ....
4. The slope of $\overleftrightarrow{AC}$ is ....

3 **Complete the following :**

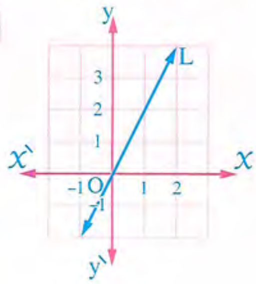
1. The slope of any horizontal straight line equals ....
2. The slope of any straight line parallel to y-axis is ....
3. The straight line whose slope = zero is parallel to ....
4. If A , B and C are collinear , then the slope of $\overleftrightarrow{AB}$ = the slope of ....

4 **Find the slope of the straight line passing through the two points in each of the following :**

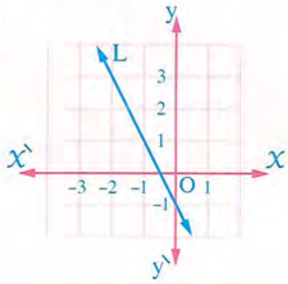
1. A (1 , 3) , B (3 , 4)
2. A (1 , 2) , B (5 , 0)
3. A (3 , 2) , B (6 , 5)
4. A (2 , – 1) , B (4 , – 1)
5. A (1 , 3) , B (2 , 3)
6. A (5 , 2) , B (5 , 4)
7. A (3 , – 1) , B (3 , 2)
8. A (3 , – 2) , B (4 , 1)
9. A (– 1 , 3) , B (2 , 1)
10. N (4 , – 2) , K (– 1 , – 7)
11. E (– 3 , – 1) , O (0 , 0)
12. A (– 6 , – 9) , B (– 1 , – 1)

5 **Find the slope of the straight line L in each of the following graphs :**

1



2



6 **In the opposite figure :**

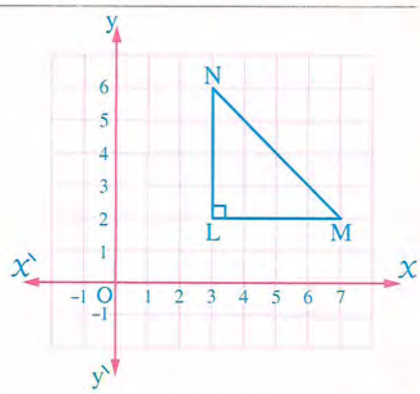
LMN is a right-angled triangle at L

, where m (∠ M) = 45°

Given that L (3 , 2) and M (7 , 2)

, find the coordinates of N

and calculate the slope of $\overleftrightarrow{MN}$



7 If A (2 , – 1) , B (10 , 3) and C (2 , 3) , find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CA}$

Draw the triangle ABC on a square grid , then mention the type of the triangle according to the measures of its angles.

8 If the slope of the straight line which passes through the two points (1 , 3) and (3 , k) equals 3 , find the value of k « 9 »

9 If the slope of the straight line which passes through the two points (3 , c) and (5 , – 2) equals – 3 , find the value of c « 4 »

10 If A (– 1 , 4) , B (x , 2) and the slope of $\overleftrightarrow{AB}$ equals – 2

, find the value of x « zero »

11 If the straight line which passes through the two points $(-2 , y)$ and $(3 , -1)$ has a slope -0.6 , find the value of y « 2 »

12 Find the value of k such that the straight line passing through the two points $(3 , 4)$ and $(2 , k)$ is parallel to X-axis. « 4 »

13 Find the value of X such that the straight line which passes through the two points $(2X , 3)$ and $(6 , 7)$ is parallel to y-axis. « 3 »

14 Find the value of y such that the straight line passing through the two points $(3 , 6)$ and $(-2 , 3y)$ is perpendicular to y-axis. « 2 »

15 Are the points $(-5 , 11)$, $(0 , 8)$ and $(5 , 5)$ collinear ?

16 Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$, where $A (2 , 1)$, $B (3 , 2)$ and $C (4 , 5)$ and represent each line graphically. What do you observe ?

17 **In each of the following , prove that the points A , B and C are collinear :**

1. $A (1 , 1)$, $B (2 , 2)$, $C (-3 , -3)$
2. $A (4 , -3)$, $B (-6 , 7)$, $C (5 , -4)$
3. $A (-2 , 12)$, $B (2 , 4)$, $C (6 , -4)$

18 **In each of the following , prove that the points A , B and C are not collinear :**

1. $A (2 , 1)$, $B (3 , 0)$, $C (5 , -1)$
2. $A (-1 , 2)$, $B (3 , 1)$, $C (7 , 2)$
3. $A (0 , -3)$, $B (2 , 2)$, $C (-3 , -3)$

19 Find the slope of the line $\overleftrightarrow{AB}$, where $A (-1 , 3)$ and $B (2 , 5)$

Is the point $C (8 , 1) \in \overleftrightarrow{AB}$? « $\frac{2}{3}$ »

20 Find the value of y such that the points $(4 , 1)$, $(-2 , 7)$ and $(3 , y)$ are collinear. « 2 »

For excellent pupils

21 If the straight line which passes through the points $(3 , -1)$, $(X , 1)$ and $(9 , y)$ has a slope $= \frac{2}{3}$, find the value of each of X and y « 6 , 3 »

EXERCISE 13

Real life applications on the slope

Interactive test

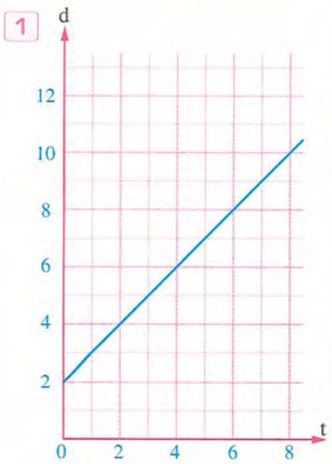
From the school book

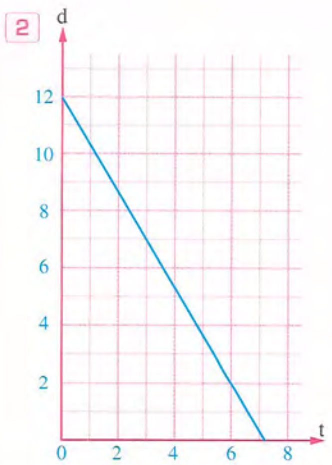
Remember   Understand   Apply   Problem Solving

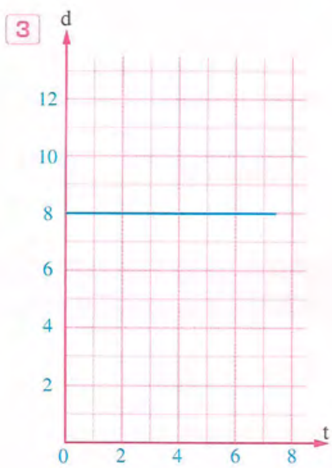
1 A car moves with uniform velocity such that it covers 180 km. per 3 hours. If the car moves for 5 hours , what is the covered distance ? « 300 km. »

2 An irrigation machine consumes 2.47 litres of diesel to work for 3 hours. If the machine works for 10 hours , how many litres of diesel will the machine consume ? « $8\frac{7}{30}$ litres »

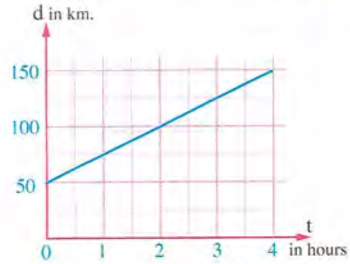
3 The following diagrams show the relation between the covered distance (in m.) and the elapsed time (in sec.) of an object. Determine the position of the object at the starting of motion and its position after 6 seconds (when t = 6 sec.) Find the slope of the line in each case and state what it represents.







4 The opposite graph represents the motion of a car moving with uniform velocity.
Determine the velocity of the car.



« 25 km./hr. »

5 Bassem drove his car from the city A to the city B
The opposite graph shows the relation between the distance d in km. and the time t in hours.

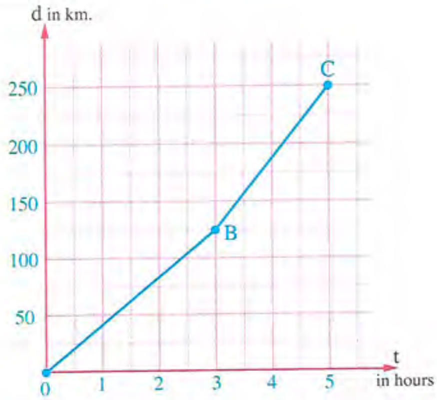
Answer the following :

1. What is the uniform velocity of the car of Bassem ?
2. Find the distance between the car and the point 0 after three hours from the moment of beginning.

« 75 km./hr. , 275 km. »

6 **The opposite graph represents the motion of a car :**

1. Find the velocity of the car within the first three hours from the beginning , then find the velocity within the next two hours.
2. Find the average velocity of the car within the total time.

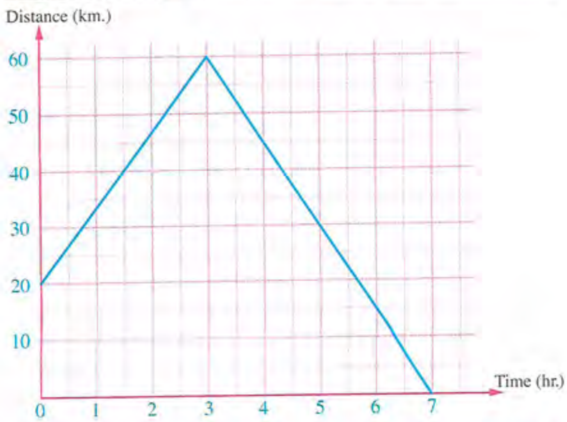


« $41\frac{2}{3}$ km./hr., $62\frac{1}{2}$km./hr., 50 km./hr. »

7 **The opposite figure represents the motion of a bicycle measured from a constant point.**
Find the regular velocity of the bicycle during :

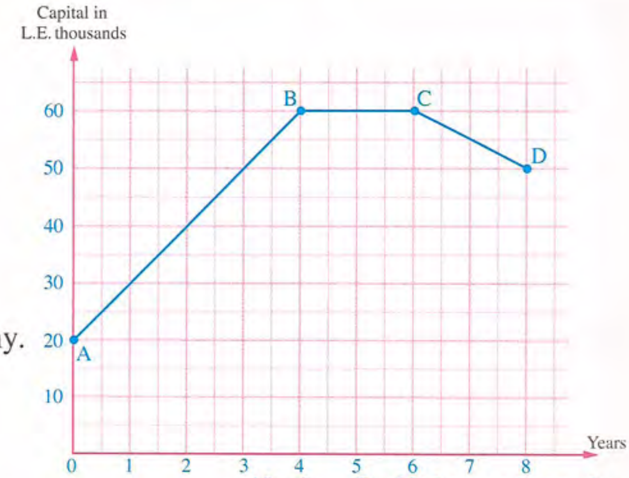
1. The first three hours.
2. The next four hours.

Find the total distance covered by the bicycle.



« $13\frac{1}{3}$km./hr., 15 km./hr. , 100 km. »

8 **The opposite figure shows the capital change of a company during 8 years :**



1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

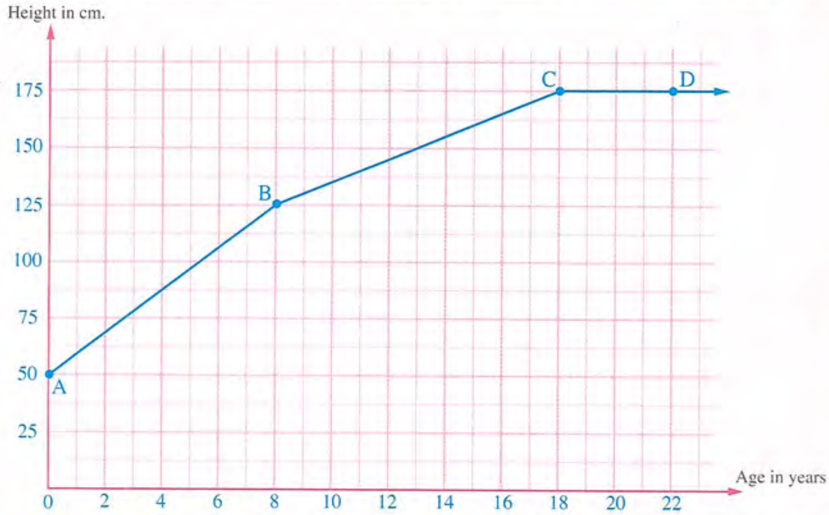
   What is the meaning of each ?

2. Find the starting capital of the company.

« 10 , 0 , – 5 , 20 thousand pounds »

---

9 **The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :**



1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

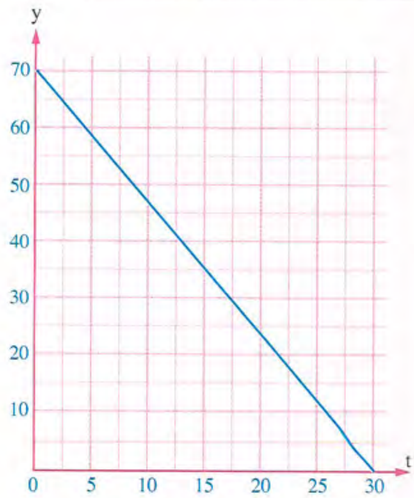
   What is the meaning of each ?

2. Calculate the difference between the height of this person when he was 8 years old and his height when he was 30 years old.

« $9\frac{3}{8}$, 5 , 0 , 50 cm. »

---

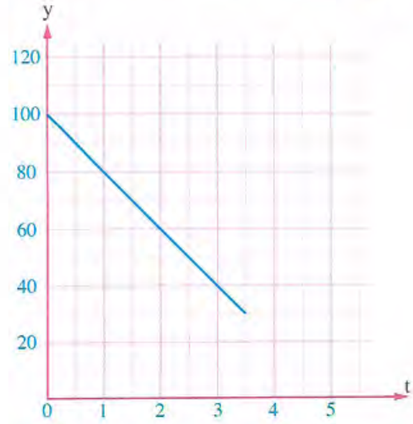
10 **Magdi filled the tank of his car by fuel. The opposite figure represents the relation between the time (t) in hours and the amount of remained fuel in the tank (y) in litres :**



1. What is the greatest capacity of the tank ?
2. When will the tank become empty ?
3. What is the amount of remained fuel after 15 hours ?
4. What is the range of consumption of fuel in each hour ?

« 70 L , 30 hr. , 35 L , $2\frac{1}{3}$ L /hr. »

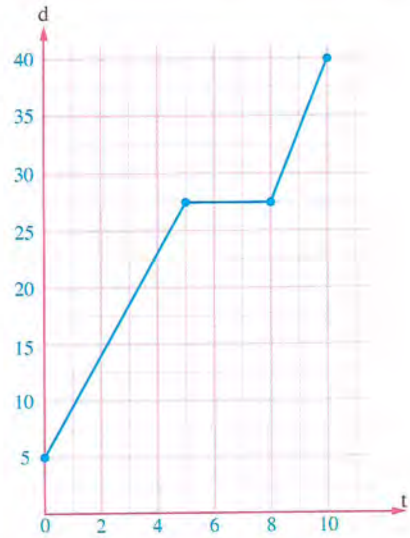
11 **A person read a book.**
The opposite graph shows the relation between the time (t) in hours and the number of remained pages (y) :



1. How many pages are remained in the beginning ?
2. Find the rate of reading pages per hour.
3. When does this person finish reading this book ?

« 100 pages , 20 pages/hr. , after 5 hours »

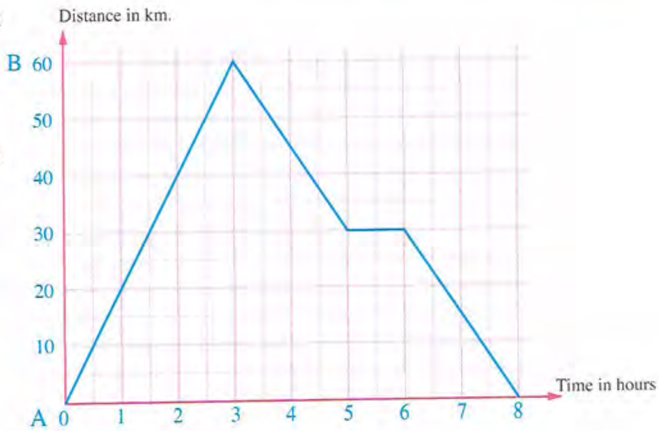
12 **A farmer wanted to complete digging a well in his farm. He rented a digging machine. The opposite graph shows the depth of the well (d) in metres after time (t) in hours , find :**



1. The depth of the well before beginning digging.
2. The depth of the well after finishing digging.
3. The total time which the machine took in digging the well.
4. The average of depth of the well which the machine digs within the first five hours.
5. The average of the depth of the well within the last two hours of digging.

« 5 m. , 40 m. , 10 hr. , 4.5 m./hr. , 6.25 m./hr. »

13 **The opposite graph shows the relation between the distance in km. and the time (t) in hours for a bicycle which moved between two towns A and B going and returning back.**
Answer the following :



1. What is the uniform velocity during the going trip ?
2. What is the average velocity during returning back ?
3. What is the meaning of the horizontal line segment in the graph ?

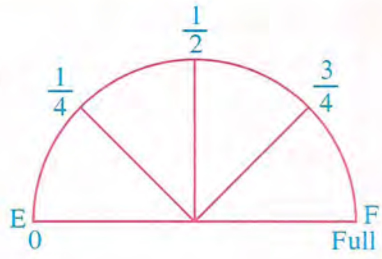
« 20 km./hr. , 12 km./hr. »

14 Hazem filled up the 40 L tank of his car.

After covering a distance of 120 km. , the fuel gauge shows that the rest of fuel is $\frac{3}{4}$ of the tank.

Draw a diagram to show the relation between the amount of fuel in the tank and the covered distance (This relation is linear).

Calculate the covered distance until the tank totally gets empty.

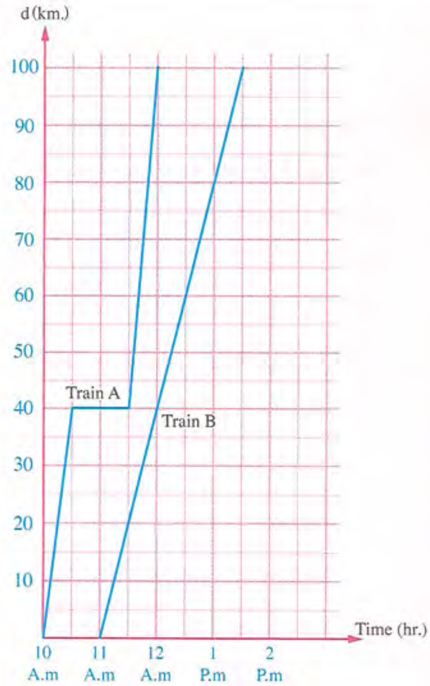


« 480 km. »

15 The opposite diagram shows the relation between the covered distance (in km.) and the elapsed time (in hr.) for two trains A and B between two railway stations.

Use the diagram to find :

1. The distance between the two railway stations.
2. The elapsed time of each train.
3. The average speed of each train.
4. The meaning of the horizontal segment in the diagram of train A

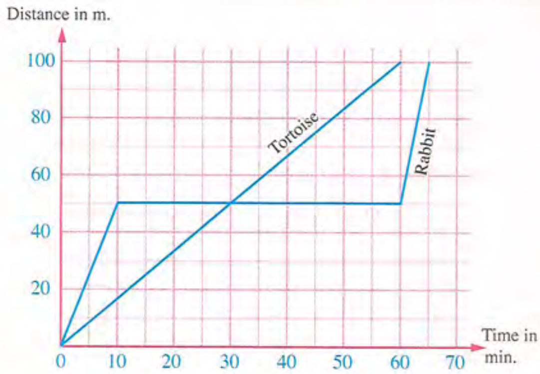


« 100 km. , 2 hr. , 2.5 hr. , 50 km./hr. , 40 km./hr. »

16 **The opposite graph shows the race of 100 metres between a rabbit and a tortoise.**

Answer the following :

1. Which of them is the winner ?
2. What is the velocity of the tortoise ?
3. What is the average velocity of the rabbit ?
4. What is the meaning of the horizontal line segment in the graph ?



« tortoise , $1\frac{2}{3}$ m./min. , $1\frac{7}{13}$ m./min. »

For excellent pupils

17 **During the motion of a bicycle with a uniform velocity in a straight line , the distances between the bicycle and a fixed point have been registered after periods measured in hours from the moment of beginning the motion in the following table :**

The distance between the bicycle and the fixed point	125	150	175	200
The passed time in hours	2	4	6	8

Graph the relation between the distance between the bicycle and the fixed point and the passed time. **From the graph , find :**

1. The velocity of the bicycle in km./hr.
2. The distance between the bicycle and the fixed point after 300 minutes.
3. The time at which the bicycle is at a distance = 187.5 km. from the fixed point.
4. The distance between the starting point of the bicycle and the fixed point.

« 12.5 km./hr. , 162.5 km. , 7 hr. , 100 km. »

A Research Project

On Unit Two

Project aims :

- Recognizing the relation between two variables of the first degree.
- Representing the relation between two variables of the first degree graphically.
- Using algebra to solve life problems.
- Associating mathematics with social studies.

Do a research project on the following topic :

"Doing sports is the start on the road of a more healthy life. Our Arab champs have achieved a lot of important achievements in many world competitions".

Discuss the following points using available resources :

1. State some achievements of our champs of Arab countries in the field of sports.

2. In football matches, a team gets three points in case of winning and one point in case of a draw. If one team scored 30 points :

   * Write the mathematical relation between (x) and (y), where (x) is the number of matches a team wins and (y) is the number of matches, the team are held to a draw. From this relation, write five different methods to score 30 points.

   * Represent this relation graphically.

UNIT 3 Statistics

Exercises of the unit :

14. Collecting and organizing data.

15. The ascending and descending cumulative frequency tables and their graphical representation.

16. Mean.

17. Median.

18. Mode.

A research project on unit three

Scan the QR code to solve an interactive test on each lesson

EXERCISE **14** ?

Collecting and organizing data

From the school book

Remember Understand Apply Problem Solving

1 **The following are the weights of 40 pupils of one class of the second year preparatory in kg. The required is forming the frequency table with sets.**
Use the subsets (25– , 30– , 35– , ...) :

36	30	42	37	25	34	35	28	30	28
29	36	38	32	44	39	34	36	35	30
30	35	30	38	27	41	33	39	31	36
36	33	37	31	43	35	40	31	39	45

2 **The following are the weekly wages of 40 workers in a factory in L.E. :**

47	71	36	94	54	64	87	89	62	57
51	61	44	52	70	66	56	32	69	36
79	48	77	90	65	99	96	67	60	55
95	75	81	84	78	38	49	94	48	59

Required : Form a frequency table with sets (use the subsets : 30– , 40– , 50– , ... , 90–)
What is the set with the highest frequency? What is the set with the lowest frequency ?

3 **The following are the scores of 30 students in a monthly math exam :**

25	35	40	20	30	37	40	33	22	38
35	36	28	37	39	28	32	26	29	37
23	34	35	36	29	38	40	35	37	31

1 From a frequency table with sets for these scores.
2 Find the total number of excellent students. The excellence rate is 36 marks or more.
« 12 students »

4 **The following are the marks of the students in a class in the second year preparatory in algebra exam. Given that their number is 40 students and the full mark is 20 marks :**

7	11	7	13	14	3	18	13	10	14
16	8	15	12	5	15	11	12	6	11
8	9	15	8	15	14	7	10	14	19
10	7	2	10	12	4	11	17	13	15

The required is forming a frequency table with sets for the marks of students in algebra using the subsets 0– , 4– , 8– , ... and so on , then find the percentage of the number of students who obtained 12 marks at least. « 47.5% »

5 **The following are the heights of 50 persons in centimetres :**

155	183	163	181	186	144	199	150	182	166
197	126	188	158	153	130	163	166	154	173
137	163	146	198	164	156	173	177	157	118
138	187	178	173	184	143	147	142	176	160
170	194	154	167	149	112	196	128	126	156

Using the previous data :

1 Find the least height in these data and the greatest height and the range in which these two heights lie. « 112 cm. , 199 cm. , 87 cm. »

2 Form a frequency table using sets of length 10 centimetres for each.

6 **In a military camp , the heights of 55 soldiers were measured in centimetres , their measures were as follows :**

169	194	200	185	165	188	166	186	181	176	173
177	179	188	170	193	180	173	173	184	192	167
182	168	186	189	171	179	172	175	175	181	166
185	177	175	165	190	172	177	178	184	166	174
178	177	172	174	175	179	195	176	189	187	189

Form a frequency table using the sets (165– , 170– , 175– , ...)

From the table , find :

1 The number of soldiers whose heights are less than 185 cm. « 39 soldiers »

2 The number of soldiers whose heights are 180 cm. at least. « 22 soldiers »

EXERCISE 15 The ascending and descending cumulative frequency tables and their graphical representation

From the school book

Remember Understand Apply Problem Solving

First Problems on the ascending cumulative frequency curve

1 **The following table shows the frequency distribution of the scores of 50 students in an experimental math exam :**

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	3	5	9	10	12	7	4	50

Graph the ascending cumulative frequency curve.

2 **The following frequency table represents the marks of 60 pupils in math :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	9	11	13	17	10	60

Graph the ascending cumulative frequency curve and if the success mark is 30 marks , find the number of failed pupils. « 20 pupils »

3 **The following table shows the frequency distribution of 100 factories according to the number of weekly work hours :**

Sets of hours	50 –	60 –	70 –	80 –	90 –	100 –	Total
Number of factories	5	16	30	22	15	12	100

1 Graph the ascending cumulative frequency curve of this distribution.

2 From the graph , find the number of factories which work less than 75 hours in the week. « 37 factories »

3 Find the percentage of the number of factories which work less than 75 hours in the week. « 37 % »

Second Problems on the descending cumulative frequency curve

4 **The following table shows the frequency distribution of the daily wages of some workers :**

Sets	5–	10–	15–	20–	25–	30–	Total
Frequency	10	14	24	30	12	10	100

Graph the descending cumulative frequency curve.

5 **A class has 50 pupils , the following table shows the distribution of studying hours among them every day :**

Sets	1 –	2 –	3 –	4 –	5 –	6 –	7 –	Total
Freq.	2	3	5	12	15	7	6	50

1 Graph the descending cumulative frequency curve of this distribution.

2 From the graph , find the number of pupils who study 6 hours or more daily.

« 13 pupils »

3 Find the percentage of the number of pupils who study 6 hours or more daily. « 26 % »

6 **The following table shows the frequency distribution of a group of 60 persons according to their weights in kg. :**

Sets of weights in kg.	55 –	60 –	65 –	70 –	75 –	80 –	85 –	Total
No. of persons	8	12	18	....	7	3	2	60

Complete the table , then graph the descending cumulative frequency curve of this distribution and from the graph , find the number of persons whose weigh 68 kg. or more for each.

« 28 persons »

Third Problems on the two curves together

7 **Graph the ascending and descending curves for the following frequency distribution :**

Sets	8–	12–	16–	20–	24–	28–	32–	36–	40–	Total
Freq.	4	7	12	18	20	19	11	6	3	100

8 **The following table shows the frequency distribution of the scores of 1000 students in a final year exam :**

Percentage	20–	30–	40–	50–	60–	70–	80–	90–	Total
Number of students	30	70	160	260	150	130	110	90	1000

1 Graph the ascending and descending cumulative frequency curves.

2 Find the number of students whose scores are less than 75% « 740 students »

3 Find the number of students whose scores are 85% or more. « 140 students »

9 **The following are the scores of 100 students in an experimental math exam :**

Sets	0–	10–	20–	30–	40–	50–	Total
Frequency	8	14	15	28	23	12	100

1 Form both the ascending and descending cumulative frequency tables.

2 Graph both the ascending and descending cumulative frequency curves on the same graph paper.

3 From the graph , find the number of students who got less than 40 marks and those who got 40 marks or more. « 65 students , 35 students »

4 Find the percentage of the number of students who succeeded given that the success mark is 20 marks. « 78 % »

5 Find the percentage of the number of students who got 45 marks or more. « 23 % »

For excellent pupils

10 **A factory has 120 workers whose experiences are from 5 years to 35 years. The opposite table shows the descending cumulative frequency distribution for those workers according to the years of experience :**

Lower boundaries of sets	Descending cumulative frequency
5 and more	120
10 and more	113
15 and more	93
20 and more	64
25 and more	27
30 and more	12
35 and more	0

1 Deduce from the table the frequency table.

2 Form the ascending cumulative frequency table.

3 Graph the ascending cumulative frequency curve.

4 From the graph , deduce the number of workers whose experience years are less than 17.5 years.

« 40 workers »

EXERCISE **16** ?

Mean

Interactive test

📖 From the school book

Remember Understand Apply Problem Solving

1 Complete the following :

1. The mean of a set of values = $\dfrac{\cdots\cdots\cdots\cdots\cdots\cdots}{\cdots\cdots\cdots\cdots\cdots\cdots}$

2. The centre of the set = $\dfrac{\cdots\cdots\cdots + \cdots\cdots\cdots}{2}$

3. The arithmetic mean of the values : 5 , 12 , 17 , 6 is ....

4. 📖 If the lower limit of a set is 8 and the upper limit of the same set is 14 , then its centre is ....

5. 📖 If the lower limit of a set is 4 and its centre is 9 , then its upper limit is ....

6. 📖 If the mean of a frequency distribution is 39.4 and the total of frequencies is 100 , then the total of the products of frequencies of the sets by their centres is ....

2 Choose the correct answer from those given :

1. The mean of the values : 2 – a , 4 , 1 , 5 , 3 + a is ....
   (a) 1 (b) 2 (c) 3 (d) 15

2. If the mean of marks of 5 pupils is 20 , then the sum of their marks is .... marks.
   (a) 4 (b) 15 (c) 25 (d) 100

3. The centre of the first set of the sets : 7 – , 13 – , 19 – , 25 – is ....
   (a) 6 (b) 7 (c) 10 (d) 13

4. If the upper limit of a set is 14 and its centre is 10 , then its lower limit is ....
   (a) 5 (b) 6 (c) 20 (d) 24

5. If the beginning of a set is 5 and its centre is 7.5 , then the length of the set is ....
   (a) 5 (b) 7.5 (c) 10 (d) 12.5

3 **Find the mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	Total
Frequency	6	8	4	2	20

« 21 »

4 **The following table shows the frequency distribution of marks of 10 students in mathematics :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	1	2	4	2	1	10

1 Calculate the mean of marks of students.

2 If the mark of success is 30 , calculate the number of failed students.

« 35 marks , 3 students »

5 **The following table shows the frequency distribution of weekly wages of 100 workers in one factory :**

Sets	16 –	20 –	24 –	28 –	32 –	36 –	Total
Frequency	10	15	22	25	20	8	100

Calculate the mean. « 28.16 »

6 **The following table shows the frequency distribution of extra wages of 30 workers :**

Sets	15 –	25 –	35 –	45 –	55 –	65 –	75 –	Total
Freq.	2	3	5	8	6	4	2	30

Find the arithmetic mean. « 51 »

7 **The following table shows the frequency distribution of the heights of 120 students in centimetres :**

Height (in cm.)	140 –	144 –	148 –	152 –	156 –	160 –	Total
Frequency	12	20	38	22	17	11	120

Find the mean. « 151.5 cm. »

8 **The following table shows the frequency distribution of number of daily studying hours of 50 pupils in a class :**

Number of hours	1 –	2 –	3 –	4 –	5 –	6 –	7 –	Total
Number of pupils	2	3	5	12	15	7	6	50

1 Calculate the mean of the number of hours of study per day.

2 Find the number of pupils who study less than 4 hours daily. « 5.1 hours , 10 pupils »

9 **The following table shows the distribution of marks of 40 students in one exam :**

Sets	5 –	15 –	……	35 –	45 –	Total
Number of students	3	……	12	10	5	40

1 Complete the table.

2 Calculate the mean.

3 Find the number of students whose marks are not less than 35 marks.

« 31 marks , 15 students »

10 **The following table shows the frequency distribution of the weights of 30 children in kg. :**

Weight (kg.)	6 –	10 –	14 –	18 –	22 –	26 –	30 –	Total
Frequency	2	3	…..	8	6	4	2	30

Complete the table , then find the mean of this distribution. « 20.4 kg. »

11 **Using the following set frequency table (given that the sets are equal in range) :**

Sets	10 –	20 –	X –	40 –	50 –	60 –	Total
Frequency	10	17	20	32	k + 2	4	100

Find :

1 The value of each of X and k

2 The mean of this distribution. « $X = 30$, k = 15 , 39.1 »

12 **The following table shows the frequency distribution of weights of 50 pupils in kg. in one school :**

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of pupils	7	3 k	4 k	10	8	4	50

1 Calculate the value of k

2 Find the mean of this distribution. « 3 , 44 kg. »

13 **The following table shows the frequency distribution of 50 workers days-off :**

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	4	5	8	k – 2	7	5	1	50

Find :

1 The value of k

2 The mean. « 22 , 15.2 days »

14 If the mean of the scores of a student during the first 5 months is 23.8 , what is the score of the 6th month if the mean of his scores is 24 marks ? « 25 marks »

15 If the mean of marks of Magdi in 4 exams is 16 marks , what is the mark which he should obtain in the fifth exam so that his mean in the five exams will be 18 marks ?

« 26 marks »

For excellent pupils

16 **The opposite table is for finding the mean of marks of m pupils in one exam :**

1 Deduce the value of each of : a , b , c , d , e , f , x , y , z and m

2 Find the mean of these marks.

Sets	Centres of sets	Frequency	Centres of sets × frequency
0 –	a	5	10
4 –	6	b	90
d –	c	30	300
12 –	e	z	y
16 –	f	10	x
Total		m	1140

EXERCISE 17 ?

Median

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 **Choose the correct answer from those given :**

1 The median of the values : 9 , 4 , 8 , 1 and 3 is ....

(a) 3 (b) 4 (c) 5 (d) 8

2 The median of the values : 3 , 7 , 2 , 9 , 5 and 11 is ....

(a) 12 (b) 7 (c) 6 (d) 5

3 The order of the median of the values : 7 , 6 , 5 , 8 and 4 is ....

(a) third. (b) fourth. (c) fifth. (d) sixth.

4 If the order of the median of a set of values is the fourth , then the number of these values equals ....

(a) 4 (b) 5 (c) 6 (d) 7

5 If the median of the values : $k + 1$, $k + 2$, $k + 5$, $k + 4$ and $k + 3$ where k is a positive integer is 13 , then $k =$ ....

(a) -10 (b) 10 (c) 13 (d) 16

6 The point of intersection of the ascending and descending cumulative frequency curves determines .... on the set-axis.

(a) the mean (b) length of the set

(c) centre of the set (d) the median

7 If the the point of intersection of the ascending and descending frequency curves is (30 , 50) , then the sum of frequencies is ....

(a) 30 (b) 50 (c) 60 (d) 100

2 **Using the ascending cumulative frequency curve , find the median of the following frequency distribution :**

Sets	0 –	2 –	4 –	6 –	Total
Frequency	1	2	2	5	10

« 6 »

3 **The following table shows the frequency distribution of 40 persons according to the percentage of intelligence of each of them :**

Sets of intelligence percentage	40 –	50 –	60 –	70 –	80 –	90 –	Total
Number of persons	1	3	8	14	10	4	40

Using the ascending cumulative frequency curve , find the madian of percentage of intelligence. « Approximately 75 % »

4 **The following table shows the frequency distribution of 100 factories according to the number of weekly working hours :**

Sets of hours	50 –	60 –	70 –	80 –	90 –	100 –	Total
Number of factories	5	8	12	28	33	14	100

Find using the descending cumulative frequency curve the median number of hours of work of these factories. « 89.5 hours »

5 **The following table shows the frequency distribution of 50 workers' wages in pounds :**

Sets of wages	300 –	400 –	500 –	600 –	700 –	Total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve , then find the median. « 520 pounds »

6 **The following table shows the frequency distribution of marks of 60 students in mathematics exam :**

Sets of marks	5 –	10 –	15 –	20 –	25 –	30 –	35 –	Total
Number of students	2	5	14	20	13	5	1	60

Find the median mark. « 22 marks »

7 **The following table shows the frequency distribution of weights of 20 children in kg. :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	4	7	4	2	20

Find the median weight in kg. using the ascending and descending cumulative frequency curves of this distribution. « 29 kg. »

8 **The following table shows the distribution of the students of a secondary school in a governorate according to their ages in years :**

Sets of ages in years	14 –	15 –	16 –	17 –	18 –	19 –	Total
Frequency	90	130	110	80	70	20	500

Graph the ascending and descending cumulative frequency curves of this distribution , then find the median age. « 16.3 years »

9 **The following table shows the frequency distribution of the marks of 90 students in a monthly exam :**

Sets of marks	10 –	14 –	18 –	22 –	26 –	30 –	34 –	Total
Number of students	8	10	24	21	12	9	6	90

Find the median mark using the ascending and descending cumulative frequency curves.

« 22.5 marks »

10 **The following table shows the frequency distribution for the scores of 50 students in an examination :**

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	3	5	9	10	12	7	4	50

Find : 1 The mean of the student's score.

2 The median. « 16.8 , 17.6 »

11 **From the following frequency table with equal sets in range :**

Sets	10 –	20 –	X –	40 –	50 –	60 –	Total
Frequency	10	17	20	32	k + 2	4	100

1 Find the value of each of X and k « $X = 30$, $k = 15$ »

2 Graph the ascending and descending cumulative curves on one figure, then calculate the median. « 41 »

EXERCISE **18** ?

Mode

Interactive test

From the school book

● Remember ● Understand ○ Apply Problem Solving

1 Choose the correct answer from those given :

1 The mode of a set of values is ....

(a) $\frac{\text{sum of values}}{\text{number of these values}}$ (b) the most common value.

(c) the middle value after rearranging the values ascendingly or descendingly.

(d) the point of intersection of the ascending and descending cumulative frequency curves.

2 The mode of the values : 5 , 3 , 8 , 5 , 9 is ....

(a) 3 (b) 5 (c) 8 (d) 9

3 The mode of the values : 8 , 7 , 8 , 7 , 6 , 5 , 8 is ....

(a) 8 (b) 7 (c) 6 (d) 5

4 If the mode of the values : 4 , a , 5 , 3 is 3 , then a = ...

(a) 5 (b) 4 (c) 3 (d) 6

5 If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then x = ...

(a) 12 (b) 11 (c) 7 (d) 6

6 If the mode of the values : 4 , 11 , 8 , $2x$ is 4 , then x = ...

(a) 1 (b) 2 (c) 4 (d) 8

7 If the mode of the values : 5 , 3 , $\sqrt{x-1}$, 4 is 3 , then x = ...

(a) 3 (b) 4 (c) 8 (d) 10

2 **A factory has 600 workers. A sample of 120 workers is taken such that it represents the all groups very well. It is found that the distribution of their ages in years is as the following table :**

Age	25–	30–	35–	40–	45–	50–	**Total**
Number of workers	12	17	18	40	25	8	**120**

Draw the histogram , then deduce the mode age. « 43 years »

3 **The following table shows the frequency distribution of marks of 100 pupils in an exam :**

Sets of marks	10–	14–	18–	22–	26–	30–	34–	**Total**
Number of pupils	2	10	15	40	25	6	2	**100**

Find the mode mark using the histogram of this distribution. « 24.5 marks »

4 **The following is the frequency distribution of 100 workers in one of the factories according to their daily wages :**

Sets of wages in pounds	10–	15–	20–	25–	30–	35–	40–	**Total**
Number of workers	6	12	16	24	20	14	8	**100**

Draw the histogram of this frequency distribution , then deduce the mode wage of the worker. « 28.5 pounds »

5 **Find the mode of the following frequency distribution for the scores of 40 students in an examination :**

Sets of marks	30–	40 –	50–	60–	70–	80–	**Total**
Frequency	3	4	12	8	7	6	**40**

« 57 »

6 **The following is the frequency distribution of ages of 45 persons :**

Sets of ages in years	12–	14–	16–	18–	20–	22–	24–	**Total**
Number of persons	5	7	8	12	6	4	3	**45**

Find the mode age. « 18.8 years »

7 **The following table shows the frequency distribution of the heights of 200 students :**

Height in cm.	110 –	115 –	120 –	125 –	130 –	135 –	140 –	Total
Number of students	10	12	28	35	60	40	15	200

Graph the frequency histogram , then find the mode height. « 132.75 cm. »

8 **The following table shows the frequency distribution of 102 cows according to the weekly amount of milk in galoons :**

Sets of milk in galoons	14 –	16 –	18 –	20 –	22 –	24 –	Total
Number of cows	8	16	28	20	18	12	102

Use the histogram of this distribution to find the mode of the weekly amount of milk.

« 19.2 galoons »

9 **The following table shows the frequency distribution of marks of 100 pupils in mathematics at the end of the year :**

Marks	15 –	20 –	25 –	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of pupils	4	6	8	12	16	20	22	7	5	100

Graph the histogram of that distribution , then find the mode mark. « 45.5 marks »

10 **The following table shows the frequency distribution of the weights of 100 children in kg. :**

Weight in kg.	10 –	14 –	18 –	22 –	26 –	30 –	Total
Frequency	5	15	30	24	17	9	100

Find the mode weight. « 20.8 kg. »

11 **The following table shows the frequency distribution of the weights of 50 students in kg. :**

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of students	$k + 4$	$3k$	$4k$	$3k + 1$	$3k - 1$	$k + 1$	50

1 Find the value of k « 3 »

2 Graph the frequency histogram , then find the mode. « 43 kg. »

12 **The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :**

Sets of wages in L.E.	70 –	80 –	90 –	100 –	x –	120 –	130 –
Number of workers	10	13	k – 4	20	16	14	11

Find : 1 The value of each of x and k « x = 110 , k = 20 »

2 The mode of wages in L.E. « 105 pounds »

13 **The following is the frequency distribution of 100 workers of building according to the number of weekly working hours :**

Sets of working hours	35 –	45 –	55 –	65 –	75 –	85 –	Total
Number of workers	15	30	23	20	8	4	100

The required is finding :

1 The mean. « 58.8 hours »

2 The median. « 57.5 hours »

3 The mode. « 52 hours »

14 **The following is the frequency distribution of the weekly bonus of 100 workers in a factory :**

Bonus in L.E.	20 –	30 –	40 –	50 –	60 –	70 –
No. of workers	10	k	22	26	20	8

1 Calculate the value of k « 14 »

2 Find the mean of this distribution. « 50.6 pounds »

3 Find the mode value of the weekly bonus using the histogram. « 54 pounds »

15 **The following table shows the frequency distribution for the weights of 50 students in kg. at a school :**

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of students	7	3 k	4 k	10	8	4	50

1 Find the value of k « 3 »

2 Calculate the mean. « 44 kg. »

3 Draw the ascending cumulative frequency curve.

4 Draw the histogram and find the mode of weights. « 43 kg. »

5 Find the median. « 43.5 kg. »

A Research Project

On Unit Three

Project aims :

- Organizing data in frequency tables with sets.
- Forming the ascending cumulative frequency table and graphing it.
- Finding the mean and the mode of some data organized in a frequency table with sets.
- Finding the median of a frequency distribution with sets.
- Appreciating the role of statistics in practical life.

Do a research project on the following topic :

"Statisticians use several measurement tools to measure the central tendency, as the mean, the median and the mode".

Discuss the following points using available resources :

1. Define the mean, the median and the mode.
2. Record the marks of your mates in class in a test of mathematics, then do the following :
   * Organize this data in a tally table, then form the frequency table with sets.
   * From the frequency table with sets, calculate the mean of the marks of your mates.
   * Using the frequency table with sets, draw the histogram and then find the mode mark.
   * Form the ascending cumulative frequency table, then represent it by the ascending cumulative frequency curve. At last, find the median mark.

TIMSS Problems

Accumulative basic skills

1 Complete the following :

1. A turtle covers 80 metres per hour , then it covers 8 metres in .... minutes.
2. The sum of the real numbers in the interval [–12 , 12] equals ....
3. If ○ + □ = 20 , ○ + ○ + □ = 35 , then ○ = ...
4. In three games of bowling , Sara gained 139 , 143 , 144 points , then the number of points she needs in the 4^{th} game so that the mean of points is 145 , is ....
5. Two boxes of apples , the sum of their weights is 54 kg. The first has 12 kg. more than the second , then the number of kilograms in the second box is .... kg.
6. $300 \div 200 = 1 \div$ ....
7. $(301 + 302 + 303 + \cdots + 325) - (1 + 2 + 3 + \cdots + 25) =$ ....
8. If four times a number is 48 , then $\frac{1}{3}$ this number is ....
9. Gamal has 3 sisters and 5 brothers , his sister Sara has $\mathcal{X}$ sisters and y brothers , then $\mathcal{X}$ y = ...
10. If a + b + c = 26 , a + b = 15 , b + c = 20 , then b = ...
11. Three girls can perform a work in 36 hours , then the needed hours for four girls to perform the same work is .... hours.
12. If

	□	□	○
	□	□	△
+	□	△	□
2	0	1	6

, then □ = ... , △ = ... , ○ = ...

2 Choose the correct answer from the given ones :

1 The number 3.015 lies on the number line between ....

(a) $\frac{5}{2}$, 3 (b) $\frac{7}{2}$, $\frac{11}{3}$ (c) 3 , $\frac{16}{5}$ (d) 3.12 , 3.15

2 Which of the following numbers lies between 0.07 , 0.08 ?

(a) 0.00075 (b) 0.0075 (c) 0.075 (d) – 0.75

3 Which of the following is different in value ?

(a) $1 \div 9 + 9 - 1$ (b) $1 + 9 \div 9 - 1$ (c) $1 - 9 + 9 \times 1$ (d) $1 \times 9 - 9 + 1$

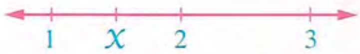
4 If x is a negative number , which of the following is a positive number ?

(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

5 The greatest number of the following is ....

(a) – 1.25 (b) – 0.125 (c) – 0.0125 (d) – 0.00125

6 The best estimation to the number opposite to x is ....



(a) 1.1 (b) 1.2 (c) 1.5 (d) 1.7

7 If 10% of x equals y , then x = ...

(a) 0.1 y (b) y (c) 9 y (d) 10 y

8 If $x = (-2)^4$, $y = -2^4$, then ....

(a) $x = y$ (b) $x > y$ (c) $x < y$ (d) $x \leq y$

9 $\sqrt{\sqrt{81 \times 81 \times 81 \times 81}}$ = ...

(a) 3 (b) 9 (c) 27 (d) 81

10 For any number k , then k + k + (k × k × k) can be written as ....

(a) $2k^2 + 3k$ (b) $5k$ (c) k^5 (d) $2k + k^3$

11 A machine produces two kinds of rods , one is red and of length (10 ± 0.5) cm. and the other is white and of length (6 ± 0.5) cm.

If we put two rods as shown in the opposite figure , then the smallest difference between their lengths may be ....

(a) 4 cm. (b) 5 cm. (c) 3 cm. (d) 8.5 cm.

12 All numbers divisible by 4 and 15 are divisible by ....

(a) 6 (b) 8 (c) 24 (d) 45

Second Geometry

UNIT 4

Medians of Triangle – Isosceles Triangle

Exercises of the unit :

1. Medians of triangle.

2. Medians of triangle "Follow".

3. The isosceles triangle.

4. The converse of the isosceles triangle theorem.

5. Corollaries of the isosceles triangle theorems.

A research project on unit four

Scan the QR code to solve an interactive test on each lesson

EXERCISE 12

Medians of triangle

Interactive test

From the school book

Remember   Understand   Apply   Problem Solving

1 Complete the following :

1. In ΔABC , if D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is called ....
2. The number of medians of the triangle is ....
3. The medians of the triangle intersect at ....
4. The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from its base.
5. The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the vertex.
6. The point of intersection of the medians of the triangle divides each of them in the ratio 2 : .... from the base.
7. The point of intersection of medians of the triangle divides each of them in the ratio .... : 8 from the vertex.

2 Choose the correct answer from those given :

1. The number of medians of the obtuse-angled triangle is ....

   (a) zero   (b) 1   (c) 2   (d) 3

2. If $\overline{YD}$ is a median in ΔXYZ , M is the point of intersection of medians , then MD = ... YM

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

3. If M is the point of intersection of medians of ΔABC , $\overline{BD}$ is a median , then BD : MD = ...

   (a) 2 : 3   (b) 1 : 3   (c) 3 : 2   (d) 3 : 1

4 If $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of medians , then AD = ... AM

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

5 If $\overline{AD}$ is a median in Δ ABC of length 9 cm. , M is the point of intersection of medians , then DM = ... cm.

(a) 3 (b) 4.5 (c) 6 (d) 9

6 If M is the point of intersection of the medians of Δ ABC , $\overline{AD}$ is a median of length 6 cm. , then AM = ... cm.

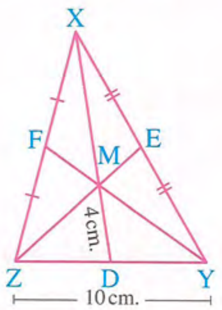
(a) 1 (b) 2 (c) 3 (d) 4

7 If M is the point of intersection of the medians of Δ ABC , D is the midpoint of $\overline{BC}$, then AD = ...

(a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

3 Using data given for each of the following figures , find the required below each figure :

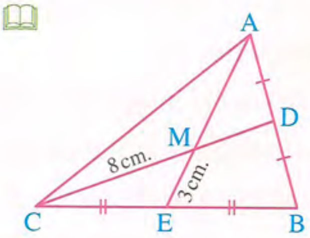
1



XM = ... cm. and

YD = ... cm.

2



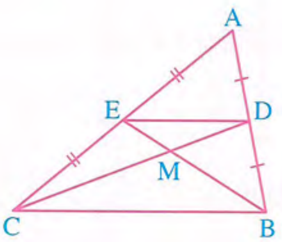
MA = ... cm. ,

MD = ... cm. ,

ME = ... AE

and MC = ... CD

3

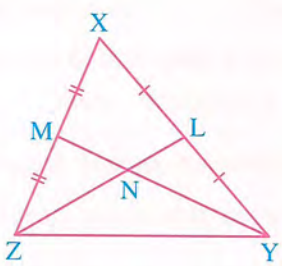


If BC = 12 cm. , BE = 9 cm. and MC = 8 cm. , then DE = ... cm. ,

ME = ... cm. and

MD = ... cm.

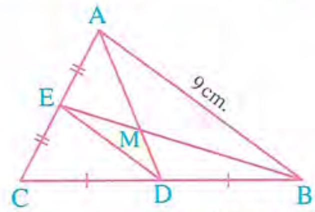
4



If LZ = 15 cm. , YM = 18 cm. and XY = 20 cm. , then NL = ... cm. ,

NY = ... cm. and the perimeter of Δ NLY = ... cm.

4 **In the opposite figure :**



ABC is a triangle in which D is the midpoint of $\overline{BC}$

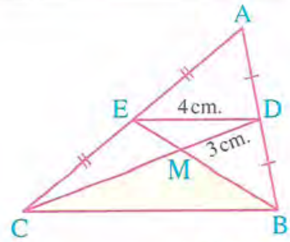
, E is the midpoint of $\overline{AC}$ and $\overline{AD} \cap \overline{BE} = \{M\}$

If AD = 6 cm. and AB = BE = 9 cm.

Calculate : The perimeter of Δ MDE

« 9.5 cm. »

5 **In the opposite figure :**



If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

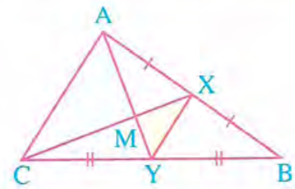
and $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm. ,

DM = 3 cm. and BE = 6 cm.

Find : The perimeter of Δ BMC

« 18 cm. »

6 **In the opposite figure :**



ABC is a triangle , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{BC}$, XY = 5 cm. and $\overline{XC} \cap \overline{AY} = \{M\}$

where CM = 8 cm. , YM = 3 cm. **Find :**

1 The perimeter of Δ MXY

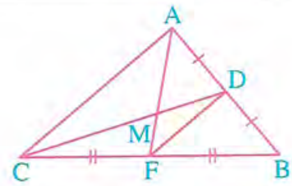
2 The perimeter of Δ MAC

« 12 cm. , 24 cm. »

7 In Δ ABC , BC = 8 cm. , F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and $\overline{BE} \cap \overline{CF} = \{M\}$ If BM = 4 cm. and CM = 6 cm. **Find :** The perimeter of Δ MFE

« 9 cm. »

8 **In the opposite figure :**



$\overline{AF}$ and $\overline{CD}$ are two medians in Δ ABC ,

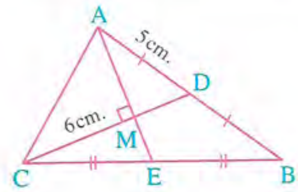
$\overline{AF} \cap \overline{CD} = \{M\}$

If the perimeter of Δ AMC = 36 cm.

Find : The perimeter of Δ MFD

« 18 cm. »

9 **In the opposite figure :**



M is the point of concurrence of the medians

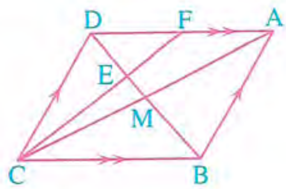
of Δ ABC , $\overline{AM} \perp \overline{CD}$

, MC = 6 cm. , AD = 5 cm.

Find : The length of $\overline{ME}$

« 2 cm. »

10 **In the opposite figure :**

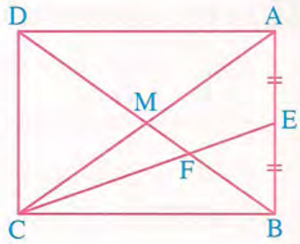


ABCD is a parallelogram, its diagonals intersect at M ,

E $\in \overline{DM}$ where DE = 2 EM , draw $\overrightarrow{CE}$ to cut $\overline{AD}$ at F

Prove that : AF = FD

11 **In the opposite figure :**



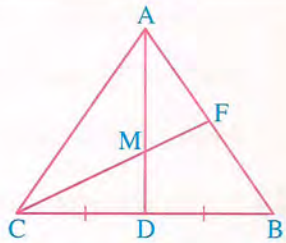
ABCD is a rectangle , its diagonals intersect at M ,

E is the midpoint of $\overline{AB}$, $\overline{CE} \cap \overline{BD} = \{F\}$

1 **Prove that :** F is the intersection point of the medians of the triangle ABC

2 If BF = 4 cm. , **find :** the length of $\overline{AM}$ « 6 cm. »

12 **In the opposite figure :**



ABC is a triangle in which D is the midpoint of $\overline{BC}$,

AB = AC , M ∈ $\overline{AD}$ where AM = $\frac{2}{3}$ AD and

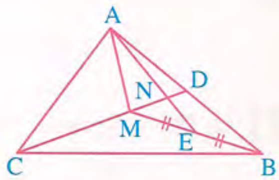
$\overrightarrow{CM} \cap \overline{AB} = \{F\}$

Prove that : BF = $\frac{1}{2}$ AC

13 ABC is a triangle where point D is the midpoint of $\overline{BC}$ and point M ∈ $\overline{AD}$, AM = 2 MD

Draw $\overrightarrow{CM}$ to intersect $\overline{AB}$ at point E If EC = 12 cm., **then find :** the length of $\overline{EM}$ « 4 cm. »

14 **In the opposite figure :**



M ∈ $\overline{CD}$, M is the point of concurrence of the medians

of Δ ABC , N ∈ $\overline{DM}$ where ND = $(X - 1)$ cm.

, MN = $(X + 3)$ cm. , $\overrightarrow{AN}$ is drawn to intersect $\overline{BM}$ at E

which is the midpoint of $\overline{BM}$

Find : The length of $\overline{MC}$ « 24 cm. »

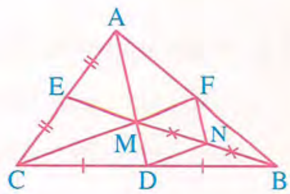
15 ABCD is a parallelogram whose diagonals intersect at M , E is the midpoint of $\overline{BC}$,

$\overline{DE}$ intersects $\overline{AC}$ at F

Prove that : 1 $\overrightarrow{BF}$ bisects $\overline{CD}$ 2 CF = $\frac{1}{3}$ AC

For excellent pupils

16 **In the opposite figure :**



$\overline{AD}$ and $\overline{BE}$ are medians in the triangle ABC intersecting at M ,

$\overrightarrow{CM} \cap \overline{AB} = \{F\}$, if N is the midpoint of $\overline{MB}$

Prove that : The figure FNDM is a parallelogram.

17 **In the opposite figure :**

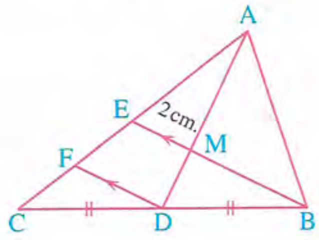
ABC is a triangle in which D is the midpoint of $\overline{BC}$

, $M \in \overline{AD}$ where AM = 2 MD

, $\overrightarrow{BM} \cap \overline{AC} = \{E\}$

, ME = 2 cm. , draw $\overrightarrow{DF} \parallel \overline{BE}$ and cut $\overline{AC}$ at F

Find : The length of $\overline{DF}$



« 3 cm. »

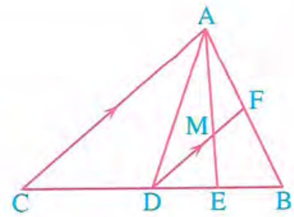
18 **In the opposite figure :**

ABC is a triangle in which D is the midpoint of $\overline{BC}$

and E is the midpoint of $\overline{BD}$, draw $\overrightarrow{DF} \parallel \overline{AC}$

and cut $\overline{AE}$ at M and $\overline{AB}$ at F

Prove that : DM = $\frac{1}{3}$ AC



19 ABC is a triangle , D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$

If $\overline{CD} \cap \overline{BE} = \{M\}$ Draw $\overrightarrow{AM}$ to intersect $\overline{BC}$ at F

Prove that : The figure DBFE is a parallelogram.

EXERCISE **2** ?

Medians of triangle "Follow"

Interactive test

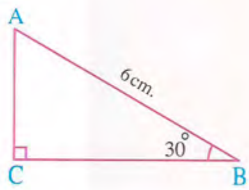
From the school book

Remember · Understand · Apply · Problem Solving

1 **Complete the following :**

1. The number of medians in the right-angled triangle is ....
2. The length of the median from the vertex of the right angle in the right-angled triangle equals ....
3. If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is ....
4. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....
5. The length of the hypotenuse in thirty and sixty triangle equals .... the length of the side opposite to the angle whose measure is 30°
6. The length of the hypotenuse in the right-angled triangle equals .... the length of the median drawn from the vertex of the right angle.

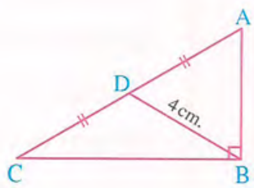
2 **Using data given for each of the following figures , find the required below each figure :**

1


AC = ... cm.

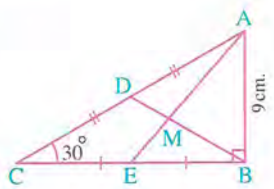
2


XZ = ... cm.

3


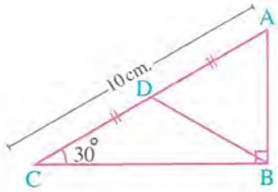
AC = ... cm.

4



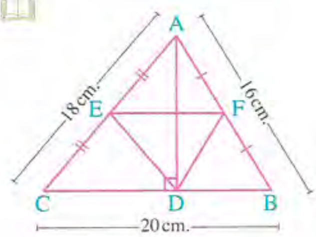
AC = ... cm. ,
BD = ... cm. ,
MD = ... BD
and MD = ... cm.

5



BD = ... cm. ,
AB = ... cm.
and the perimeter of
Δ ABD = ... cm.

6



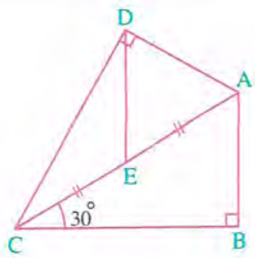
DF = ... cm. ,
DE = ... cm. ,
FE = ... cm.
and the perimeter of
Δ DEF = ... cm.

3 Choose the correct answer from those given :

1. In the right-angled triangle , the ratio between the length of the median drawn from the vertex of the right angle and the length of the hypotenuse is ....
   (a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 3 : 2
2. In the thirty-sixty triangle , the ratio between the length of the hypotenuse and the length of the side opposite to the angle of measure 30° is ....
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3
3. In the thirty-sixty triangle , the ratio between the length of the median drawn from the vertex of the right angle and the length of the side opposite to the angle of measure 30° is ....
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 2 : 3
4. ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, then BD = ...
   (a) $\frac{1}{2}$ AC (b) AC (c) $\frac{1}{2}$ BC (d) AB
5. ABC is a triangle in which m ($\angle$ A) = 90° , AC = $\frac{1}{2}$ BC , then m ($\angle$ C) = ...
   (a) 30° (b) 60° (c) 90° (d) 120°
6. In Δ ABC , m ($\angle$ B) = 90° , if 2 AB – AC = 0 , then m ($\angle$ C) = ...
   (a) 30° (b) 60° (c) 90° (d) 120°

4 In the opposite figure :

m ($\angle$ ABC) = m ($\angle$ ADC) = 90° ,
m ($\angle$ ACB) = 30° and
E is the midpoint of $\overline{AC}$
Prove that : AB = DE



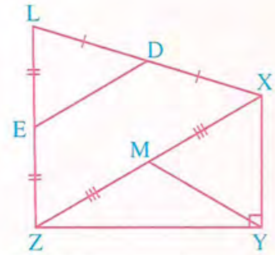
5 **In the opposite figure :**

m (∠ XYZ) = 90° , D is the midpoint of $\overline{XL}$,

E is the midpoint of $\overline{ZL}$ and

M is the midpoint of $\overline{XZ}$

Prove that : DE = YM



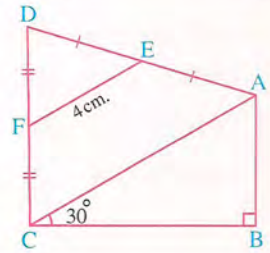
6 **In the opposite figure :**

ABCD is a quadrilateral in which m (∠ B) = 90° ,

E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$,

m (∠ ACB) = 30° and EF = 4 cm.

Find by proof : The length of $\overline{AB}$ « 4 cm. »



7 **In the opposite figure :**

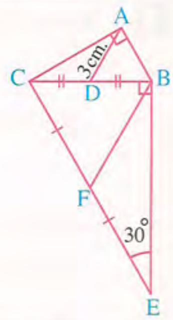
m (∠ BAC) = m (∠ CBE) = 90°

, m (∠ BEC) = 30°

, D and F are the midpoints

of $\overline{BC}$ and $\overline{CE}$ respectively and AD = 3 cm.

Find : The length of $\overline{BF}$ « 6 cm. »



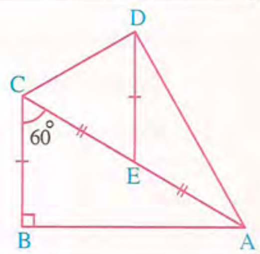
8 **In the opposite figure :**

ABC is a right-angled triangle at B , m (∠ ACB) = 60° ,

E is the midpoint of $\overline{AC}$ and

DE = BC

Prove that : m (∠ ADC) = 90°



9 **In the opposite figure :**

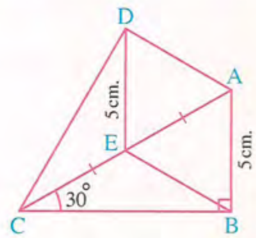
ABC is a right-angled triangle at B ,

m (∠ ACB) = 30° , AB = 5 cm. and

E is the midpoint of $\overline{AC}$

If DE = 5 cm. ,

prove that : m (∠ ADC) = 90°



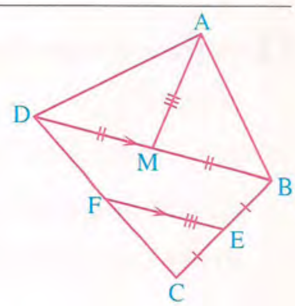
10 **In the opposite figure :**

ABD is a triangle , M is the midpoint of $\overline{BD}$,

E is the midpoint of $\overline{BC}$,

F ∈ $\overline{CD}$, $\overline{EF}$ // $\overline{BD}$ and AM = EF

Prove that : m (∠ BAD) = 90°



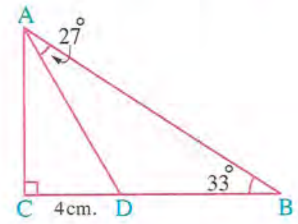
11 **In the opposite figure :**

ABC is a triangle in which m (∠ B) = 33°

, m (∠ C) = 90° , D ∈ $\overline{BC}$ where CD = 4 cm.

, m (∠ BAD) = 27°

Find : The length of $\overline{AD}$ « 8 cm. »

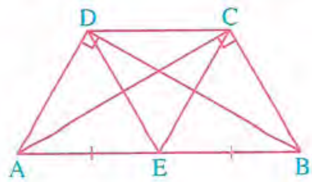


12 **In the opposite figure :**

ADB is a right-angled triangle at D ,

ACB is a right-angled triangle at C and E is the midpoint of $\overline{AB}$

Prove that : Δ CED is an isosceles triangle.



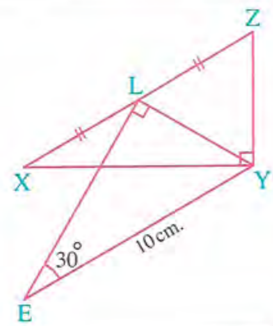
13 **In the opposite figure :**

m (∠ YLE) = 90° , m (∠ E) = 30° , YE = 10 cm. ,

m (∠ XYZ) = 90° and

L is the midpoint of $\overline{XZ}$

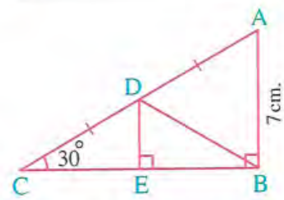
Find by proof : The length of $\overline{XZ}$ « 10 cm. »



14 **In the opposite figure :**

ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, $\overline{DE}$ ⊥ $\overline{BC}$, AB = 7 cm. and m (∠ C) = 30°

Find the length of each of : $\overline{BD}$ and $\overline{DE}$



« 7 cm. , 3.5 cm. »

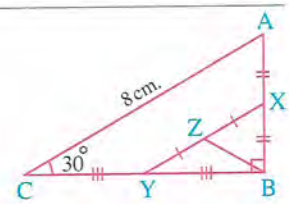
15 **In the opposite figure :**

ABC is a triangle in which m (∠ ABC) = 90° , m (∠ C) = 30° ,

X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{XY}$

respectively and AC = 8 cm.

Find the length of each of : $\overline{AB}$, $\overline{XY}$ and $\overline{BZ}$



« 4 cm. , 4 cm. , 2 cm. »

16 **In the opposite figure :**

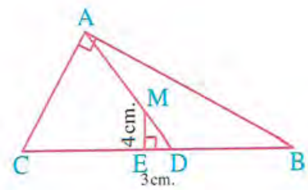
ABC is a right-angled triangle at A

, M is the point of concurrence of its medians

, E ∈ $\overline{DC}$ where $\overline{ME}$ ⊥ $\overline{DC}$, DE = 3 cm.

and ME = 4 cm.

Find : The length of $\overline{BC}$



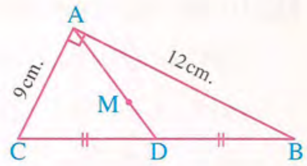
« 30 cm. »

17 **In the opposite figure :**

$m(\angle BAC) = 90°$, $AB = 12$ cm. , $AC = 9$ cm.

$\overline{AD}$ is a median of ΔABC and M is the point of concurrence of the medians of ΔABC

Find : The length of $\overline{AM}$



« 5 cm. »

---

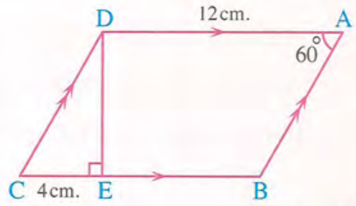
18 **In the opposite figure :**

ABCD is a parallelogram in which

$m(\angle A) = 60°$, $\overline{DE} \perp \overline{BC}$

, $AD = 12$ cm. and $EC = 4$ cm.

Find : The perimeter of the parallelogram ABCD



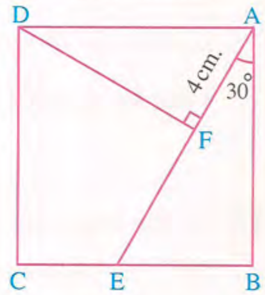
« 40 cm. »

---

19 **In the opposite figure :**

ABCD is a square , $E \in \overline{BC}$ where $m(\angle BAE) = 30°$ and $\overline{DF} \perp \overline{AE}$ If $AF = 4$ cm.

Calculate : The area of the square ABCD



« 64 cm^2 »

---

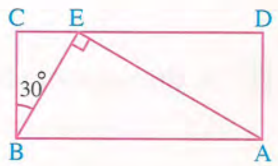
20 **In the opposite figure :**

ABCD is a rectangle , $E \in \overline{DC}$

where $m(\angle CBE) = 30°$

and $m(\angle AEB) = 90°$

Prove that : $CE = \frac{1}{4} AB$



---

21 **In the opposite figure :**

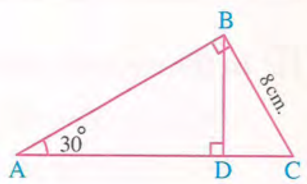
ABC is a right-angled triangle at B ,

$m(\angle A) = 30°$,

$D \in \overline{AC}$ such that $\overline{BD} \perp \overline{AC}$

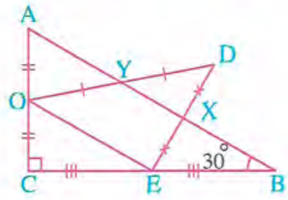
If $BC = 8$ cm.

Find : The length of $\overline{AD}$



« 12 cm. »

22 **In the opposite figure :**



ABC is a right-angled triangle at C in which m ($\angle$ B) = 30°

, E , O , X , Y are the midpoints of $\overline{BC}$, $\overline{AC}$

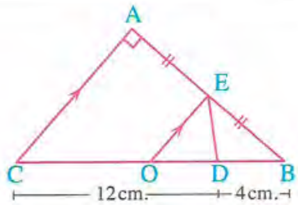
, $\overline{DE}$, $\overline{DO}$ respectively

Prove that : XY = $\frac{1}{2}$ AC

23 ABC is a triangle in which AB = AC and $\overrightarrow{AD}$ is drawn to be perpendicular to $\overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$ If E and F are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively,

prove that : DE + DF = AB

24 **In the opposite figure :**



ABC is a right-angled triangle at A

, E is the midpoint of $\overline{AB}$, O $\in \overline{BC}$

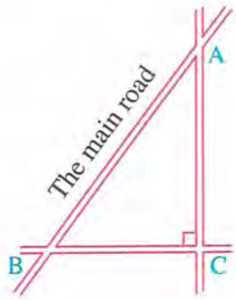
where $\overline{EO}$ // $\overline{AC}$, D $\in \overline{BO}$ where BD = 4 cm. , DC = 12 cm.

Find : The length of $\overline{DE}$

« 4 cm. »

Life Application

25 The opposite figure is a sketch for three towns A , B and C such that the distance between the towns A and C is 40 km. and the distance between the towns B and C is 30 km.

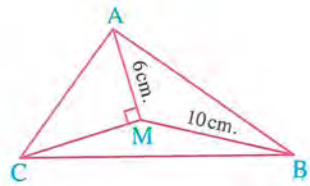


If we want to build a service station lying on the main road at the half-way between the towns A and B , also we want to build a road linking this station to the town C , then how long will this road be ?

« 25 km. »

For excellent pupils

26 **In the opposite figure :**



M is the point of concurrence of the medians of Δ ABC

, AM = 6 cm. , BM = 10 cm.

, m ($\angle$ AMC) = 90°

Find by proof : 1 The length of $\overline{AC}$ 2 The length of $\overline{MC}$

« 10 cm. , 8 cm. »

27 ABCD is a parallelogram , X is an interior point in it such that $\overrightarrow{DX}$ bisects $\angle$ ADC , $\overrightarrow{CX}$ bisects $\angle$ DCB , if the point Y is the midpoint of $\overline{DC}$

, **prove that :** XY = YC

EXERCISE 3 ?

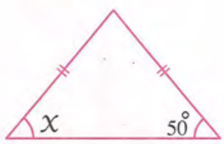
The isosceles triangle

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

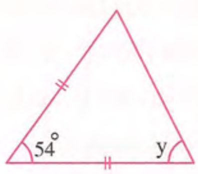
1 **In each of the following , find the value of the symbol used for the measure of the angle :**

1 

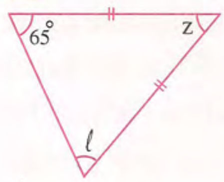
$x =$ ....°

2 

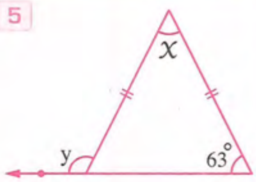
$x =$ ....°

3 

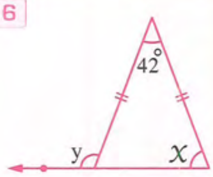
y = ...°

4 

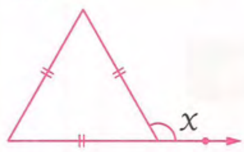
$\ell =$ ....° ,
z = ...°

5 

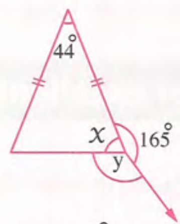
$x =$ ....° ,
y = ...°

6 

$x =$ ....° ,
y = ...°

7 

$x =$ ....°

8 

$x =$ ....° ,
y = ...°

2 **Complete the following :**

- 1 The base angles of the isosceles triangle are ....
- 2 The measure of each angle in the equilateral triangle equals ....°
- 3 In Δ DEF , if DE = DF , then m (∠ E) = m (∠ ....)
- 4 In the isosceles triangle , if the measure of one of the two base angles is 65° , then the measure of its vertex angle equals ....°

5 In the isosceles triangle , if the measure of the vertex angle equals 40° , then the measure of one of the two base angles equals ....°

6 An isosceles triangle , the measure of its vertex angle is 80° , if the measure of one of its base angles is (X + 30°) , then X = ...

3 Choose the correct answer from those given :

1 In Δ XYZ , if XY = YZ = XZ , then m (∠ X) = ...

(a) 30° (b) 60° (c) 90° (d) 180°

2 The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 90° (c) 120° (d) 180°

3 LMN is a triangle in which LM = MN , m (∠ M) = 70° , m (∠ N) = ...

(a) 20° (b) 35° (c) 55° (d) 70°

4 In Δ ABC , AB = AC , m (∠ C) = 65° , then m (∠ A) = ...

(a) 30° (b) 50° (c) 55° (d) 130°

5 In Δ XYZ , ZY = ZX , m (∠ Z) = 120° , then m (∠ X) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

6 If Δ ABC is right-angled at A and AB = AC , then m (∠ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

7 XYZ is an isosceles triangle in which , m (∠ Y) = 100° , then m (∠ Z) = ...

(a) 100° (b) 80° (c) 50° (d) 40°

8 If the measure of one of the two base angles in the isosceles triangle is 30° , then the triangle is ....

(a) obtuse-angled. (b) acute-angled.

(c) right-angled. (d) equilateral.

9 In Δ ABC , AB = AC , m (∠ B) = 6 X ° , m (∠ A) = 3 X ° , then X = ...

(a) 30° (b) 12° (c) 60° (d) 90°

10 In Δ XYZ , if XY = XZ , then the exterior angle at the vertex Z is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

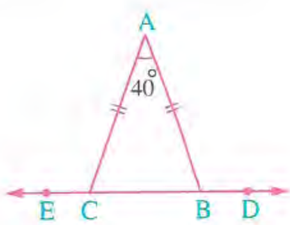
4 In the opposite figure :

ABC is an isosceles triangle in which AB = AC ,

m (∠ A) = 40° and D ∈ $\overrightarrow{CB}$, E ∈ $\overrightarrow{BC}$

1 **Find :** m (∠ ABC)

2 **Prove that :** ∠ ABD ≡ ∠ ACE



« 70° »

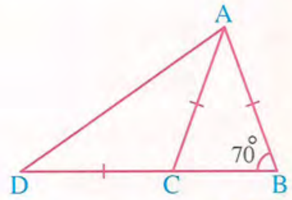
5 **In the opposite figure :**

AB = AC = CD

and m (∠ B) = 70°

Find by proof :

m (∠ BAD)



« 75° »

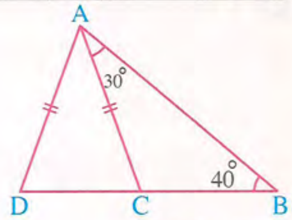
6 **In the opposite figure :**

m (∠ B) = 40° , m (∠ BAC) = 30°

and AC = AD

Find by proof :

1 m (∠ D) 2 m (∠ CAD)



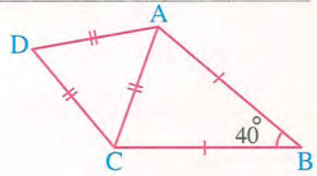
« 70° , 40° »

7 **In the opposite figure :**

AD = DC = AC , AB = BC

and m (∠ ABC) = 40°

Find : m (∠ BAD)



« 130° »

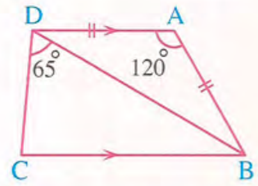
8 **In the opposite figure :**

AB = AD , $\overline{AD}$ // $\overline{BC}$,

m (∠ BAD) = 120° and m (∠ BDC) = 65°

Find :

1 m (∠ ADB) 2 m (∠ C)



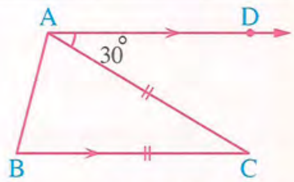
« 30° , 85° »

9 **In the opposite figure :**

ABC is a triangle in which AC = BC ,

$\overrightarrow{AD}$ // $\overline{BC}$ and m (∠ DAC) = 30°

Find : The measures of the angles of Δ ABC



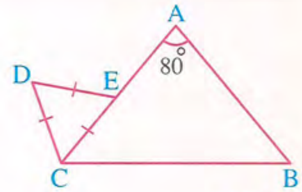
« 30° , 75° , 75° »

10 **In the opposite figure :**

AB = AC , m (∠ BAC) = 80°

and CE = ED = CD

Find by proof : m (∠ BCD)



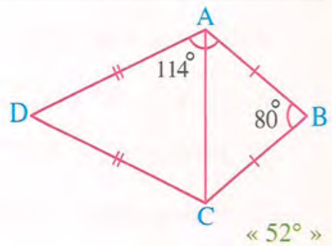
« 110° »

11 **In the opposite figure :**

AB = BC , AD = CD , m (∠ BAD) = 114°

and m (∠ B) = 80°

Find : m (∠ ADC)



« 52° »

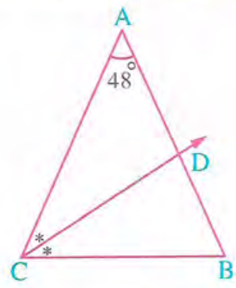
12 **In the opposite figure :**

AB = AC , m (∠ BAC) = 48° , $\overrightarrow{CD}$ bisects ∠ BCA

and intersects $\overline{AB}$ at D

Find :

1 m (∠ B)   2 m (∠ BCD)



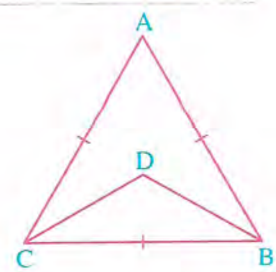
« 66° , 33° »

13 **In the opposite figure :**

ABC is an equilateral triangle and the two bisectors of

∠ B and ∠ C intersect together at D

Find : m (∠ BDC)



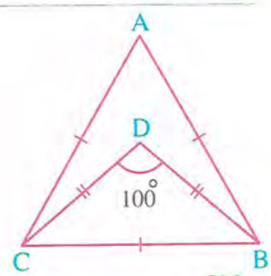
« 120° »

14 **In the opposite figure :**

ABC is an equilateral triangle , DB = DC

and m (∠ BDC) = 100°

Find by proof : m (∠ ABD)



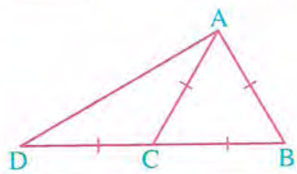
« 20° »

15 **In the opposite figure :**

ABC is an equilateral triangle.

D ∈ $\overrightarrow{BC}$ such that BC = CD

Prove that : $\overline{BA} \perp \overline{AD}$



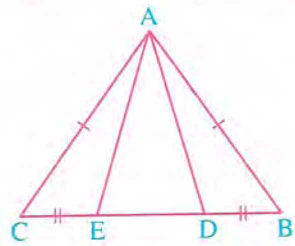
16 **In the opposite figure :**

ABC is an isosceles triangle in which AB = AC , D ∈ $\overline{BC}$

and E ∈ $\overline{BC}$, such that BD = EC

Prove that : 1 Δ ADE is an isosceles triangle.

2 ∠ AED ≡ ∠ ADE

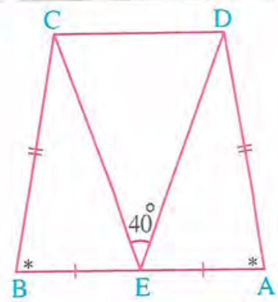


17 **In the opposite figure :**

E is the midpoint of $\overline{AB}$, AD = BC , m (∠ A) = m (∠ B)

and m (∠ DEC) = 40°

Find : m (∠ EDC)



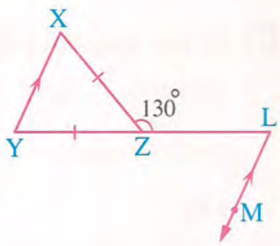
« 70° »

18 **In the opposite figure :**

$Z \in \overline{LY}$, $XZ = YZ$, m ($\angle$ LZX) = 130°

and $\overrightarrow{LM} \,//\, \overline{XY}$

Find : m ($\angle$ MLY)



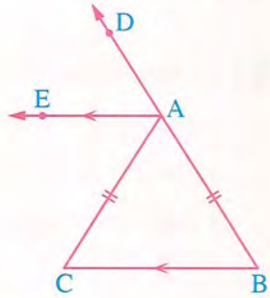
« 65° »

19 **In the opposite figure :**

$A \in \overrightarrow{BD}$, $AB = AC$ and $\overrightarrow{AE} \,//\, \overline{BC}$

Prove that :

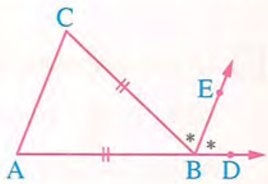
$\overrightarrow{AE}$ bisects $\angle$ DAC



20 **In the opposite figure :**

$AB = BC$ and $\overrightarrow{BE}$ bisects $\angle$ CBD

Prove that : $\overrightarrow{BE} \,//\, \overline{AC}$



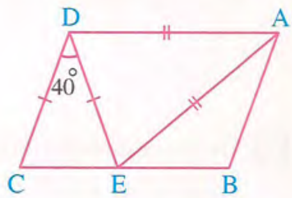
21 **In the opposite figure :**

ABCD is a parallelogram , $E \in \overline{BC}$,

where $AE = AD$, $DE = DC$ and m ($\angle$ EDC) = 40°

Find : 1 m ($\angle$ AED)

2 m ($\angle$ BAE)



« 70° , 30° »

22 **In the opposite figure :**

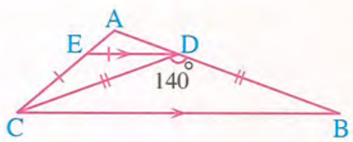
ABC is a triangle in which

$D \in \overline{AB}$, $E \in \overline{AC}$

where $\overline{DE} \,//\, \overline{BC}$, $DE = EC$

, $DB = DC$ and m ($\angle$ BDC) = 140°

Find : m ($\angle$ A)



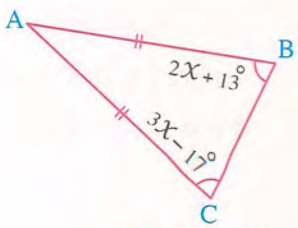
« 120° »

23 **In the opposite figure :**

$AB = AC$, m ($\angle$ B) = $2x + 13°$

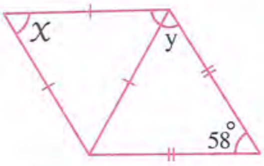
and m ($\angle$ C) = $3x - 17°$

Find : The measures of the angles of Δ ABC

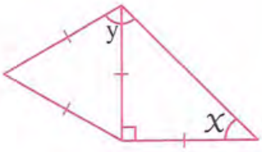


« 73° , 73° , 34° »

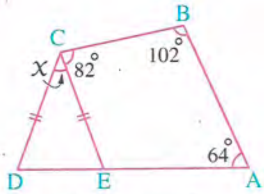
24 **In each of the following figures , find the value of the symbol used for the measure of the angle :**

1


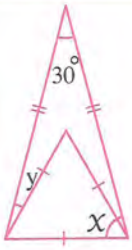
x = ...° ,
y = ...°

2


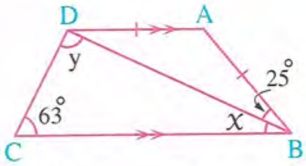
x = ...° ,
y = ...°

3


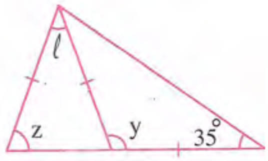
x = ...°

4


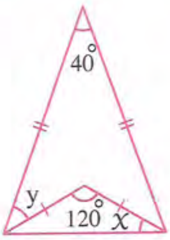
x = ...° ,
y = ...°

5


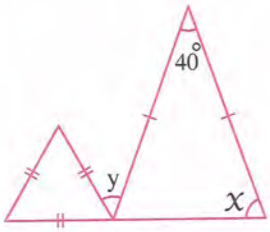
x = ...° ,
y = ...°

6


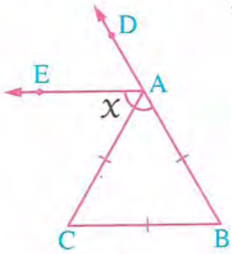
y = ...° , ℓ = ...° ,
z = ...°

7


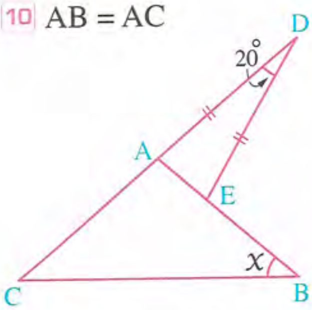
x = ...° ,
y = ...°

8


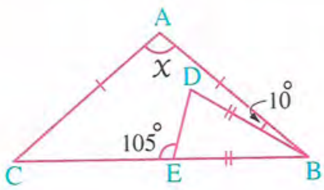
x = ...° ,
y = ...°

9 $\overrightarrow{AE}$ bisects ∠ CAD


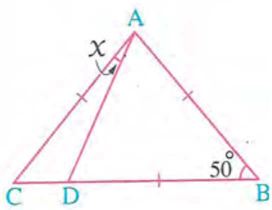
x = ...°

10 AB = AC


x = ...°

11


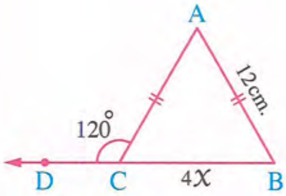
x = ...°

12


x = ...°

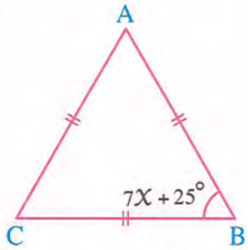
25 **Find the value of x in each of the following figures :**

1



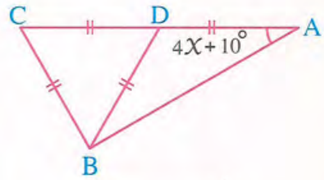
x = ... cm.

2



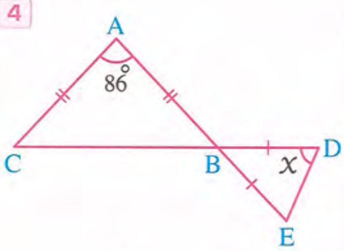
x = ...°

3



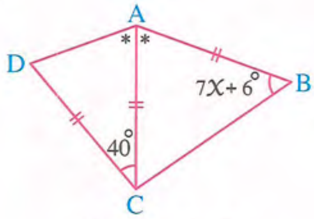
x = ...°

4



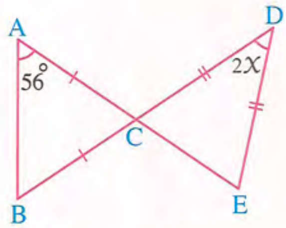
x = ...°

5



x = ...°

6



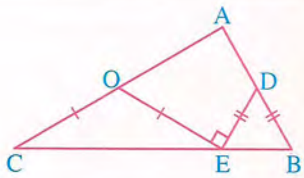
x = ...°

26 **In the opposite figure :**

ABC is a triangle in which D $\in \overline{AB}$, E $\in \overline{BC}$, O $\in \overline{AC}$

where m ($\angle$ DEO) = 90° , DB = DE and OE = OC

Find : m ($\angle$ A)



« 90° »

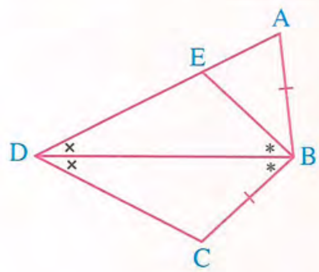
27 **In the opposite figure :**

BA = BC , E $\in \overline{AD}$

and $\overleftrightarrow{BD}$ bisects each

of $\angle$ CBE and $\angle$ CDE

Prove that : m ($\angle$ A) + m ($\angle$ C) = 180°



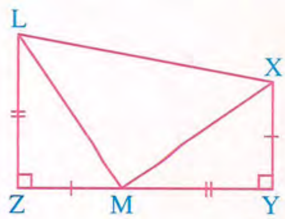
For excellent pupils

28 **In the opposite figure :**

m ($\angle$ Y) = m ($\angle$ Z) = 90°

, XY = MZ and YM = ZL

Find by proof : m ($\angle$ MXL)



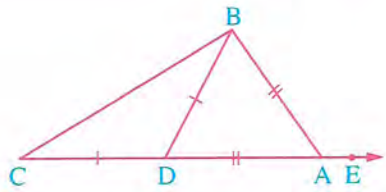
« 45° »

29 **In the opposite figure :**

ABC is a triangle , $D \in \overline{AC}$ such that BD = DC

AD = AB and $E \in \overrightarrow{CA}$

Prove that : m ($\angle$ BAE) = 4 m ($\angle$ BCD)

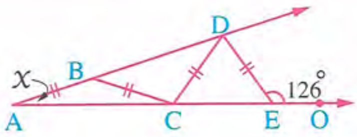


30 **In the opposite figure :**

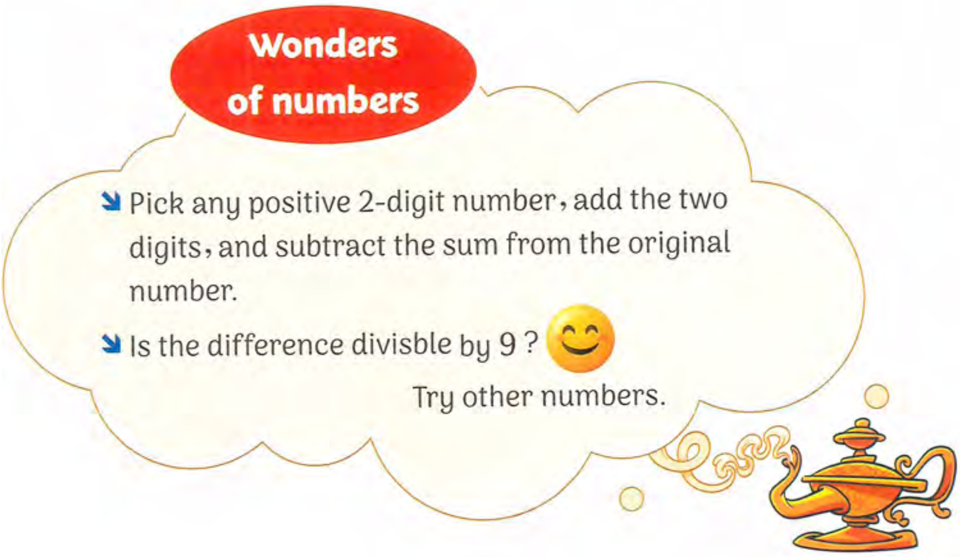
m ($\angle$ A) = $x°$, AB = BC = CD = DE

and m ($\angle$ DEO) = 126°

Find : The value of x



« 18° »



EXERCISE 4

The converse of the isosceles triangle theorem

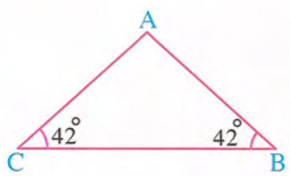
Interactive test

From the school book

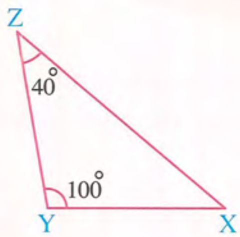
● Remember ● Understand ● Apply ● Problem Solving

1 In each of the following figures , write the equal sides in length :

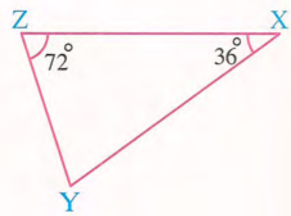
1



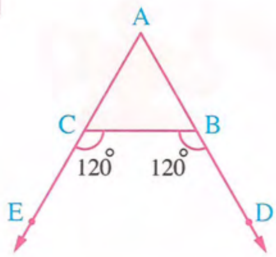
2



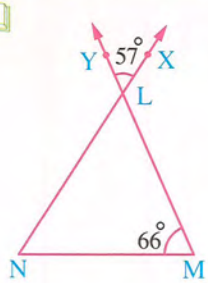
3



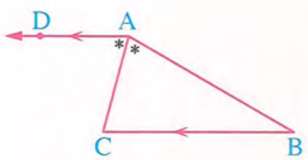
4



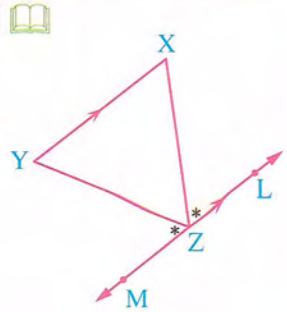
5



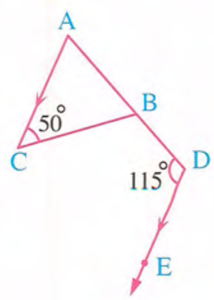
6



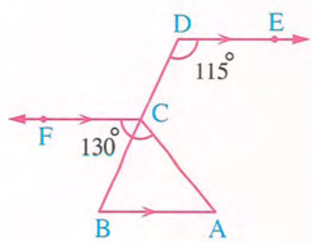
7



8



9



2 **Complete the following :**

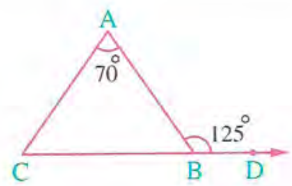
- 1 If two angles in the triangle are congruent , then the two sides opposite to these two angles are .... and the triangle is ....
- 2 If the three angles in the triangle are congruent , then the triangle is ....
- 3 In Δ ABC , if m ($\angle$ A) = 50° and m ($\angle$ B) = 80° , then the triangle is ....
- 4 If the measure of one angle in the right-angled triangle is 45° , then the triangle is ....
- 5 If the measure of one angle of an isosceles triangle is 60° , then the triangle is ....
- 6 ABC is a triangle in which AB = AC and m ($\angle$ A) = 60°
  If its perimeter = 18 cm. , then BC = ... cm.
- 7 In Δ ABC , CA = CB , m ($\angle$ C) = m ($\angle$ A) , then m ($\angle$ B) = ...°

3 **In the opposite figure :**

D $\in \overrightarrow{CB}$, m ($\angle$ ABD) = 125°

and m ($\angle$ A) = 70°

Prove that : Δ ABC is an isosceles triangle.

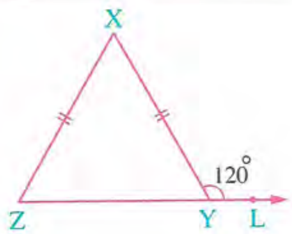


4 **In the opposite figure :**

XY = XZ , m ($\angle$ XYL) = 120°

and L $\in \overrightarrow{ZY}$

Prove that : Δ XYZ is an equilateral triangle.

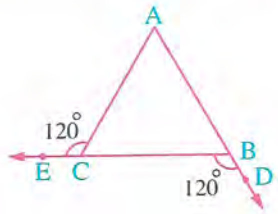


5 **In the opposite figure :**

D $\in \overrightarrow{AB}$, E $\in \overrightarrow{BC}$ and

m ($\angle$ CBD) = m ($\angle$ ACE) = 120°

Prove that : Δ ABC is an equilateral triangle.

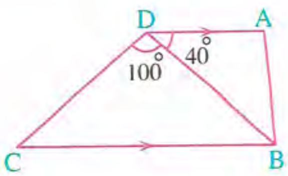


6 **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, m ($\angle$ ADB) = 40°

and m ($\angle$ BDC) = 100°

Prove that : Δ DBC is an isosceles triangle.



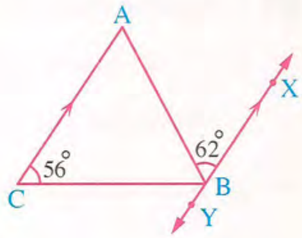
7 **In the opposite figure :**

$B \in \overleftrightarrow{XY}$, $\overleftrightarrow{XY} \parallel \overline{AC}$

, m ($\angle$ ABX) = 62° and

m ($\angle$ C) = 56°

Prove that : AC = BC



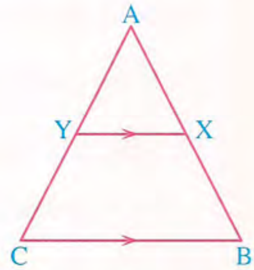
8 **In the opposite figure :**

ABC is a triangle in which AB = AC , $X \in \overline{AB}$,

$Y \in \overline{AC}$ and $\overline{XY} \parallel \overline{BC}$

Prove that : 1 Δ AXY is an isosceles triangle.

2 XB = YC



9 ABC is a triangle in which $D \in \overline{AB}$ and $E \in \overline{BC}$ such that BD = BE

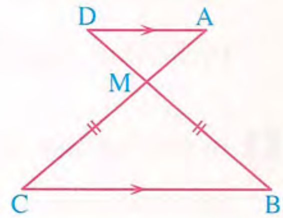
So if $\overline{DE} \parallel \overline{AC}$, **prove that :** AB = BC

10 **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{M\}$,

MB = MC and $\overline{AD} \parallel \overline{BC}$

Prove that : MA = MD

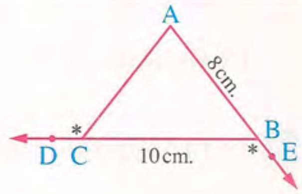


11 **In the opposite figure :**

$B \in \overrightarrow{AE}$, $C \in \overrightarrow{BD}$, AB = 8 cm. ,

BC = 10 cm. and m ($\angle$ EBC) = m ($\angle$ ACD)

Find : The perimeter of Δ ABC



« 26 cm. »

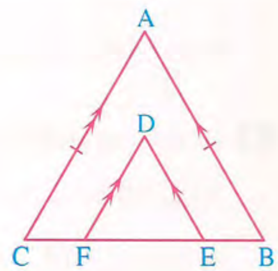
12 **In the opposite figure :**

AB = AC , $\overline{DE} \parallel \overline{AB}$ and $\overline{DF} \parallel \overline{AC}$

Prove that :

1 DE = DF

2 m ($\angle$ BAC) = m ($\angle$ EDF)

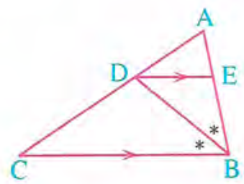


13 **In the opposite figure :**

ABC is a triangle

, $\overrightarrow{BD}$ bisects $\angle$ ABC and $\overline{ED} \parallel \overline{BC}$ where E $\in \overline{AB}$

Prove that : Δ EBD is an isosceles triangle.

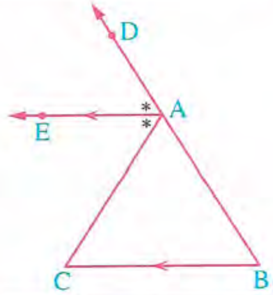


14 **In the opposite figure :**

A $\in \overrightarrow{BD}$, $\overrightarrow{AE} \parallel \overline{BC}$

and $\overrightarrow{AE}$ bisects $\angle$ CAD

Prove that : AB = AC

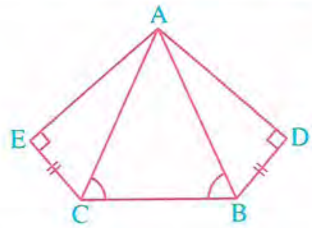


15 **In the opposite figure :**

BD = CE , m ($\angle$ ABC) = m ($\angle$ ACB)

and m ($\angle$ D) = m ($\angle$ E) = 90°

Prove that : m ($\angle$ DAB) = m ($\angle$ CAE)



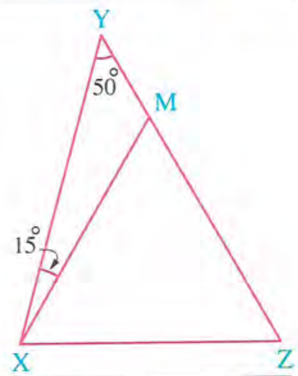
16 **In the opposite figure :**

YZX is a triangle in which YZ = YX

, m ($\angle$ Y) = 50°

and m ($\angle$ YXM) = 15°

Prove that : Δ MZX is an isosceles triangle.

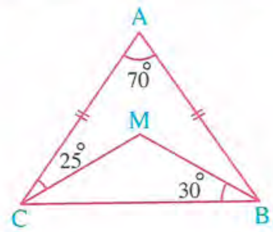


17 **In the opposite figure :**

ABC is a triangle in which AB = AC , m ($\angle$ A) = 70°

, m ($\angle$ MCA) = 25° and m ($\angle$ MBC) = 30°

Prove that : Δ MBC is an isosceles triangle.

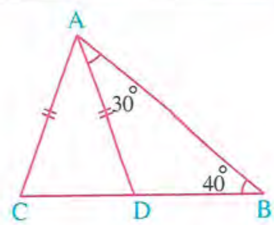


18 **In the opposite figure :**

AD = AC , m ($\angle$ B) = 40°

and m ($\angle$ BAD) = 30°

Prove that : AB = CB



19 ABC is a triangle in which AB = AC , $\overrightarrow{BD}$ bisects ∠ ABC and $\overrightarrow{CD}$ bisects ∠ ACB

Prove that : Δ DBC is an isosceles triangle.

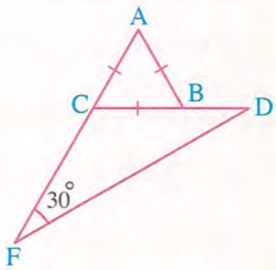
---

20 **In the opposite figure :**

ABC is an equilateral triangle , F ∈ $\overrightarrow{AC}$,

D ∈ $\overrightarrow{CB}$ and m (∠ DFC) = 30°

Prove that : Δ DCF is an isosceles triangle.



---

21 **In the opposite figure :**

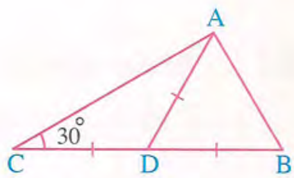
D ∈ $\overline{BC}$ such that DA = DB = DC

and m (∠ C) = 30°

Prove that :

1 Δ ABD is an equilateral triangle.

2 Δ ABC is a right-angled triangle.



---

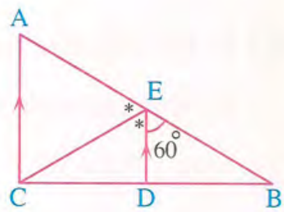
22 **In the opposite figure :**

ABC is a triangle in which E ∈ $\overline{AB}$,

$\overline{ED}$ // $\overline{AC}$, m (∠ BED) = 60°

and $\overrightarrow{EC}$ bisects ∠ AED

Prove that : Δ AEC is an equilateral triangle.



---

23 **In the opposite figure :**

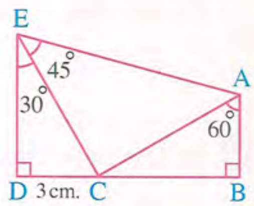
C ∈ $\overline{BD}$, m (∠ B) = m (∠ D) = 90° ,

m (∠ CED) = 30°

, m (∠ AEC) = 45° , m (∠ BAC) = 60°

and CD = 3 cm.

Find : The length of $\overline{AC}$



« 6 cm. »

---

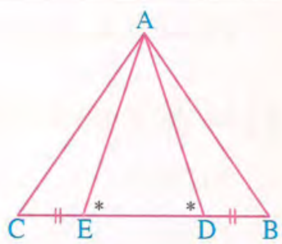
24 **In the opposite figure :**

∠ ADE ≡ ∠ AED

, B , D , E , C are collinear

and BD = CE

Prove that : Δ ABC is an isosceles triangle.

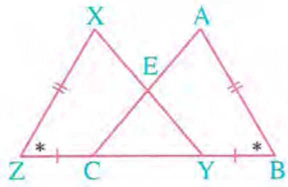


25 **In the opposite figure :**

$Y \in \overline{BZ}$, $C \in \overline{BZ}$, $AB = XZ$,

$BY = CZ$, $\overline{XY} \cap \overline{AC} = \{E\}$ and $m(\angle B) = m(\angle Z)$

Prove that : Δ EYC is an isosceles triangle.

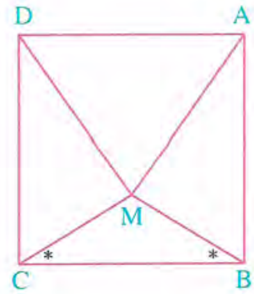


26 **In the opposite figure :**

ABCD is a square.

M is a point inside it such that : $m(\angle MBC) = m(\angle MCB)$

Prove that : Δ AMD is an isosceles triangle.



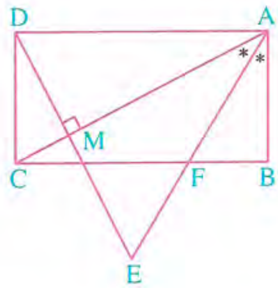
27 **In the opposite figure :**

ABCD is a rectangle in which

$\overline{AC}$ is a diagonal , $\overrightarrow{AE}$ bisects $\angle$ BAC

and $\overline{DE} \perp \overline{AC}$ where $\overrightarrow{AE} \cap \overrightarrow{DE} = \{E\}$, $\overline{AC} \cap \overline{DE} = \{M\}$

Prove that : DA = DE



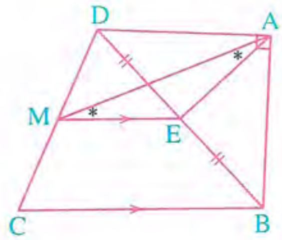
28 **In the opposite figure :**

ABCD is a quadrilateral in which

$m(\angle BAD) = 90°$, E is the midpoint of $\overline{BD}$ and $M \in \overline{DC}$

such that $\overline{EM} \parallel \overline{BC}$ and $m(\angle EAM) = m(\angle EMA)$

Prove that : BD = BC

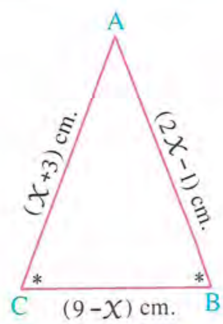


29 **In the opposite figure :**

ABC is a triangle in which :

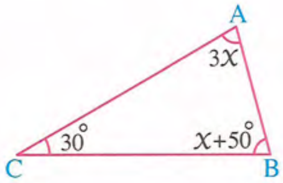
$m(\angle B) = m(\angle C)$

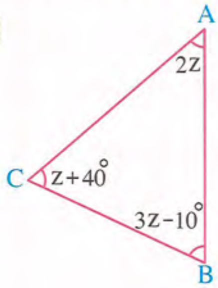
Find : The perimeter of the triangle.

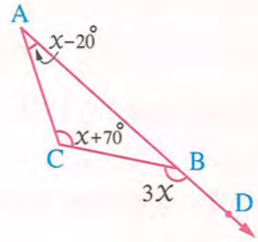


« 19 cm. »

30 **In each of the following figures , write the equal sides in length showing the steps of solution :**

1


2


3


For excellent pupils

31 **Choose the correct answer from those given :**

1 If the sum of measures of two congruent angles in a triangle = $\frac{2}{3}$ the sum of measures of its angles , then the triangle is ....

(a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.

2 ABC is a triangle in which m ($\angle$ A) = 30° and m ($\angle$ B) : m ($\angle$ C) = 1 : 4 , then Δ ABC is ....

(a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.

Wonders of numbers

- Pick any positive 2-digit number.
- Interchange the two digits to get a new number.
- Subtract the smaller number from the bigger number.
- Is the difference divisible by 9 ?

Do the exercise again using different numbers.

EXERCISE 5 ?

Corollaries of the isosceles triangle theorems

Interactive test

From the school book

Remember   Understand   Apply   Problem Solving

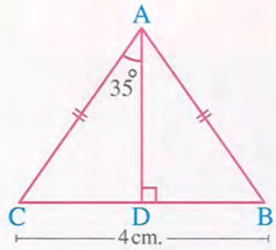
1 Complete the following :

1. The straight line drawn from the vertex of the isosceles triangle perpendicular to the base is called ....
2. The number of axes of symmetry in the equilateral triangle equals ....
3. The number of axes of symmetry in the isosceles triangle equals ....
4. The number of axes of symmetry in the scalene triangle equals ....
5. The median of the isosceles triangle drawn from the vertex angle ....
6. The bisector of the vertex angle of the isosceles triangle ....
7. The straight line passing through the vertex angle of the isosceles triangle perpendicular to its base ....
8. The axis of the line segment is ....
9. Any point belonging to the axis of a line segment is .... from its two terminals.
10. In Δ ABC , if $m(\angle A) = m(\angle B) = 60°$, then the number of axes of symmetry of Δ ABC is ....
11. In Δ ABC , if $m(\angle A) = m(\angle B) \neq 60°$, then the number of axes of symmetry of Δ ABC is ....
12. In Δ ABC , if AB = AC , $m(\angle A) = 60°$, then the number of axes of symmetry of Δ ABC is ....

2 **In the opposite figure :**

If AB = AC , $\overline{AD} \perp \overline{BC}$, BC = 4 cm. and

m (∠ DAC) = 35° , **complete the following :**



1 m (∠ BAD) = ...°

2 m (∠ BAC) = ...°

3 m (∠ B) = ...°

4 BD = ... cm.

5 The axis of symmetry of Δ ABC is ....

3 **Choose the correct answer from those given :**

1 If C ∈ the axis of symmetry of $\overline{AB}$, then AC – BC = ...

(a) zero (b) 1 (c) 2 (d) 4

2 In Δ XYZ , XY = XZ , $\overline{XE}$ is a median , if m (∠ YXE) = 30°
, then m (∠ YXZ) = ...

(a) 15° (b) 30° (c) 60° (d) 90°

3 In Δ LMN , LM = LN , E ∈ $\overline{MN}$ where $\overline{LE} \perp$ MN , if ME = 4 cm.
, then MN = ... cm.

(a) 12 (b) 8 (c) 4 (d) 2

4 If the measure of one angle in the right-angled triangle is 45° , then the number of axes of symmetry of the triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

5 In Δ ABC , m (∠ A) = 40° , m (∠ C) = 100° , then the number of axes of symmetry of the triangle is ....

(a) 1 (b) 2 (c) 3 (d) infinite number.

6 The triangle in which the measures of two angles in it are 45° , 65° , then the number of axes of symmetry of the triangle is ....

(a) zero (b) 3 (c) 2 (d) 1

7 An isosceles triangle , the measure of one of its angles is 60° , then the number of its axes of symmetry is ....

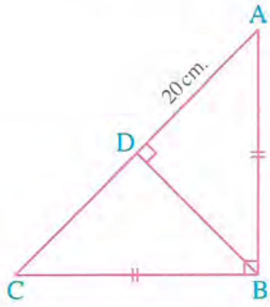
(a) 4 (b) 3 (c) 2 (d) 1

8 If Δ ABC has 1 axis of symmetry , m (∠ ABC) = 120° , m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

4 **In the opposite figure :**

ABC is a right-angled triangle at B and it is also an isosceles triangle , $\overline{BD} \perp \overline{AC}$ and AD = 20 cm. Find the length of $\overline{AC}$ and m (∠ DBC) , then deduce that Δ BDC is an isosceles triangle.



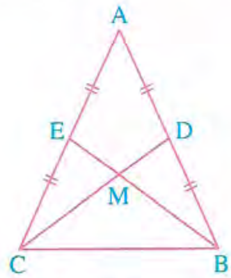
« 40 cm. , 45° »

5 **In the opposite figure :**

AB = AC , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and $\overline{BE} \cap \overline{CD} = \{M\}$

Prove that :

1. $\overrightarrow{AM} \perp \overline{BC}$
2. $\overrightarrow{AM}$ bisects ∠ BAC

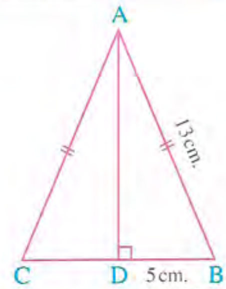


6 **In the opposite figure :**

In Δ ABC , AB = AC , $\overline{AD} \perp \overline{BC}$, AB = 13 cm. and BD = 5 cm.

Find :

1. The length of $\overline{BC}$
2. The area of Δ ABC

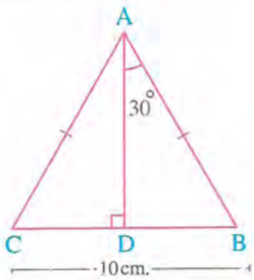


« 10 cm. , 60 cm.^{2} »

7 **In the opposite figure :**

AB = AC , BC = 10 cm. , m (∠ BAD) = 30° and $\overline{AD} \perp \overline{BC}$

1. Find the length of each of : $\overline{BD}$ and $\overline{AD}$
2. How many axes of symmetry are there at Δ ABC ?
3. Find the area of Δ ABC



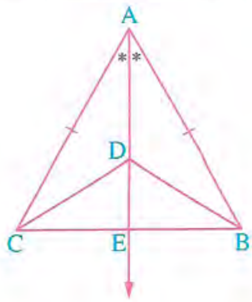
« 5 cm. , $5\sqrt{3}$ cm. , $25\sqrt{3}$ cm.^{2} »

8 **In the opposite figure :**

ABC is a triangle in which AB = AC , $\overrightarrow{AE}$ bisects ∠ BAC , $\overrightarrow{AE} \cap \overline{BC} = \{E\}$ and D ∈ $\overrightarrow{AE}$

Prove that :

1. $BE = \frac{1}{2} BC$
2. BD = CD



9 **In the opposite figure :**

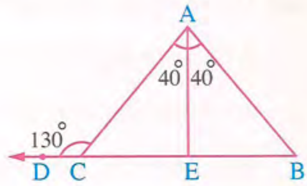
$C \in \overline{BD}$, m ($\angle$ ACD) = 130°

and m ($\angle$ BAE) = m ($\angle$ CAE) = 40°

Prove that :

1 $\overline{AE} \perp \overline{BC}$

2 E is the midpoint of $\overline{BC}$



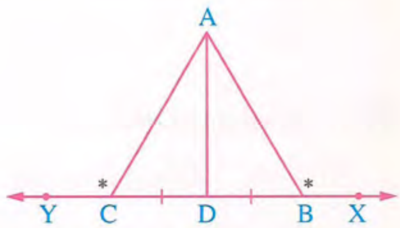
10 **In the opposite figure :**

X , B , C , D and Y are collinear points ,

$\overline{AD}$ is a median of Δ ABC and

m ($\angle$ ABX) = m ($\angle$ ACY)

Prove that : $\overline{AD} \perp \overline{BC}$



11 **In the opposite figure :**

ABCD is a quadrilateral in which

$\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ bisects $\angle$ ABC and

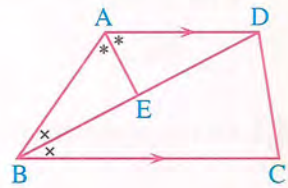
$\overrightarrow{AE}$ bisects $\angle$ BAD

Prove that :

1 AB = AD

2 $\overline{AE} \perp \overline{BD}$

3 BE = ED



12 **In the opposite figure :**

ABC is a triangle in which

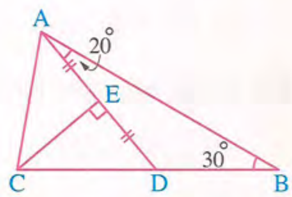
m ($\angle$ B) = 30° , $D \in \overline{BC}$

where m ($\angle$ BAD) = 20°

, E is the midpoint of $\overline{AD}$ and $\overline{CE} \perp \overline{AD}$

Find : m ($\angle$ ACE)

« 40° »



13 **In the opposite figure :**

ABC is a triangle in which

m ($\angle$ C) = 70° , $D \in \overline{BC}$

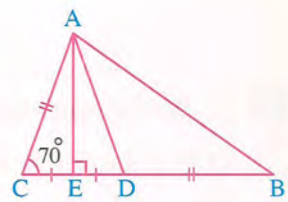
where BD = AC

, E is the midpoint of $\overline{DC}$

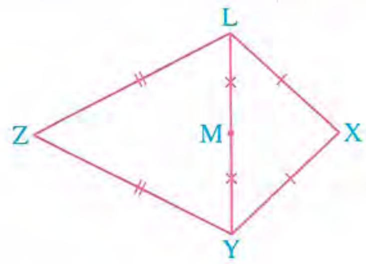
and $\overline{AE} \perp \overline{DC}$

Find : m ($\angle$ B)

« 35° »



14 **In the opposite figure :**

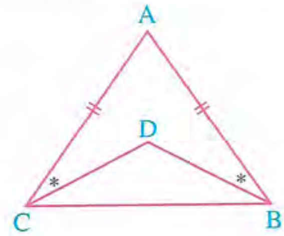


XY = XL , ZY = ZL and LM = YM

Prove that :

X , M and Z are on the same straight line.

15 **In the opposite figure :**

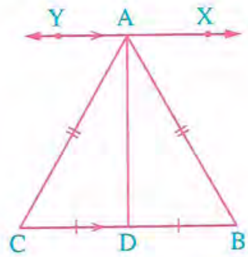


ABC is a triangle , D is a point inside it such that

m (∠ ABD) = m (∠ ACD) and AB = AC

Prove that : $\overleftrightarrow{AD}$ is the axis of symmetry of $\overline{BC}$

16 **In the opposite figure :**



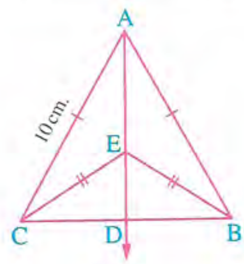
ABC is a triangle in which AB = AC ,

D is the midpoint of $\overline{BC}$ and $\overleftrightarrow{XY}$ passes through the vertex A

such that $\overleftrightarrow{XY}$ // $\overline{BC}$

Prove that : $\overline{AD} \perp \overleftrightarrow{XY}$

17 **In the opposite figure :**



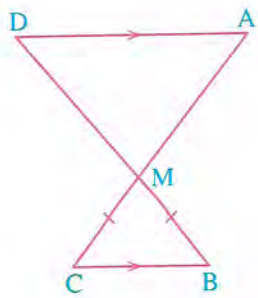
AB = AC = 10 cm. , EB = EC and $\overrightarrow{AE} \cap \overline{BC} = \{D\}$

Prove that : BD = DC and if BC = 6 cm.

Find the length of each of : $\overline{CD}$ and $\overline{AD}$

« 3 cm. , $\sqrt{91}$ cm. »

18 **In the opposite figure :**

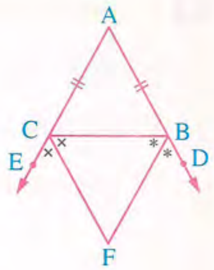


$\overline{AC} \cap \overline{BD} = \{M\}$, $\overline{AD}$ // $\overline{BC}$ and MB = MC

Prove that :

1. Δ AMD is an isosceles triangle.
2. The axis of symmetry of Δ AMD is the same of Δ BMC

19 **In the opposite figure :**



$AB = AC$, $D \in \overrightarrow{AB}$, $E \in \overrightarrow{AC}$,

$\overrightarrow{BF}$ bisects $\angle$ DBC and $\overrightarrow{CF}$ bisects $\angle$ BCE

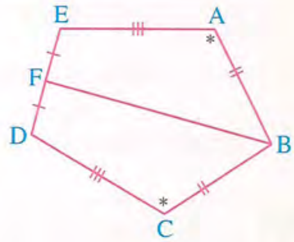
Prove that :

1 Δ BFC is an isosceles triangle.

2 $\overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$

---

20 **In the opposite figure :**



$AB = BC$, $AE = CD$,

$m(\angle BAE) = m(\angle BCD)$

and F is the midpoint of $\overline{DE}$

Prove that : $\overline{BF} \perp \overline{DE}$

---

21 **Choose the correct answer from those given :**

1 If ABCD is a quadrilateral in which $AB = AD$ and $BC = DC$, then $\overleftrightarrow{AC}$ is .... $\overline{BD}$

(a) parallel to (b) equal to

(c) the axis of symmetry of (d) congruent to

2 The triangle whose sides lengths are 2 cm. , $(X + 3)$ cm. and 5 cm. becomes an isosceles triangle when $X =$ .... cm.

(a) 1 (b) 2 (c) 3 (d) 4

3 If the length of any side in a triangle $= \frac{1}{3}$ of the perimeter of the triangle , then the number of axes of symmetry of the triangle equals ....

(a) 1 (b) 2 (c) 3 (d) zero

4 If $\overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$, then ....

(a) $AX = BY$ (b) $AX = BX$ (c) $BY = XY$ (d) $AY = BX$

5 In the rhombus ABCD , the axis of symmetry of $\overline{AC}$ is ....

(a) $\overleftrightarrow{BD}$ (b) $\overleftrightarrow{AB}$ (c) $\overleftrightarrow{AD}$ (d) $\overleftrightarrow{CD}$

6 In the square ABCD , $\overleftrightarrow{BD}$ is the axis of symmetry of ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{CD}$

For excellent pupils

22 **In the opposite figure :**

ABCD is a quadrilateral in which

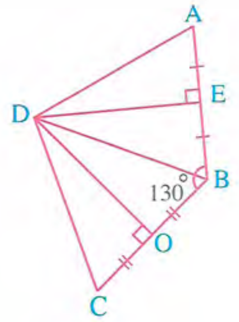
m (∠ ABC) = 130°

, E is the midpoint of $\overline{AB}$

, O is the midpoint of $\overline{BC}$

, $\overline{DE} \perp \overline{AB}$ and $\overline{DO} \perp \overline{BC}$

Find : m (∠ ADC)

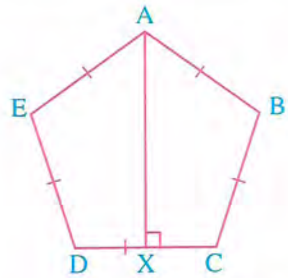


« 100° »

23 **In the opposite figure :**

ABCDE is a regular pentagon and $\overline{AX} \perp \overline{CD}$

Find : m (∠ DAX)



« 18° »

Wonders of numbers

Choose an integer from 1 to 9, multiply it by 9, then multiply the product by 123456789.

Where do you stand ?

A Research Project

On Unit Four

Project aims :

- Using geometrical instruments to make art designs.
- Using the properties of the equilateral triangle.
- Calculating the area of an equilateral triangle.
- Calculating the area of a square.
- Calculating the area of a geometrical shape consisting of a group of shapes.
- Associating geometry with arts.
- Associating geometry with science.

Do a research project on the following topic :

"Geometry is used in many fields of life. One of these is making art designs".

Discuss the following points using available resources :

1. Using geometrical instruments, design a logo of a fossil museum which consists of a square. On each side, draw an equilateral triangle.
2. Calculate the area of the resulted shape.
3. Decorate the logo with colours of your choice and stick a picture of one fossil inside the square.
4. Write a short note on the kinds of fossils and how to be formed, and mention an example of each kind.

UNIT 5

Inequality

Exercises of the unit :

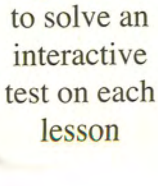
6. Inequality.

7. Comparing the measures of angles in a triangle.

8. Comparing the lengths of sides in a triangle.

9. Triangle inequality.

A research project on unit five



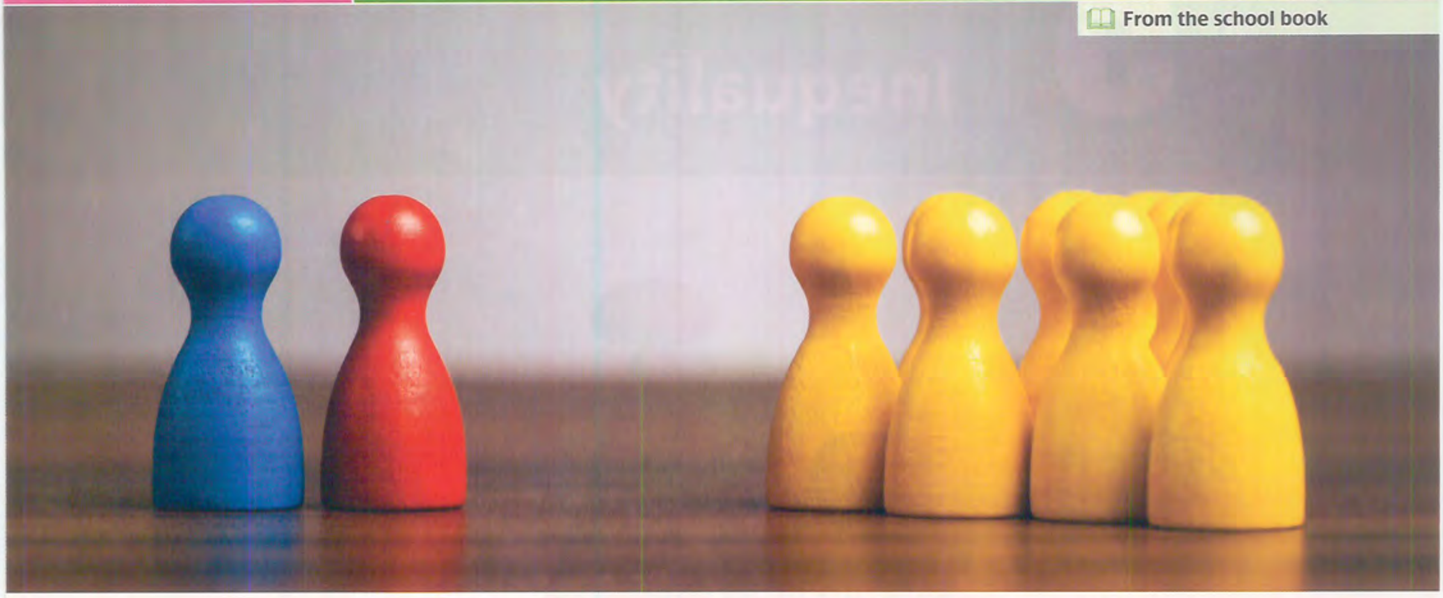
Scan the QR code to solve an interactive test on each lesson

EXERCISE **6**

Inequality

Interactive test

From the school book

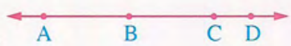


● Remember ● Understand ● Apply ● Problem Solving

1 **Complete each of the following using > or < :**

1 **In the opposite figure :**

If C and B belong to $\overleftrightarrow{AD}$ such that DC < BA , then AC .... DB



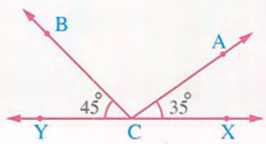
2 **In the opposite figure :**

If B and C belong to $\overleftrightarrow{AD}$ where AB > CD , then AC .... BD

3 **In the opposite figure :**

If C $\in \overleftrightarrow{XY}$, m (∠ ACX) = 35°

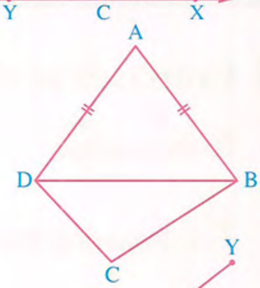
and m (∠ BCY) = 45° , then m (∠ XCB) .... m (∠ ACY)



4 **In the opposite figure :**

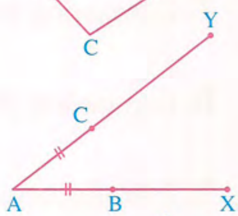
AB = AD , m (∠ DBC) < m (∠ CDB)

, then m (∠ ABC) .... m (∠ ADC)



5 **In the opposite figure :**

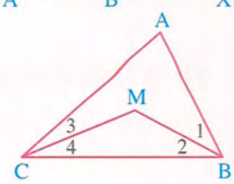
If AB = AC and AY > AX , then BX .... CY



6 **In the opposite figure :**

m (∠ 1) > m (∠ 3) , m (∠ 2) > m (∠ 4)

, then m (∠ ABC) .... m (∠ ACB)

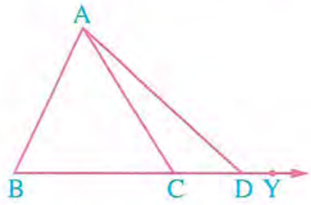


7 **In the opposite figure :**

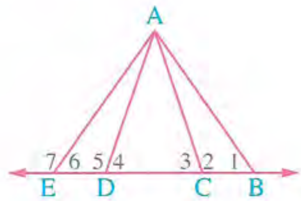
ABC is a triangle , $C \in \overline{BD}$ and $Y \in \overrightarrow{CD}$

, then m (∠ ADY) .... m (∠ DAC)

, m (∠ ABC) .... m (∠ ADY)



2 **Use the opposite figure to arrange the given measures ascendingly , where B , C , D and E are collinear :**



1 m (∠ 1) , m (∠ 3)

2 m (∠ 2) , m (∠ 4)

3 m (∠ 5) , m (∠ 3)

4 m (∠ 2) , m (∠ 6)

5 m (∠ 3) , m (∠ 1) , m (∠ 5)

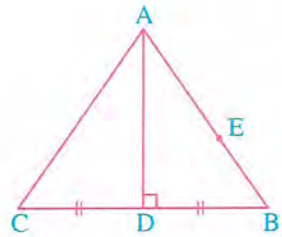
6 m (∠ 3) , m (∠ 5) , m (∠ 7)

7 m (∠ 3) , m (∠ 1) , m (∠ 7) , m (∠ 5)

3 **In the opposite figure :**

$E \in \overline{AB}$, $\overline{AD} \perp \overline{BC}$ and D is the midpoint of $\overline{BC}$

Prove that : AC > AE



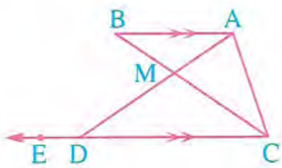
4 **In the opposite figure :**

$\overline{AB} \parallel \overrightarrow{CD}$, $\overline{AD} \cap \overline{BC} = \{M\}$, $E \in \overrightarrow{CD}$ and $E \notin \overline{CD}$

Prove that :

1 m (∠ ACD) > m (∠ ABC)

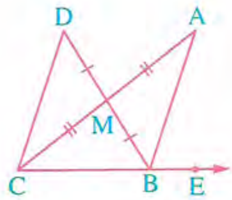
2 m (∠ ADE) > m (∠ ABC)



5 **In the opposite figure :**

$E \in \overrightarrow{CB}$ and M is the midpoint of each of $\overline{AC}$ and $\overline{BD}$

Prove that : m (∠ ABE) > m (∠ ACD)

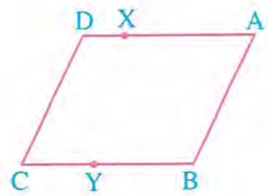


6 **In the opposite figure :**

ABCD is a parallelogram , $X \in \overline{AD}$ and $Y \in \overline{BC}$

such that DX < BY

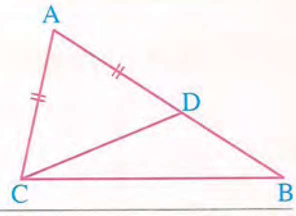
Prove that : AX + AB > CY + CD



7 **In the opposite figure :**

$D \in \overline{AB}$ where $AD = AC$

Prove that : $m(\angle ACB) > m(\angle B)$

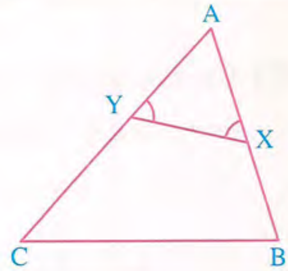


8 **In the opposite figure :**

ABC is a triangle in which : $AC > AB$, $X \in \overline{AB}$

and $Y \in \overline{AC}$ where $m(\angle AXY) = m(\angle AYX)$

Prove that : $YC > XB$

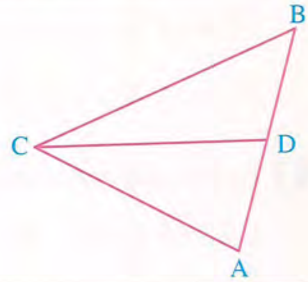


9 **In the opposite figure :**

ABC is a triangle in which :

$\overline{AB} \equiv \overline{AC}$ and $D \in \overline{AB}$

Prove that : $m(\angle ADC) > m(\angle ACB)$



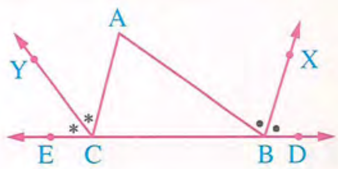
10 **In the opposite figure :**

$B \in \overleftrightarrow{DE}$, $C \in \overleftrightarrow{DE}$ such that

$m(\angle ACB) > m(\angle ABC)$

, $\overrightarrow{BX}$ bisects $\angle ABD$ and $\overrightarrow{CY}$ bisects $\angle ACE$

Prove that : $m(\angle ABX) > m(\angle ACY)$



11 M is a point inside the triangle ABC

Prove that : $m(\angle AMB) > m(\angle ACB)$

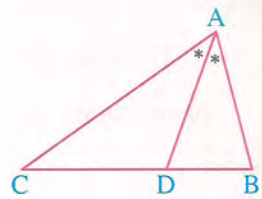
For excellent pupils

12 **In the opposite figure :**

ABC is a triangle in which : $m(\angle B) > m(\angle C)$, $D \in \overline{BC}$

such that $\overrightarrow{AD}$ bisects $\angle BAC$

Prove that : $\angle ADC$ is an obtuse angle.

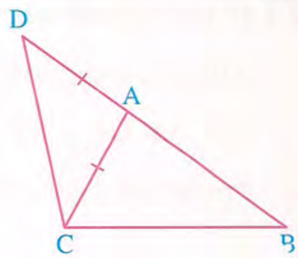


13 **In the opposite figure :**

ABC is a triangle in which : $m(\angle ACB) > m(\angle ABC)$

, $A \in \overline{BD}$ such that $AC = AD$

Prove that : $\angle BCD$ is an obtuse angle.



EXERCISE 7 ? Comparing the measures of angles in a triangle

Interactive test

From the school book

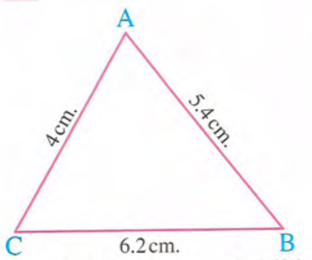
● Remember ● Understand ● Apply ● Problem Solving

1 **Complete the following :**

1. The lengths of two sides in a triangle are not equal , then the greater side in length is opposite to ....
2. In Δ ABC , AB = 7 cm. , BC = 5 cm. and AC = 6 cm. , then the smallest angle in measure is ....
3. In Δ DEF , if DE > EF , then m (∠ F) > ....
4. In any triangle ABC , if AB > AC > BC , then m (∠ ....) < m (∠ ....) < m (∠ ....)

2 **In each of the following figures , complete using (> or <) :**

1

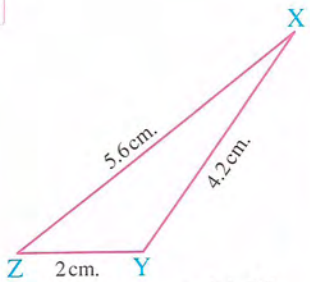


m (∠ A) .... m (∠ B)

m (∠ A) .... m (∠ C)

m (∠ B) .... m (∠ C)

2

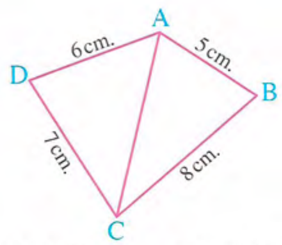


m (∠ Z) .... m (∠ Y)

m (∠ X) .... m (∠ Y)

m (∠ Z) .... m (∠ X)

3



m (∠ BAC) .... m (∠ BCA)

m (∠ DAC) .... m (∠ DCA)

m (∠ BAD) .... m (∠ BCD)

3 **Arrange the measures of the angles of Δ ABC in each of the following cases ascendingly :**

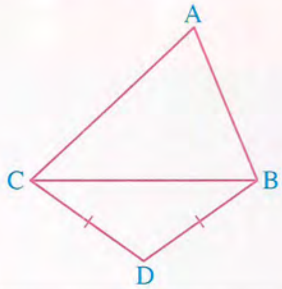
1. If AB = 12 cm. , BC = 15 cm. and AC = 10 cm.
2. If AB = 5.7 cm. , BC = 8.5 cm. and AC = 6 cm.

4 **In the opposite figure :**

AC > AB and DB = DC

Prove that :

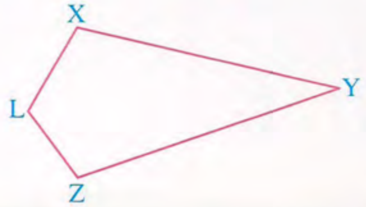
m (∠ ABD) > m (∠ ACD)



5 **In the opposite figure :**

XY > XL and YZ > ZL

Prove that : m (∠ XLZ) > m (∠ XYZ)

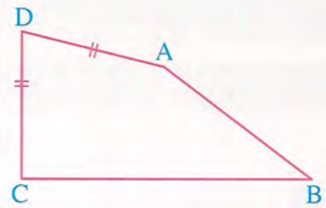


6 **In the opposite figure :**

ABCD is a quadrilateral in which :

AD = DC and BC > AB

Prove that : m (∠ A) > m (∠ C)



7 ABCD is a quadrilateral in which : $\overline{AB}$ is the longest side , $\overline{CD}$ is the shortest one

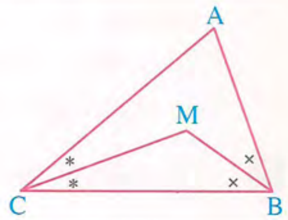
Prove that : m (∠ BCD) > m (∠ BAD)

8 **In the opposite figure :**

ABC is a triangle , $\overrightarrow{BM}$ bisects ∠ ABC and $\overrightarrow{CM}$ bisects ∠ ACB

If MC > MB

, prove that : m (∠ ABC) > m (∠ ACB)



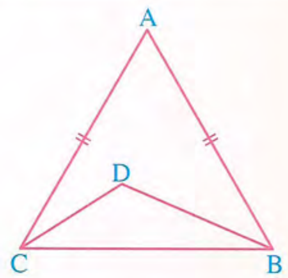
9 **In the opposite figure :**

ABC is a triangle in which :

AB = AC and DB > DC

Prove that :

m (∠ ABD) > m (∠ ACD)



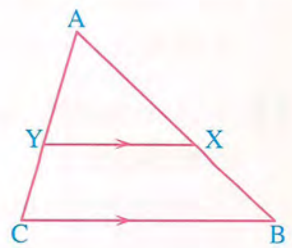
10 **In the opposite figure :**

ABC is a triangle ,

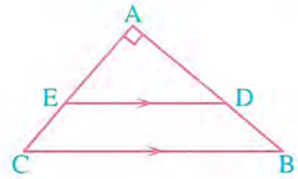
AB > AC and $\overline{XY}$ // $\overline{BC}$

Prove that :

m (∠ AYX) > m (∠ AXY)



11 **In the opposite figure :**

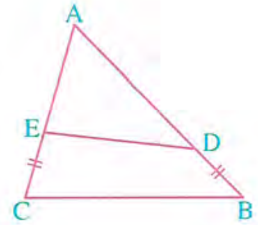


ABC is a triangle in which : m (∠ A) = 90° , AB > AC ,

D ∈ $\overline{AB}$, E ∈ $\overline{AC}$ and $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ AED) > 45°

12 **In the opposite figure :**



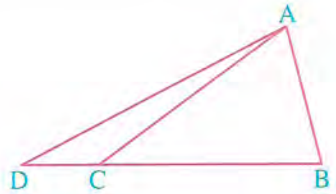
ABC is a triangle in which :

AB > AC , D ∈ $\overline{AB}$ and

E ∈ $\overline{AC}$ where BD = CE

Prove that : m (∠ AED) > m (∠ ADE)

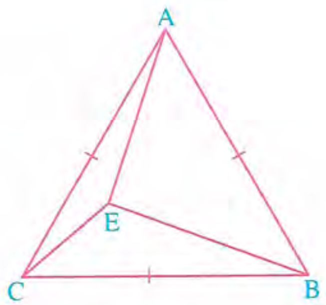
13 **In the opposite figure :**



C ∈ $\overline{BD}$ such that AC > AB

Prove that : m (∠ ABD) > m (∠ D)

14 **In the opposite figure :**



ABC is an equilateral triangle ,

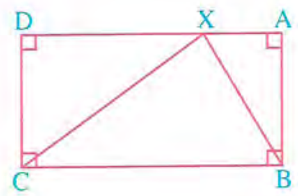
E is a point inside it ,

m (∠ ECB) > m (∠ EBC)

Prove that : 1 m (∠ ABE) > m (∠ ACE)

2 m (∠ A) > m (∠ ABE) > m (∠ ACE)

15 **In the opposite figure :**

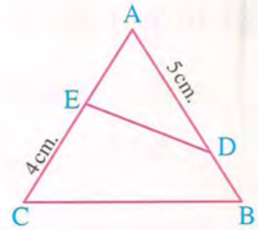


ABCD is a rectangle , X ∈ $\overline{AD}$

such that XC > XB

Prove that : m (∠ ABX) < m (∠ XCD)

16 **In the opposite figure :**



ABC is an equilateral triangle

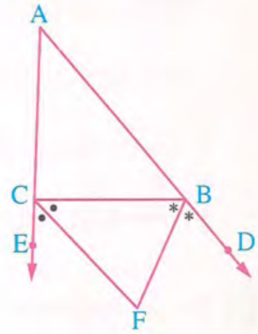
whose side length = 7 cm. , D $\in \overline{AB}$ such that

AD = 5 cm. and E $\in \overline{AC}$ such that CE = 4 cm.

Prove that : m ($\angle$ AED) > 60°

---

17 **In the opposite figure :**



ABC is a triangle in which :

AB > AC , D $\in \overrightarrow{AB}$, E $\in \overrightarrow{AC}$,

$\overrightarrow{BF}$ bisects $\angle$ DBC and $\overrightarrow{CF}$ bisects $\angle$ BCE

Prove that : m ($\angle$ FBC) > m ($\angle$ BCF)

---

18 ABC is a triangle , D is a point inside it. If DA > DB > DC

Prove that : m ($\angle$ ACB) > m ($\angle$ DAC) + m ($\angle$ DBC)

---

19 ABC is a triangle , $\overline{AD}$, $\overline{CE}$ are two medians intersecting at M If MD > ME

Prove that : m ($\angle$ CAM) < m ($\angle$ MCA)

---

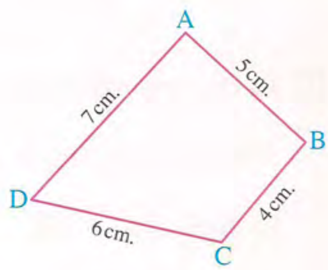
20 ABC is a triangle in which : AB > AC , D is the midpoint of $\overline{AB}$

Draw $\overrightarrow{DE}$ // $\overline{AC}$ to meet $\overline{BC}$ at E

Prove that : m ($\angle$ CAE) > m ($\angle$ DAE)

---

21 **In the opposite figure :**



ABCD is a quadrilateral in which :

AB = 5 cm. , BC = 4 cm. , CD = 6 cm. and DA = 7 cm.

Prove that : 1 m ($\angle$ ABC) > m ($\angle$ ADC)

2 m ($\angle$ BCD) > m ($\angle$ BAD)

3 m ($\angle$ B) + m ($\angle$ C) > 180°

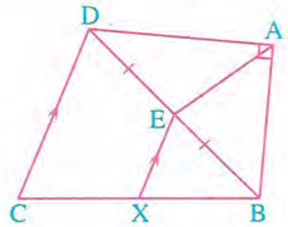
22 **In the opposite figure :**

ABCD is a quadrilateral in which : m ($\angle$ A) = 90° ,

$\overline{AE}$ is a median of Δ ABD , $\overrightarrow{EX}$ // $\overline{DC}$ and

$\overline{EX} \cap \overline{BC} = \{X\}$ If AE > EX

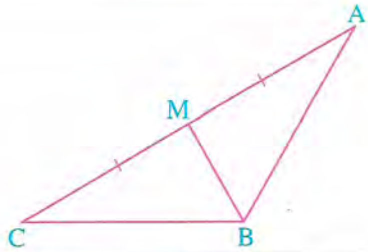
Prove that : m ($\angle$ C) > m ($\angle$ DBC)



23 **In the opposite figure :**

$\overline{BM}$ is a median in the triangle ABC and BM < AM

Prove that : $\angle$ ABC is an obtuse angle.

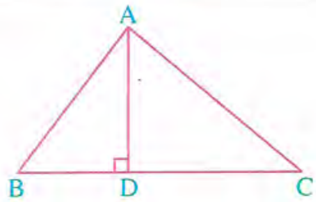


24 **In the opposite figure :**

ABC is a triangle in which : AC > AB , $\overline{AD} \perp \overline{BC}$

and intersects it at D

Prove that : m ($\angle$ BAD) < m ($\angle$ CAD)



25 ABC is a triangle , $\overrightarrow{AD}$ bisects $\angle$ A and intersects $\overline{BC}$ at D , if AC > AB

Prove that : $\angle$ ADC is an obtuse angle.

26 ABCD is a parallelogram in which : AC > BD

Prove that : $\angle$ D is an obtuse angle.

For excellent pupils

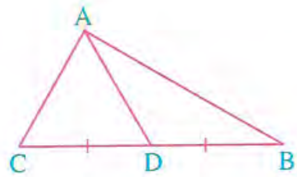
27 ABC is a triangle , D is the midpoint of $\overline{BC}$, if the perimeter of Δ ACD > the perimeter of Δ ABD

Prove that : m ($\angle$ B) > m ($\angle$ C)

28 **In the opposite figure :**

AB > AC and DB = DC

Prove that : m ($\angle$ BAD) < m ($\angle$ CAD)



EXERCISE **8** ?

Comparing the lengths of sides in a triangle

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 Complete the following :

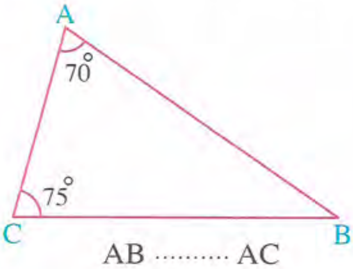
- 1 If two angles in a triangle are unequal in measure , then the greater angle in measure is opposite to .... and if the two lengths of two sides in a triangle are unequal , then the greater side in length is opposite to the angle which is ....
- 2 The smallest angle of a triangle (in measure) is opposite to ....
- 3 The longest side in the right-angled triangle is ....
- 4 The shortest distance between a given point and a given straight line is ....
- 5 ABC is a triangle in which : $m (\angle C) = 110^\circ$, then its longest side is ....
- 6 In Δ ABC : If $m (\angle A) = 50^\circ$, $m (\angle B) = 30^\circ$, then the shortest side in the triangle is ....
- 7 In Δ ABC : If $m (\angle A) = m (\angle B) + m (\angle C)$, then the longest side in the triangle is ....

2 Choose the correct answer from those ones :

- 1 In Δ ABC , if $m (\angle B) > m (\angle C)$, then ....
  (a) AB > AC (b) BC > AC (c) AC > AB (d) AB > BC
- 2 In Δ ABC , if $m (\angle B) = 90^\circ$, then ....
  (a) AC > CB (b) AB > AC (c) BC > AC (d) AB = AC
- 3 In Δ ABC , if $m (\angle A) = 40^\circ$ and $m (\angle B) = 70^\circ$, then ....
  (a) AB < AC (b) AB > AC (c) $\overline{AB} \perp \overline{AC}$ (d) AB = AC
- 4 In Δ XYZ , if $m (\angle X) = 110^\circ$, $m (\angle Y) = 40^\circ$, then XY .... XZ
  (a) < (b) > (c) = (d) //

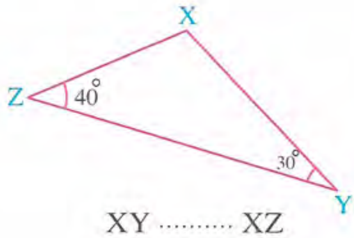
3 **In the following figures , complete using > , < or = :**

[1]



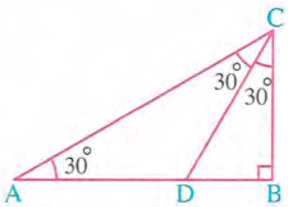
AB .... AC
AB .... BC
AC .... BC

[2]



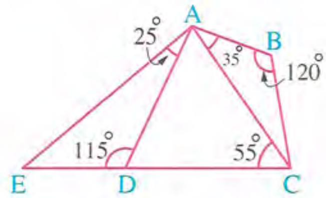
XY .... XZ
YZ .... XY
YZ .... XZ

[3]



AC .... BC
BC .... DB
AC .... BD
CA .... DC

[4]



BC .... AB
CD .... CA
AD .... AE
CD .... AD

4 XYZ is a triangle in which : m (∠ X) = 45° , m (∠ Y) = 85° and m (∠ Z) = 50°

Arrange the lengths of the sides of the triangle ascendingly.

5 ABC is a triangle in which : m (∠ A) = 40° and m (∠ B) = 75°

Order the lengths of the sides of the triangle descendingly.

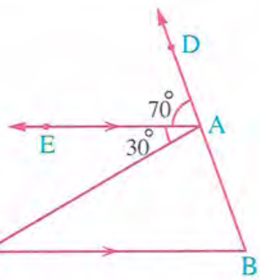
6 **In the opposite figure :**

$\overrightarrow{AE} \parallel \overline{BC}$,

m (∠ DAE) = 70°

and m (∠ EAC) = 30°

Prove that : AC > AB



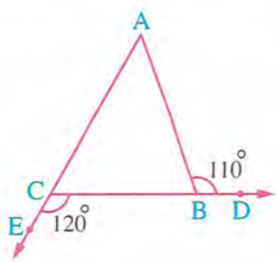
7 **In the opposite figure :**

ABC is a triangle , D ∈ $\overrightarrow{CB}$,

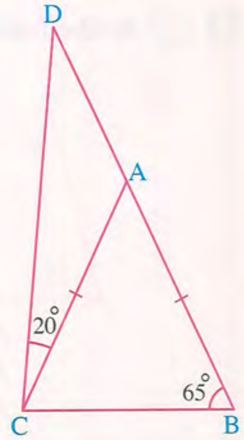
E ∈ $\overrightarrow{AC}$, m (∠ ABD) = 110°

and m (∠ BCE) = 120°

Prove that : AB > BC



8 **In the opposite figure :**

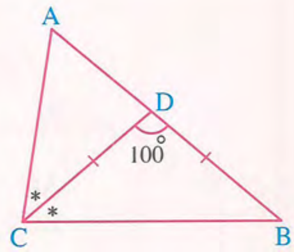


AB = AC , m (∠ ABC) = 65°

, m (∠ ACD) = 20° , A ∈ $\overline{BD}$

Prove that : AB > AD

9 **In the opposite figure :**

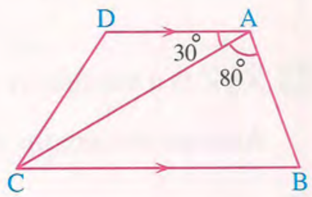


ABC is a triangle , $\overrightarrow{CD}$ bisects ∠ C and intersects $\overline{AB}$ at point D

, m (∠ BDC) = 100° and DB = DC

Prove that : AC > DB

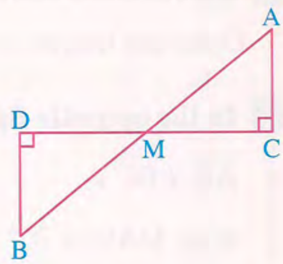
10 **In the opposite figure :**



$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 80° and m (∠ DAC) = 30°

Prove that : BC > AB

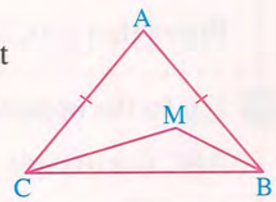
11 **In the opposite figure :**



$\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$ and $\overline{BD} \perp \overline{CD}$

Prove that : AB > CD

12 **In the opposite figure :**



ABC is a triangle in which : AB = AC , M is a point inside it such that

m (∠ ABM) < m (∠ ACM)

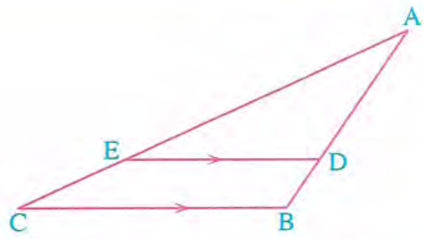
Prove that : MC > MB

13 **In the opposite figure :**

ABC is an obtuse-angled triangle at B

, $\overline{DE}$ // $\overline{BC}$

Prove that : AE > AD

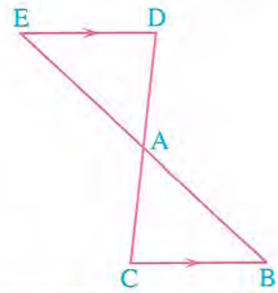


14 **In the opposite figure :**

AB > AC , $\overline{DE}$ // $\overline{BC}$ and

$\overline{DC} \cap \overline{BE} = \{A\}$

Prove that : AE > AD

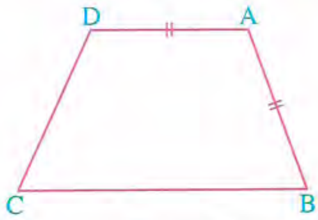


15 **In the opposite figure :**

ABCD is a quadrilateral , AB = AD

and m (∠ D) > m (∠ B)

Prove that : BC > CD



16 **In the opposite figure :**

ABC is a triangle in which : AB > AC , D $\in \overrightarrow{AB}$, E $\in \overrightarrow{AC}$

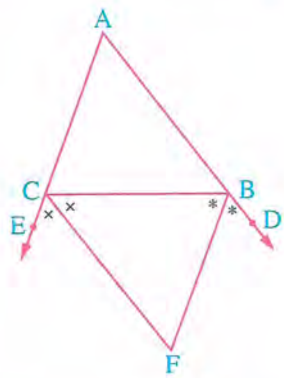
, $\overrightarrow{BF}$ bisects ∠ DBC and $\overrightarrow{CF}$ bisects ∠ BCE

, $\overrightarrow{BF} \cap \overrightarrow{CF} = \{F\}$

Prove that :

1 m (∠ FBC) > m (∠ BCF)

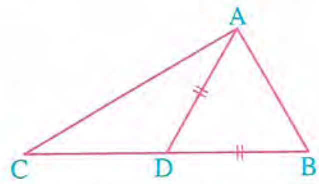
2 CF > BF



17 **In the opposite figure :**

ABC is a triangle and D $\in \overline{BC}$ where BD = AD

Prove that : BC > AC



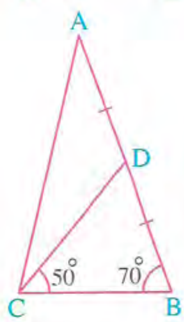
18 **In the opposite figure :**

D is the midpoint of $\overline{AB}$, m (∠ B) = 70° and m (∠ DCB) = 50°

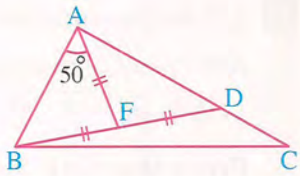
Prove that :

1 m (∠ A) > m (∠ ACD)

2 ∠ ACB is an acute angle.



19 **In the opposite figure :**

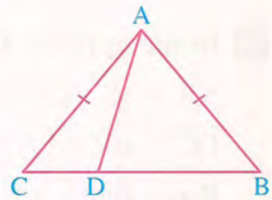


AF = BF = DF and m (∠ FAB) = 50°

Prove that : 1 AD > AB   2 BC > AC

---

20 **In the opposite figure :**

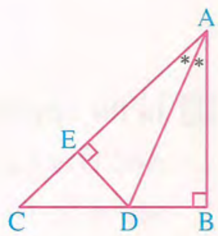


ABC is a triangle in which : AB = AC and D $\in \overline{BC}$

Prove that : AB > AD

---

21 **In the opposite figure :**



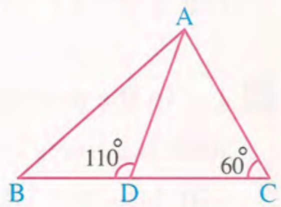
m (∠ B) = 90° , $\overline{DE} \perp \overline{AC}$ and $\overrightarrow{AD}$ bisects ∠ BAE

Prove that : 1 BD = DE

2 DC > DB

---

22 **In the opposite figure :**



m (∠ ADB) = 110° and m (∠ C) = 60°

Prove that : AB + AC > 2 AD

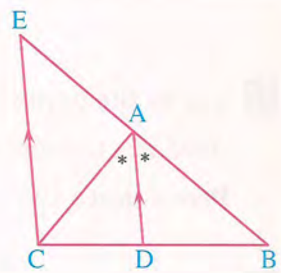
---

23 ABC is a right-angled triangle at B

Prove that : AB + BC < 2 AC

---

24 **In the opposite figure :**



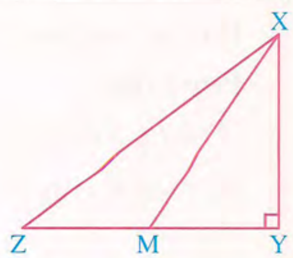
ABC is a triangle , $\overrightarrow{AD}$ bisects ∠ BAC

, $\overrightarrow{CE}$ // $\overline{DA}$ and cuts $\overrightarrow{BA}$ at E

Prove that : BE > BC

---

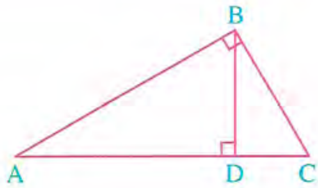
25 **In the opposite figure :**



XYZ is a right-angled triangle at Y and M $\in \overline{YZ}$

Prove that : XZ > XM

26 **In the opposite figure :**



m (∠ ABC) = 90° , $\overline{BD} \perp \overline{AC}$

and AB > BC

Prove that : AD > BD

27 ABC is a triangle , $\overrightarrow{CD}$ bisects ∠ C , $\overrightarrow{CD} \cap \overline{AB} = \{D\}$

Prove that : BC > BD

28 ABC is a right-angled triangle at B , D $\in \overline{AC}$ and E $\in \overline{BC}$ where AD = BE

Prove that : m (∠ CED) > m (∠ CDE)

29 ABC is a triangle in which : AB = AC and X $\in \overline{AC}$, draw $\overleftrightarrow{XY}$ to cut $\overline{AB}$ at Y and cut $\overrightarrow{CB}$ at Z

Prove that : AY > AX

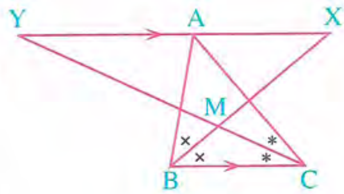
30 ABC is a triangle in which : m (∠ A) = $(5\,X + 2)°$,

m (∠ B) = $(6\,X - 10)°$ and m (∠ C) = $(X + 20)°$

Order the lengths of sides of the triangle ascendingly.

For excellent pupils

31 **In the opposite figure :**



AB < AC , ∠ B and ∠ C are bisected by two bisectors meeting at M , $\overrightarrow{BM}$ and $\overrightarrow{CM}$ intersect the straight line drawn from A parallel to $\overline{BC}$ at X and Y respectively.

Prove that : BX < CY

EXERCISE **9**

Triangle inequality

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 **Is it possible to draw a triangle whose side lengths are as follows ? Give reasons :**

1. 3 cm. , 4 cm. , 9 cm.
2. 5 cm. , 7 cm. , 8 cm.
3. 10 cm. , 6 cm. , 4 cm.
4. 13 cm. , 8 cm. , 6 cm.
5. 5 cm. , 3 cm. , 4 cm.
6. 9 cm. , 9 cm. , 19 cm.

2 **Find the interval to which the length of the third side of the triangle belongs in each of the following triangles if the lengths of the two other sides are :**

1. 6 cm. , 9 cm.
2. 3 cm. , 3 cm.
3. 2.9 cm. , 3.2 cm.
4. 5.7 cm. , 7.3 cm.

3 **Choose the correct answer from those given :**

1. The sum of lengths of any two sides in a triangle is .... the length of the third side.

   (a) less than (b) greater than (c) equal (d) half

2. The length of any side in a triangle .... the sum of lengths of the other two sides.

   (a) $>$ (b) $<$ (c) $=$ (d) twice

3. Which of the following numbers cannot be the lengths of sides of a triangle ?

   (a) 7 , 7 , 5 (b) 9 , 9 , 9 (c) 3 , 6 , 12 (d) 3 , 4 , 5

4. If the lengths of two sides in a triangle are 7 cm. and 4 cm. , then the length of the third side can be ....

   (a) 1 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

5 If the lengths of two sides of an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side is ....

(a) 7 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.

6 A triangle has one axis of symmetry, the lengths of two sides in it are 4 cm. and 8 cm. , then its perimeter = ...

(a) 16 cm. (b) 20 cm. (c) 24 cm. (d) 30 cm.

7 In Δ ABC , if AB = 3 cm. , BC = 5 cm. and AC = x cm. , then $x \in$ ....

(a) $]3, 5[$ (b) $]2, 5[$ (c) $]5, 8[$ (d) $]2, 8[$

8 If the lengths of two sides of a triangle are 5 cm. and 10 cm. , then the length of the third side belongs to ....

(a) $[10, 15[$ (b) $]5, 15[$ (c) $]5, 10]$ (d) $[10, 15]$

9 In Δ ABC : AB + BC – AC ....

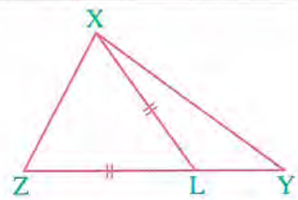
(a) > zero (b) < zero

(c) = zero (d) = the perimeter of the triangle ABC

10 In Δ ABC , $\frac{AB + BC}{AC}$ 1

(a) > (b) < (c) = (d) ≤

4 **In the opposite figure :**



XYZ is a triangle in which $L \in \overline{YZ}$ such that XL = LZ

Prove that : YZ > XY

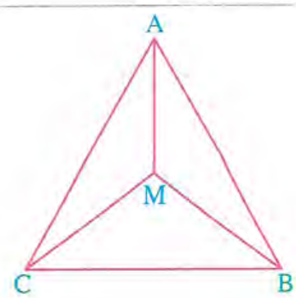
5 ABC is a triangle in which $\overline{BC}$ is the longest side , $D \in \overline{BC}$ such that CD = CA

Prove that : AB > BD

6 ABC is a triangle , $\overrightarrow{AD}$ is drawn to cut $\overline{BC}$ at D

Prove that : BD + DC + 2 AD > AB + AC

7 **In the opposite figure :**



ABC is a triangle in which M is a point inside it.

Prove that :

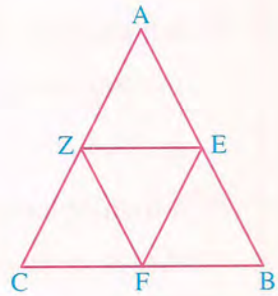
MA + MB + MC > $\frac{1}{2}$ the perimeter of the triangle ABC

8 **In the opposite figure :**

ABC is a triangle in which $E \in \overline{AB}$, $F \in \overline{BC}$ and $Z \in \overline{AC}$

Prove that :

The perimeter of Δ ABC > the perimeter of Δ EFZ

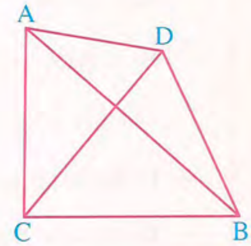


9 **In the opposite figure :**

ABC is a triangle and D is a point outside it.

Prove that :

The perimeter of Δ ABC < 2 (DA + DB + DC)

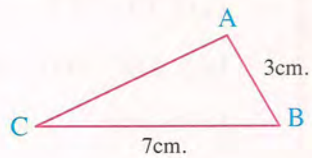


10 **In the opposite figure :**

ABC is a triangle in which :

AB = 3 cm. , BC = 7 cm.

Prove that: m ($\angle$C) < m ($\angle$B)



11 Prove that the length of any side in a triangle is less than half of the perimeter.

12 ABCD is a quadrilateral.

Prove that : AB + BC + CD > AD

13 Prove that the sum of the lengths of two diagonals in a convex quadrilateral is less than its perimeter.

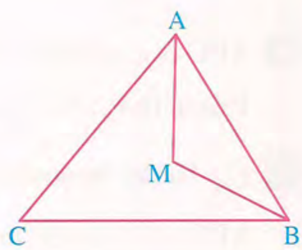
14 Prove that the perimeter of any quadrilateral is less than twice the sum of lengths of its diagonals.

For excellent pupils

15 **In the opposite figure :**

M is a point inside the triangle ABC

Prove that : AM + MB < AC + BC



16 ABC is a triangle and F is the midpoint of $\overline{BC}$ **Prove that :**

1. AB + AC > 2 AF
2. AB + AC > AF + BF

A Research Project

On Unit Five

Project aims :

- Using the triangle inequality to determine three numbers can be side lengths of a triangle.
- Drawing a triangle knowing the lengths of its sides.
- Recognizing the type of a triangle according to the lengths of its sides.
- Recognizing the type of a triangle according to the measures of its angles.
- Comparing between the measures of angles of a triangle.
- Associating geometry with history.

Do a research project on the following topic :

"Many Arab scientists excelled in the field of geometry".

Discuss the following points using available resources :

1. Write a short note about some Arab scientists and their contributions in geometry.
2. Select three numbers can be side lengths of a triangle.
3. Use a ruler and a compass to draw that triangle.
4. Determinc the type of that triangle according to the lengths of its sides and according to the measures of its angles.
5. Arrange the measures of the angles of that triangle in a descending order.

TIMSS Problems

Accumulative basic skills

1 Complete the following :

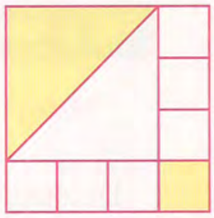
1. A lamppost of height 4.5 metres is 2 metres far from a building of height 10.5 metres , then the distance between the top of the lamppost and the top of the building is .... metres.

2. The ratio between the lateral and the total areas of a cube is ....

3. A cuboid is of lateral area 200 cm^2. and the dimensions of its base are 8 cm. and 12 cm. , then its height equals .... cm.

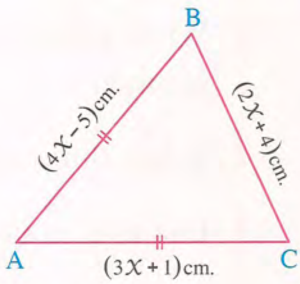
4. The measure of the angle between the two hands of the clock at 7 o'clock in degrees is ....°

5. **In the opposite figure :**



A square divided into 7 small congruent squares and two congruent triangles. If the area of the coloured square = 4 cm^2. , then the area of the coloured triangle is .... cm^2.

6. **In the opposite figure :**



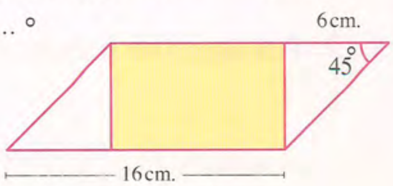
ABC is a triangle in which : AB = $(4X - 5)$ cm. , BC = $(2X + 4)$ cm. , AC = $(3X + 1)$ cm. , AB = AC , then the perimeter of Δ ABC = ... cm.

7. A rectangle its length is X cm. , its width is y cm. and its perimeter is P cm. , then the relation between X , y and P is X = ...

8. If the side length of an equilateral triangle is 10 cm. , then its height is .... cm.

9. The measure of the angle of the regular pentagon is .... °

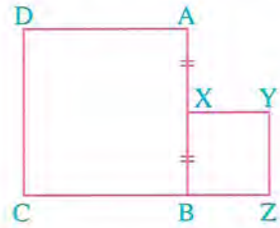
10. The opposite figure shows a coloured rectangle inside a parallelogram , then the area of the rectangle equals .... cm^2.



11 **In the opposite figure :**

If the perimeter of the square ABCD = 24 cm.

, then the area of the square XYZB is .... cm^2.



12 A cuboid is of total area 148 cm^2. and its lateral area is 110 cm^2. , then the area of its base is .... cm^2.

2 **Choose the correct answer from the given ones :**

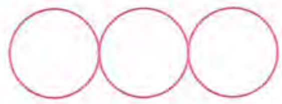
1 The acute angle supplements .... angle.

(a) an acute (b) an obtuse (c) a right (d) a reflex

2 The number of diagonals of the hexagon equals ....

(a) 3 (b) 6 (c) 9 (d) 12

3 The number of axes of symmetry of the opposite shape is ....



(a) 1 (b) 2 (c) 3 (d) 4

4 A wire in the shape of an equilateral triangle of side length 4 cm. is reshaped as a square, then the side length of the square is .... cm.

(a) 12 (b) 16 (c) 4 (d) 3

5 **In the opposite figure :**

A circle of radius length 2 cm. touches two sides of a square , then the area of the coloured part is .... cm^2.

(a) $4 - \pi$ (b) $\pi - 2$ (c) $\frac{\pi}{2}$ (d) 2π

6 The ratio between the area of a square region of side length ℓ cm. and the area of a square region of side length 2ℓ cm. is ....

(a) 1 : 2 (b) ℓ : 4 (c) 1 : 4 (d) 4 : 1

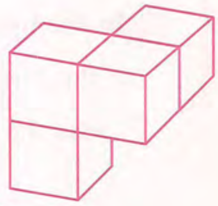
7 On a map , each 1 cm. represents 5 km. If the distance between two places is $\frac{1}{2}$ km. , then the distance between them on the map is ....

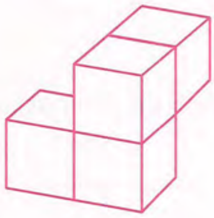
(a) 0.1 cm. (b) 10 cm. (c) 2.5 cm. (d) 0.4 cm.

8 If the area of the base of a cuboid is 12 cm^2. and the areas of two side faces are 6 cm^2. and 8 cm^2. , then the volume of the cuboid is .... cm^3.

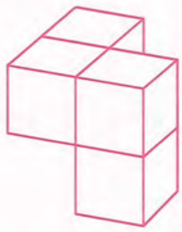
(a) 9 (b) 576 (c) 24 (d) 32

9 This solid will be rotated to another position. Which of the following may be a position of the solid after rotation ?

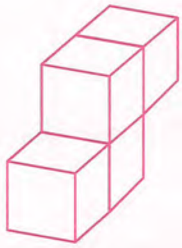




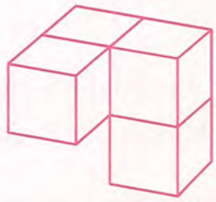
(a)



(b)

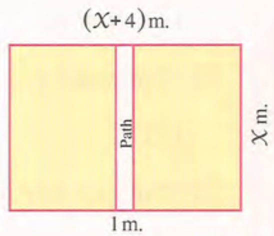


(c)



(d)

10 **In the opposite figure :**



A rectangular garden with a rectangular path of width 1 metre. Which expression shows the area of the coloured part of the garden in square metres ?

(a) $x^2 + 3x$ (b) $x^2 + 4x$

(c) $x^2 + 4x - 1$ (d) $x^2 + 3x - 1$

11 The opposite figure represents a quarter of a circle of radius length 2 cm. , then the perimeter of the figure in centimetres is ....

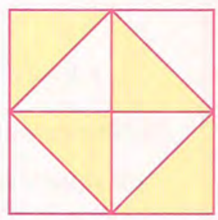
(a) 2π (b) 5π

(c) $\pi + 4$ (d) $4\pi + 4$

12 The area of a square whose side length is an integer may be .... cm^2.

(a) 600 (b) 900 (c) 800 (d) 700

13 **In the opposite figure :**

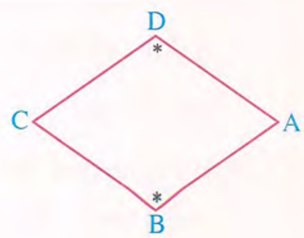


A square of perimeter 32 cm. divided into 8 congruent triangles , then the area of the coloured region is .... cm^2.

(a) 4 (b) 8

(c) 16 (d) 32

14 **In the opposite figure :**



If m ($\angle$ A) + m ($\angle$ C) = 140°

, m ($\angle$ B) = m ($\angle$ D)

, then m ($\angle$ B) = ...

(a) 50° (b) 55°

(c) 110° (d) 220°

EL-MOASSER

By a group of supervisors

NOTEBOOK

- Accumulative Tests
- Important Questions
- Final Revision
- Final Examinations

2nd PREP.

FIRST TERM

Maths

GPS

AL TALABA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
e-mail: info@elmoasserbooks.com
www.elmoasserbooks.com
15014

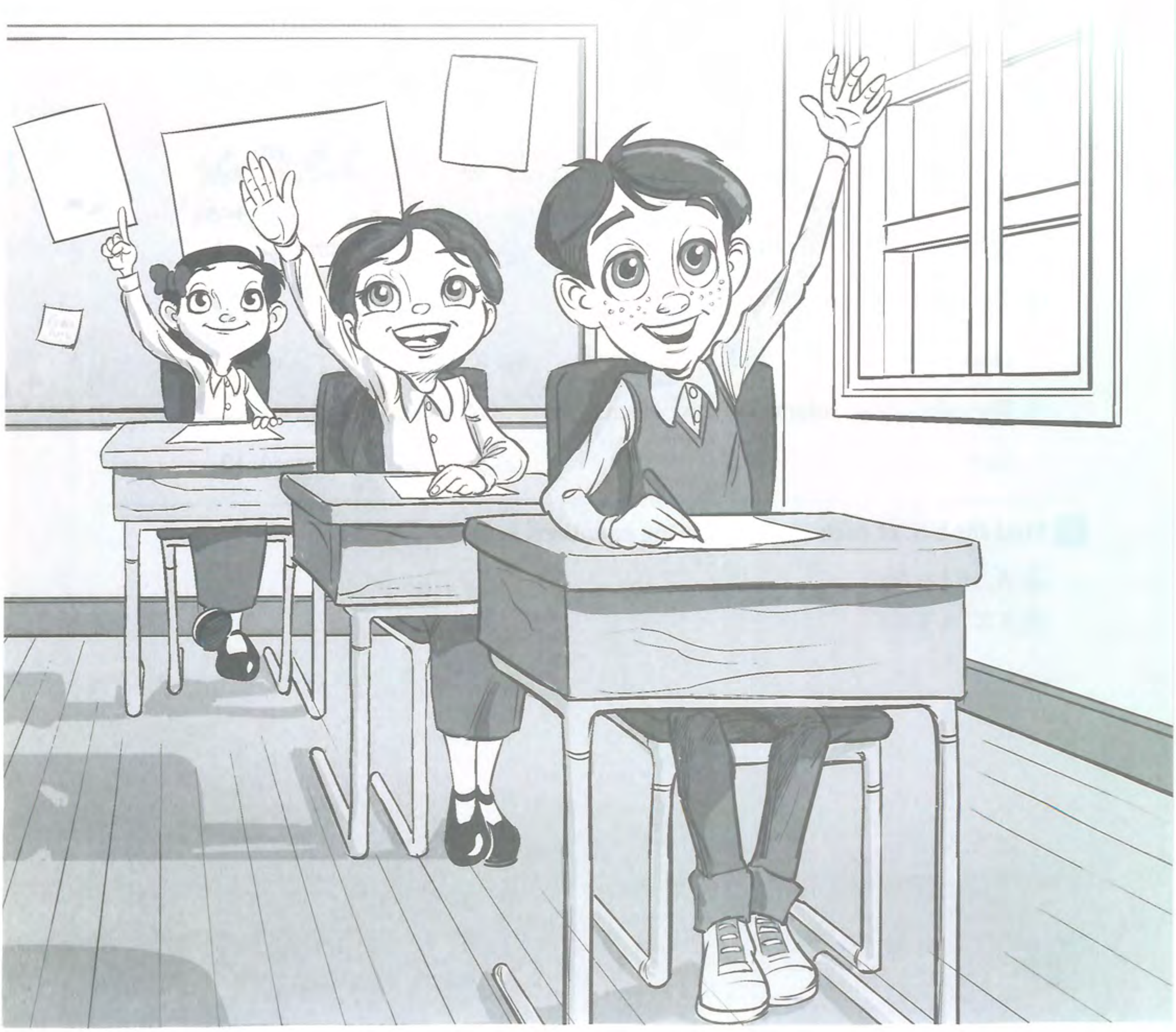
/ElMoasser.eg

Contents

First Algebra and Statistics

Accumulative Tests

on Algebra and Statistics



Accumulative test 1 on lesson 1 – unit 1

1 **Choose the correct answer from the given ones :**

1. $\sqrt[3]{2\frac{10}{27}} = \ldots\ldots\ldots\ldots$
   (a) $\frac{3}{4}$ (b) $\frac{10}{3}$ (c) $\frac{4}{3}$ (d) $\frac{20}{27}$
2. $\sqrt{25} - \sqrt[3]{125} = \ldots\ldots\ldots\ldots$
   (a) 10 (b) 5 (c) zero (d) – 5
3. $\sqrt{4} = \sqrt[3]{\ldots\ldots\ldots\ldots}$
   (a) 2 (b) 4 (c) 8 (d) 16
4. $\sqrt[3]{\ldots\ldots\ldots\ldots} + \sqrt[3]{27} = \sqrt{64}$
   (a) 25 (b) – 125 (c) 125 (d) 5
5. If $\sqrt[3]{x} = \frac{1}{4}$, then $x = \ldots\ldots\ldots\ldots$
   (a) $\frac{1}{2}$ (b) $\frac{1}{16}$ (c) $\frac{1}{64}$ (d) $\frac{1}{12}$
6. If $\sqrt[3]{x^2} = 4$, then $x = \ldots\ldots\ldots\ldots$
   (a) 8 (b) ± 8 (c) 4 (d) ± 4
7. $\sqrt[3]{x^6} = \sqrt{\ldots\ldots\ldots\ldots}$
   (a) x (b) x^2 (c) x^3 (d) x^4
8. The cube whose volume is 1 cm^3 , then the sum of all its edge lengths = ………… cm.
   (a) 1 (b) 6 (c) 8 (d) 12

---

2 **Find the S.S. of each of the following equations in $\mathbb{Q}$:**

1. $x^3 + 1 = \text{zero}$
2. $8x^3 + 7 = 8$

Accumulative test 2 till lesson 2 – unit 1

1 **Choose the correct answer from the given ones :**

1 $\sqrt{6} \in$ ....

(a) $\mathbb{N}$ (b) $\mathbb{Q}$ (c) $\grave{\mathbb{Q}}$ (d) $\mathbb{Z}$

2 The irrational number located between 2 and 3 is ....

(a) $\sqrt{7}$ (b) $\sqrt{10}$ (c) 2.5 (d) $\sqrt{3}$

3 If the volume of a cube is 125 cm^3, then the area of one of its faces is ....

(a) 25 cm^2 (b) 50 cm^2 (c) 100 cm^2 (d) 125 cm^2

4 The nearest integer to $\sqrt[3]{-28}$ is ....

(a) -4 (b) -30 (c) -3 (d) 3

5 If $x = \sqrt{2}$, $y = 2$, then which of the following does not represent a rational number ?

(a) $x^2 + y$ (b) $x + y^2$ (c) $\sqrt{x^2 y}$ (d) $\sqrt{2}\, x y$

6 If $x < \sqrt{7} < x + 1$, then $x =$ ....

(a) 4 (b) 3 (c) 2 (d) 5

7 $\sqrt[3]{-8} + \sqrt{4} =$ ....

(a) 4 (b) 2 (c) zero (d) 13

8 The S.S. of the equation : $x^2 - 16 =$ zero in $\grave{\mathbb{Q}}$ is ....

(a) $\varnothing$ (b) $\{-4\}$ (c) $\{-4, 4\}$ (d) $\{4\}$

---

2 **[a] Prove that :** $\sqrt{5}$ is included between 2.2 and 2.3

[b] Without using the calculator , prove that :

$\sqrt[3]{15}$ is included between 2.4 and 2.5

Accumulative test 3 till lesson 3 – unit 1

Choose the correct answer from the given ones :

1 The set of real numbers $\mathbb{R} = \cdots\cdots$

(a) $\mathbb{R}_+ \cup \mathbb{R}_-$ (b) $\mathbb{R}^* - \mathbb{R}_+$ (c) $\mathbb{Q} \cup \grave{\mathbb{Q}}$ (d) $\mathbb{Q} \cap \grave{\mathbb{Q}}$

2 $\mathbb{Q} \cap \grave{\mathbb{Q}} = \cdots\cdots$

(a) $\mathbb{Q}$ (b) $\grave{\mathbb{Q}}$ (c) $\mathbb{R}$ (d) $\varnothing$

3 $\mathbb{R}_+ \cup \mathbb{R}_- = \cdots\cdots$

(a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$ (c) $\mathbb{R}^*$ (d) $\mathbb{R}$

4 The irrational number located between 4 and 5 is $\cdots\cdots$

(a) $\sqrt{8}$ (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{10}$

5 $\sqrt[3]{9} \cdots\cdots \sqrt{4}$

(a) $>$ (b) $<$ (c) $=$ (d) $\leq$

6 Which of the following rational numbers is located between $\frac{1}{5}$ and $\frac{2}{5}$?

(a) $\frac{2}{10}$ (b) $\frac{1}{10}$ (c) 0.3 (d) -0.3

7 The S.S. of : $x^2 + 25 = $ zero in $\mathbb{Q}$ is $\cdots\cdots$

(a) $\varnothing$ (b) $\{-5, 5\}$ (c) $\{5\}$ (d) $\{-5\}$

8 The S.S. of the equation : $x^3 + 8 = 0$ in $\mathbb{R}$ is $\cdots\cdots$

(a) $\varnothing$ (b) $\{2\}$ (c) $\{-8\}$ (d) $\{-2\}$

9 $\sqrt[3]{64} - \sqrt{64} = \cdots\cdots$

(a) 4 (b) 8 (c) zero (d) -4

10 If $\frac{1}{a}$, $\frac{a}{\sqrt{5}}$ are two real numbers included between zero and 1 , then a can equal $\cdots\cdots$

(a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2

Accumulative test 4 till lesson 4 – unit 1

1 **Choose the correct answer from the given ones :**

1. The interval that represents : $X = \{x : x \in \mathbb{R}, -1 < x \leq 4\}$ is ……………

   (a) $[-4, 1]$ (b) $]-4, 1[$ (c) $[-1, 4[$ (d) $]-1, 4]$

2. $\{$The multiplicative identity element $, 3\}$ …………… $[0, 3]$

   (a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

3. $\mathbb{R} =$ ……………

   (a) $\mathbb{R}_+ \cup \mathbb{R}_-$ (b) $]-\infty, \infty[$ (c) $]-\infty, 0]$ (d) $[0, \infty[$

4. If $\sqrt{4} - \sqrt[3]{x} = 5$, then $x =$ ……………

   (a) 125 (b) 27 (c) -27 (d) 3

5. The square whose area is 10 cm^2 , then its side length is …………… cm.

   (a) 5 (b) -5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$

6. $[-5, 3] -]-5, 3[=$ ……………

   (a) $\{3\}$ (b) $\{-5\}$ (c) $\{-5, 3\}$ (d) $\{-3\}$

7. If x is a negative number , then which of the following numbers is positive ?

   (a) x^3 (b) $2x$ (c) x^2 (d) $\frac{x}{2}$

8. $]-\infty, 1[\cup]1, \infty[=$ ……………

   (a) $\mathbb{R}$ (b) $\{1\}$ (c) $\varnothing$ (d) $\mathbb{R} - \{1\}$

---

2 If $X = [2, 5]$, $Y = [0, 3]$

1. Write X using the description method.
2. Represent X , Y on the number line.
3. Find : X – Y as an interval by using the number line. Is $\sqrt{29} \in X - Y$?

---

3 If $X = [-1, 4]$, $Y = [3, \infty[$

, find using the number line each of : $X \cup Y$, $X \cap \grave{Y}$, $Y - X$

Accumulative test 5 till lesson 5 – unit 1

1 **Choose the correct answer from the given ones :**

1. $\frac{6}{\sqrt{3}} = \text{...............}$

   (a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{3}$ (d) $6\sqrt{3}$

2. $\left(2\sqrt[3]{2}\right)^3 = \text{...............}$

   (a) 4 (b) 8 (c) 16 (d) 40

3. The multiplicative inverse of $\frac{\sqrt{2}}{2}$ is ....

   (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 2 (d) $-\frac{\sqrt{2}}{2}$

4. $[-1 , 5] - \{-1 , 5\} = \text{...............}$

   (a) $\{-1 , 5\}$ (b) $]-1 , 5[$ (c) $]-1 , 5]$ (d) $[-1 , 5[$

5. $\mathbb{Q} \cup \mathbb{Q}' = \text{...............}$

   (a) zero (b) $\mathbb{Z}$ (c) $\mathbb{R}$ (d) $\varnothing$

6. The cube whose volume is 8 cm.^{3} , its total area = ... cm.^{2}

   (a) 16 (b) 24 (c) 64 (d) 8

7. The rectangle whose dimensions are $(\sqrt{7} - 1)$ cm. , $(\sqrt{7} + 1)$ cm. , its area is .... cm.^{2}

   (a) 8 (b) 7 (c) 6 (d) $2\sqrt{7}$

8. If $x = \sqrt{2} + 3$, $y = \sqrt{2} - 3$, then $x^2 - y^2 = \text{...............}$

   (a) $2\sqrt{3}$ (b) $12\sqrt{2}$ (c) $6\sqrt{5}$ (d) $3\sqrt{6}$

---

2 If $y = \sqrt{2 + \sqrt{3}}$, **find the value of :** $y^4 - 2y^2 + 1$

---

3 If $a = 5 - \sqrt{3}$, $b = 5 + \sqrt{3}$, **find in the simplest form "Showing steps"**

1. $a\,b$
2. $a^2 + b^2$

Accumulative test 6 till lesson 6 – unit 1

1 **Choose the correct answer from the given ones :**

1. The multiplicative inverse of the number $\sqrt{32}$ is ....

(a) $4\sqrt{2}$ (b) $\frac{\sqrt{2}}{8}$ (c) $2\sqrt{5}$ (d) $\frac{\sqrt{3}}{2}$

2. $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ... "In the same pattern"

(a) $\sqrt{75}$ (b) $\sqrt{90}$ (c) $\sqrt{112}$ (d) $\sqrt{125}$

3. $\sqrt{75} - \sqrt{27} - \sqrt{12} =$ ....

(a) $\sqrt{3}$ (b) zero (c) $-\sqrt{3}$ (d) $-2\sqrt{3}$

4. The additive inverse of the number $-\sqrt{5}$ is ....

(a) $\sqrt{5}$ (b) 5 (c) -5 (d) $\frac{-1}{\sqrt{5}}$

5. $]-1 , 3] \cap [-3 , -1] =$ ....

(a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$

6. The S.S. of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\varnothing$ (c) $\{-3\}$ (d) $\{-3 , 3\}$

7. $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} =$ ....

(a) $2\sqrt{2}$ (b) 1 (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

8. If $x = 2\sqrt{2} - \sqrt{7}$, $y = 2\sqrt{2} + \sqrt{7}$, then $x\,y - 1 =$ ....

(a) 5 (b) zero (c) -4 (d) 7

---

2 **If $A =]-\infty , 3[$, $B = [-2 , 5]$, find using the number line each of :**

1. $A \cap B$ 2. $B - A$

---

3 **Simplify each of the following to the simplest form :**

1. $\sqrt{18} + \sqrt{54} - 3\sqrt{2} - \frac{1}{2}\sqrt{24}$

2. $\sqrt{128} - \frac{14}{\sqrt{2}} + 6\sqrt{\frac{1}{2}} - \left(\sqrt{2}\right)^5$

Accumulative test 7 till lesson 7 – unit 1

1 **Choose the correct answer from the given ones :**

1. The conjugate number of the number : $\sqrt{3}+\sqrt{2}$ is ....

(a) $\sqrt{3}+\sqrt{2}$ (b) $\sqrt{3}-\sqrt{2}$ (c) $\sqrt{3}$ (d) $-\sqrt{2}$

2. The multiplicative inverse of the number : $1-\sqrt{2}$ is ....

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$ (c) $-\sqrt{2}-1$ (d) $1+\sqrt{2}$

3. If $X = 2+\sqrt{5}$, $y = 2-\sqrt{5}$, then $(X-y)^2 =$ ....

(a) $2\sqrt{8}$ (b) 20 (c) $4\sqrt{5}$ (d) -1

4. $]-3 , 2[\cap \mathbb{Z}_+ =$ ....

(a) $\{1\}$ (b) $\{1 , 2\}$ (c) $\{2 , -3\}$ (d) $\varnothing$

5. $\sqrt{16}-\sqrt[3]{-64} =$ ....

(a) zero (b) 12 (c) 8 (d) -8

6. The irrational number included between 3 and 6 is ....

(a) $\sqrt{5}$ (b) $\sqrt{10}$ (c) $\sqrt{25}$ (d) $\sqrt[3]{27}$

7. The multiplicative inverse of the number : $\frac{\sqrt{5}}{5}$ is ....

(a) $5\sqrt{5}$ (b) $-\sqrt{5}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$

8. If $X = \sqrt{7}+\sqrt{3}$ and y is the conjugate number of X , then $X y =$ ....

(a) 10 (b) 4 (c) 40 (d) 58

---

2 **[a]** If $X y = 1$, $y = 2+\sqrt{3}$, **find the value of :** $X^2+\sqrt{48}$ in its simplest form.

[b] Without using the calculator , simplify the following to the simplest form :

$$2\sqrt{5}\left(\sqrt{5}-2\right)+\sqrt{20}-10\sqrt{\frac{1}{5}}$$

---

3 If $X = \sqrt{5}+2$, $y =$ the multiplicative inverse of X , prove that X and y are conjugate numbers , **then find the value of :** $\left(\frac{X-y}{X+y}\right)^2$

Accumulative test 8 till lesson 8 – unit 1

1 **Choose the correct answer from the given ones :**

1 $\sqrt[3]{16} - \sqrt[3]{2} =$ ....

(a) $\sqrt[3]{14}$ (b) $\sqrt[3]{2}$ (c) $3\sqrt[3]{2}$ (d) 8

2 $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} =$ ....

(a) 0 (b) 1 (c) – 1 (d) 2

3 $\sqrt[3]{2} + \sqrt[3]{2} =$ ....

(a) $\sqrt[3]{16}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{4}$ (d) $\sqrt[3]{2}$

4 The S.S. of the equation : $x^3 = 27$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{0\}$

5 The set of non-positive real numbers are ....

(a) $[0 , \infty[$ (b) $]0 , \infty[$ (c) $]-\infty , 0]$ (d) $]-\infty , 0[$

6 $x = \sqrt[3]{3} + 1$, $y = \sqrt[3]{3} - 1$, then $x + y =$ ....

(a) $3\sqrt[3]{6}$ (b) $2\sqrt[3]{3}$ (c) 3 (d) 0

7 $\sqrt{12} + \sqrt{3} =$ ....

(a) 3 (b) $\sqrt{15}$ (c) $3\sqrt{3}$ (d) $3\sqrt{2}$

8 $\frac{5}{4}\sqrt[3]{5} + \frac{3}{4}\sqrt[3]{5} =$ ....

(a) 5 (b) $\sqrt[3]{20}$ (c) $\sqrt[3]{5}$ (d) $\sqrt[3]{40}$

---

2 **Simplify each of the following to the simplest form :**

1 $\sqrt[3]{54} + 4\sqrt[3]{\frac{1}{4}} - \sqrt[3]{-2}$

2 $\sqrt[3]{32} + 4\sqrt[3]{\frac{1}{2}} - \left(2\sqrt[3]{-2}\right)^2 + \left(\sqrt{2}\right)^{\text{zero}} - \left(\frac{2}{\sqrt{2}}\right)^2$

---

3 **[a]** If $X = [-2 , 3]$, $Y =]-\infty , 1]$

, find using the number line each of :

1 $X \cap Y$ 2 $X - Y$

[b] If $x = \frac{6}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2} - 1}$ **, find the value of :** $\left(y - \frac{1}{3}x\right)^2$

Accumulative test 9 till lesson 9 – unit 1

1 **Choose the correct answer from the given ones :**

1. If the volume of a sphere = $\frac{4}{3}\pi$ cm^3, then its radius = ... cm.
   (a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\left(\frac{4}{3}\right)^{zero}$ (d) π

2. The volume of a cube is 512 cm^3, then the perimeter of one face = ... cm.
   (a) 8 (b) 64 (c) 32 (d) 16

3. If the dimensions of a cuboid are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. and $\sqrt{6}$ cm.
   , then its volume = ... cm^3
   (a) 6 (b) 36 (c) $6\sqrt{6}$ (d) $18\sqrt{2}$

4. $\sqrt[3]{2}+\sqrt[3]{2}$ = ...
   (a) $\sqrt[3]{16}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{4}$ (d) $\sqrt[3]{2}$

5. If $a=\sqrt{7}+4$, $b=\sqrt{7}-4$, then a b = ...
   (a) 3 (b) 16 (c) 9 (d) -9

6. $\mathbb{R}_+ \cap [-1 , 3]$ = ...
   (a) $[0 , 3]$ (b) $]0 , 3]$ (c) $[0 , 3[$ (d) $]0 , 3[$

7. A right circular cylinder whose base area is 20 cm^2 and its volume is 80 cm^3
   , then its height = ... cm.
   (a) 3 (b) 4 (c) 5 (d) 100

8. A sphere and a cylinder are equal in volume and there radii are equal in length
   , then the height of the cylinder = ... the radius of the sphere.
   (a) 3 (b) 4 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

---

2 **[a]** Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 64π cm^3

[b] If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$, $y=\sqrt{7}-\sqrt{3}$

, **prove that :** x , y are two conjugate numbers , **then find the value of :** x y

---

3 **[a]** The volume of a sphere is 36π cm^3 Calculate its surface area in terms of π

[b] Simplify to the simplest form : $\sqrt{125}-\sqrt[3]{250}+\frac{1}{2}\sqrt[3]{16}+\sqrt{20}$

Accumulative test 10 till lesson 10 – unit 1

1 **Choose the correct answer from the given ones :**

1. The S.S. of the equation : $x + 5 = |-5|$ in $\mathbb{N}$ is ....

(a) $\varnothing$ (b) $\{0\}$ (c) $\{10\}$ (d) $\{-10\}$

2. The S.S. of the inequality : $-2x \geq 6$ in $\mathbb{R}$ is the interval ....

(a) $]-\infty , -3]$ (b) $[3 , \infty[$ (c) $]3 , \infty[$ (d) $]-\infty , -3[$

3. The S.S. of the equation : $\sqrt{3}\,x - 1 = 2$ in $\mathbb{R}$ is ....

(a) $\{2\sqrt{3}\}$ (b) $\{\sqrt{3}\}$ (c) $\{2\}$ (d) $\{2\sqrt{2}\}$

4. If three quarters of the volume of a sphere is 8π cm.^{3} , then its radius length is .... cm.

(a) 64 (b) 8 (c) 4 (d) 2

5. The S.S. of the equation : $(x^2 + 9)(x^3 + 1) =$ zero in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) 1 (c) $\{-1\}$ (d) $\{3 , 1\}$

6. The irrational number included between 2 and 3 is ....

(a) $2\frac{1}{2}$ (b) $\sqrt{10}$ (c) $\sqrt{7}$ (d) $\sqrt{3}$

7. The S.S. of the inequality : $x + 3 < 3$ in $\mathbb{R}$ is ....

(a) $[0 , \infty[$ (b) $\mathbb{R}_-$ (c) $]-\infty , 0]$ (d) $\mathbb{R}_+$

8. The S.S. of the inequality : $-2 < 3x + 7 \leq 10$ in $\mathbb{R}$ is ....

(a) $]-3 , 1]$ (b) $]1 , 3]$ (c) $[-3 , 1]$ (d) $[-3 , 1[$

---

2 **[a]** The volume of a sphere is $\frac{99000}{7}$ cm.^{3} , calculate its radius length. $\left(\pi = \frac{22}{7}\right)$

[b] **Find the S.S. of the inequality :** $-3 \leq 2x + 1 < 7$ in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line.

---

3 **[a]** If $X = [-1 , 4[$, $Y = [2 , 6]$ **using the number line find each of the following :**

1. $X \cup Y$ 2. $X \cap Y$

[b] **Find in $\mathbb{R}$ the S.S. of the inequality :** $x - 1 < 3 - x \leq x + 5$ in the form of an interval and represent it on the number line.

Accumulative test 11 till lesson 1 – unit 2

1 **Choose the correct answer from the given ones :**

1 If $(2, -5)$ satisfies the relation : $3X - y + k = 0$, then $k =$ ....

(a) 1 (b) 11 (c) -1 (d) -11

2 The relation : $2X + y = 6$ is represented by a straight line intersects the y-axis at the point ....

(a) $(0, -6)$ (b) $(0, 6)$ (c) $(6, 0)$ (d) $(3, 0)$

3 The relation : $2X = 3y$ is represented by a straight line passing through the point ....

(a) $(2, 3)$ (b) $\left(0, \frac{3}{2}\right)$ (c) $(0, 0)$ (d) $\left(\frac{2}{3}, 0\right)$

4 The S.S. of the equation : $X + 9 = |-5|$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\varnothing$ (c) $\{-4\}$ (d) $\{4\}$

5 The volume of a sphere is $\frac{32}{3}\pi$ cm.^{3}, then its radius length = ... cm.

(a) 2 (b) 4 (c) 8 (d) 32

6 The simplest form of the expression : $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$ is ....

(a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$

7 The point $(3k, 2k)$ lies on the straight line : $X - 3y = 9$, then $k =$ ....

(a) -3 (b) 1 (c) 0 (d) 2

8 The X-axis is the graphical representation of the relation $y =$ ....

(a) 1 (b) -1 (c) zero (d) X

---

2 **[a] Find four ordered pairs satisfying the relation :** $y + 2X = 5$

[b] Without using the calculator, simplify the following to the simplest form : "Showing steps"

$$\sqrt{12} + \sqrt[3]{54} + 3\sqrt{\frac{1}{3}} - 6\sqrt[3]{\frac{1}{4}}$$

---

3 **[a] Find in $\mathbb{R}$ the S.S. of the inequality :**

$-2X + 5 \le X - 4$ and represent it on the number line.

[b] Graph the relation : $X - 4y = 4$ and if the straight line representing the relation intersects the X-axis at the point A and the y-axis at the point B, find the area of the triangle OAB where O is the origin point.

Accumulative test 12 till lesson 2 – unit 2

1 **Choose the correct answer from the given ones :**

1 The slope of the straight line which passes through the two points (2 , 3) and (3 , 4) is ....

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) – 1

2 Which of the following ordered pairs satisfies the relation : $y = -x - 3$?

(a) (1 , – 4) (b) (– 2 , – 5) (c) (1 , – 2) (d) (3 , 0)

3 $\sqrt{16} - \sqrt[3]{-64} =$ ....

(a) zero (b) 12 (c) 8 (d) – 8

4 The slope of the straight line perpendicular on y-axis is ....

(a) positive. (b) negative. (c) zero. (d) undefined.

5 The slope of the vertical straight line is ....

(a) zero (b) 1 (c) – 1 (d) undefined.

6 The lateral area of the cube whose volume is 216 cm^3 equals .... cm^2

(a) 36 (b) 6 (c) 144 (d) 216

7 If the straight line which passes through the two points (2 , 3) and (5 , y) is parallel to x-axis , then y = ...

(a) 3 (b) – 3 (c) zero (d) $\frac{1}{3}$

8 If the slope of the straight line representing the relation : $x + m y = 5$ is undefined , then m = ...

(a) 1 (b) – 1 (c) 5 (d) zero

---

2 **[a]** Represent graphically , then find the slope of the straight line that represents the relation : $x + y = 7$

[b] Find the S.S. in $\mathbb{R}$ for the inequality :

$2x + 3 \leq 5x + 3 \leq 2x + 9$, then represent it on the number line.

---

3 **[a]** Prove that the points A , B and C are collinear where A (2 , – 3) , B (4 , – 5) and C (0 , –1)

[b] Simplify to the simplest form : $5\sqrt{8} + 4\sqrt[3]{\frac{1}{4}} - 2\sqrt{50} - \sqrt[3]{16}$

Accumulative test 13 till lesson 3 – unit 2

1 Choose the correct answer from the given ones :

1. If 100 gm. of food have 300 calories , then the number of calories that exist in 30 gm. of the same food equals .... calories.
   (a) 90 (b) 100 (c) 900 (d) 9000

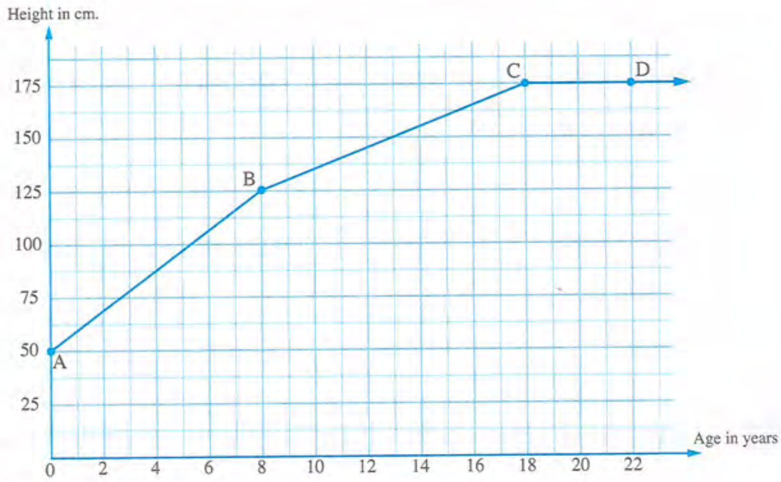
2. The slope of any straight line parallel to X-axis is ....
   (a) positive. (b) negative. (c) zero. (d) undefined.

3. The slope of the straight line which passes through the two points : (2 , k) and (4 , 7) equals 3 , then the value of k = ...
   (a) 5 (b) 4 (c) 3 (d) 1

4. The straight line which represents the relation : $4X = 3y$ is passing through the point ....
   (a) (4 , 3) (b) (3 , 4) (c) (4 , 0) (d) (0 , 3)

5. $\sqrt{9}$ .... $]-3 , \infty[$
   (a) $\subset$ (b) $\not\subset$ (c) $\in$ (d) $\notin$

2 The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :



1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$
2. Calculate the difference between the height of this person when he was 8 years old and his height when he was 30 years old.

3 [a] A right circular cylinder , its diameter length is 14 cm. and its height is 10 cm. , find the lateral area and the volume of the cylinder. $\left(\pi = \frac{22}{7}\right)$

[b] Represent graphically the relation : $y - 2X = 1$, then find the points of intersection of the straight line with the two axes.

4 If $\mathbb{R}_+$ is the set of the positive real numbers and $Z = [-2 , 3]$, **find :**

1. $\mathbb{R}_+ \cap Z$
2. $\mathbb{R}_+ \cup Z$
3. $\grave{\mathbb{R}}_+$

Accumulative test 14 till lesson 1 – unit 3

1 **Choose the correct answer from the given ones :**

1 If (5 , a) satisfies the relation : $x + y = 3$, then a = ...

(a) 2 (b) zero (c) – 2 (d) 8

2 The irrational number included between 3 and 4 is ....

(a) 1.5 (b) $\sqrt{5}$ (c) $\sqrt{11}$ (d) 3.5

3 The slope of the straight line passing through (3 , 2) and (4 , 2) is ....

(a) undefined. (b) $\frac{4}{7}$ (c) zero (d) $\frac{1}{7}$

4 $\sqrt{50} - \sqrt{8} =$ ....

(a) $\sqrt{42}$ (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) $\sqrt{58}$

---

2 **The following table shows the marks obtained by 30 students in an examination :**

5	9	11	4	9	9	16	7	8	12	2	10	7	12	5
8	15	13	13	9	7	14	19	3	11	14	3	12	13	7

Form the frequency table to these data.

---

3 **[a] Find the S.S. in $\mathbb{R}$ of the following inequality as an interval :** $1 < 3x - 2 < 13$

[b] Find three ordered pairs satisfying the relation :

$y = x + 2$, then represent it graphically.

---

4 **[a] Simplify to the simplest form :** $4\sqrt{\frac{1}{2}} + \sqrt{32} - \sqrt{72}$

[b] If $x = \sqrt{5} + 2$, $x y = 1$

, **find :** y , then prove that x and y are two conjugate numbers.

Accumulative test 15 till lesson 2 – unit 3

1 **Choose the correct answer from the given ones :**

1. The slope of the straight line passes through (1 , 5) and (5 , – 3) is ....

   (a) 4 (b) – 2 (c) zero (d) undefined.

2. If $X = 2 + \sqrt{5}$, $y = 2 - \sqrt{5}$, then $(X - y)^2 =$ ....

   (a) $2\sqrt{8}$ (b) 20 (c) $4\sqrt{5}$ (d) – 1

3. The figure 

represents the interval ....

   (a) [3 , 7[(b) [3 , 7] (c)]3 , 7[(d)]3 , 7]

4. If the point (3 , a) lies on the straight line : $y + 2X = 5$, then a = ...

   (a) 1 (b) – 1 (c) 11 (d) zero

---

2 **The following table is the frequency distribution of wages of 100 workers weekly :**

Sets	50 –	60 –	70 –	80 –	90 –	Total
Frequency	5	15	30	40	10	100

1. Find the number of workers whose wages are less than 70 pounds weekly.
2. Graph the ascending cumulative frequency curve.

---

3 **Find the S.S. in $\mathbb{R}$ of each of the following :**

1. $2\sqrt{2}\,X - 1 = 3$
2. $-5 \le 2X - 3 < 7$ and represent the S.S. on the number line.

---

4 **[a]** Find the slope of $\overleftrightarrow{AB}$, where : A (– 1 , 3) and B (2 , 5) , is the point C (8 , 1) $\in \overleftrightarrow{AB}$?

[b] If X = [2 , ∞[and y =]–2 , 3[, **find using the number line :**

1. $X \cap Y$ 2. $X \cup Y$ 3. $X - Y$

Accumulative test 16 till lesson 3 – unit 3

1 Choose the correct answer from the given ones :

1 The arithmetic mean of a frequency distribution equals ....

(a) $\frac{\text{sum of } (X \times f)}{\text{sum of } f}$ (b) $\frac{\text{sum of } (X + f)}{\text{sum of } f}$

(c) $\frac{\text{sum of } f \times \text{sum of } X}{\text{sum of } f}$ (d) sum of $(f + X) \times \frac{2}{\text{sum of } f}$

2 If the lower limit of a set is 15 and its centre is 20 , then its upper limit is ....

(a) 5 (b) 15 (c) 35 (d) 25

3 The arithmetic mean of the values : 18 , 23 , 2 k – 1 , 29 , k is 18 , then k = ...

(a) 1 (b) 7 (c) 29 (d) 90

4 The slope of any straight line parallel to X-axis is ....

(a) zero (b) undefined. (c) 1 (d) – 1

5 $\sqrt{a} + \sqrt{18} = 4\sqrt{2}$ if a = ...

(a) $\sqrt{2}$ (b) zero (c) 2 (d) 3

6 The conjugate of the number : $\sqrt{2} - \sqrt{3}$ is ....

(a) $\sqrt{2} + \sqrt{3}$ (b) $\sqrt{3} - 2$ (c) $\sqrt{2} - 3$ (d) $-\sqrt{2} + \sqrt{3}$

7 If the arithmetic mean of the lengths of a triangle equals 12 cm.
, then its perimeter = ... cm.

(a) 4 (b) 36 (c) 24 (d) 48

8 The mean of the values : $\sqrt{5}$ and $\sqrt{45}$ is ....

(a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) $3\sqrt{5}$ (d) $4\sqrt{5}$

---

2 The following table shows the frequency distribution of extra wages weekly for 100 workers in a factory :

Extra wages in pounds	20 –	30 –	40 –	50 –	X –	70 –
Number of workers	10	k	22	26	20	8

1 Calculate the value of each of X and k

2 Find the arithmetic mean of this distribution.

---

3 **[a]** A right circular cylinder of volume is 924 cm^3. and its height 6 cm.
Calculate the diameter length of its base $\left(\pi = \frac{22}{7}\right)$

[b] If $X = \frac{4}{\sqrt{7} - \sqrt{3}}$, $y = \frac{4}{\sqrt{7} + \sqrt{3}}$, put X and y in the simplest form
, then find the value of : $X^2 y^2$

Accumulative test 17 till lesson 4 – unit 3

1 Choose the correct answer from the given ones :

1 The median of the values : 8 , 4 , 5 , 3 and 7 is ....

(a) 8 (b) 5 (c) 3 (d) 7

2 The median of the values : 34 , 23 , 25 , 40 , 22 and 4 is ....

(a) 22 (b) 23 (c) 24 (d) 25

3 The order of the median of the values : 5 , 7 , 6 , 4 and 8 is ....

(a) third. (b) fourth. (c) fifth. (d) sixth.

4 If the order of the median of a set of values is the fourth , then the number of these values equals ....

(a) 3 (b) 5 (c) 7 (d) 9

5 The arithmetic mean of five numbers is 7 , then the sum of these numbers equals ....

(a) 12 (b) 35 (c) 21 (d) 18

6 (3 , 2) does not satisfy the relation ....

(a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $2y - x = 1$

7 The volume of the cuboid whose dimensions , are $\sqrt{2}$ cm. , $\sqrt{5}$ cm. and $\sqrt{10}$ cm. is .... cm^3

(a) 20 (b) 100 (c) 50 (d) 10

8 The intersection point of the ascending and descending cumulative curves is (30 , 50) , then the sum of frequencies is ....

(a) 30 (b) 50 (c) 100 (d) 60

---

2 [a] Graph the relation : $y = 2 - x$

[b] The following table shows a frequency distribution :

Sets	20 –	30 –	40 –	50 –	60 –	70 –	Total
Frequency	10	k	22	25	20	8	100

Find : 1 The value of k

2 The median using the descending cumulative frequency.

---

3 [a] Find the S.S. of the inequality : $-3 \le 2 - 5x \le 12$ where $x \in \mathbb{R}$

[b] If $X =]3 , 7]$, $Y = [5 , \infty[$ **by using the number line find :**

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

Accumulative test 18 till lesson 5 – unit 3

1 Choose the correct answer from the given ones :

1 The most common value or the most repeated value of a set of values is ....

(a) the arithmetic mean. (b) the median.

(c) the mode. (d) the range.

2 The mode of the values : 3 , 4 , 5 , 4 , 3 , 4 , 7 is ....

(a) 3 (b) 4 (c) 5 (d) 7

3 The mode of the values : 11 , 8 , $3X+2$, 11 , 5 is 11 , then $X=$ ....

(a) 2 (b) 1 (c) 4 (d) 3

4 The arithmetic mean of the values : k , – k , 3 k equals ....

(a) 3 k (b) 2 k (c) – k (d) k

5 If (k , 2 k) satisfies the relation : $3X-y=1$, then k = ...

(a) 1 (b) – 1 (c) $\frac{1}{2}$ (d) 5

6 $\sqrt{4}-\sqrt[3]{8}=$ ....

(a) 4 (b) – 2 (c) zero (d) – 4

7 If the volume of a sphere is 36π cm^3 , then its radius length = ...

(a) 3 (b) $\sqrt{3}$ (c) $\sqrt[3]{3}$ (d) 6

8 The mode of the values : 8 , $\sqrt{8}$, $\sqrt[3]{8}$, $2\sqrt{2}$ is ....

(a) 8 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

---

2 [a] Find the S.S. of the equation : $\sqrt{7}\,X+1=8$ in $\mathbb{R}$

[b] Reduce to the simplest form : $\left(\sqrt{5}-\sqrt{2}\right)^2+\sqrt{40}$

---

3 [a] Find the value of y such that the straight line passing through the two points (3 , 4) and (2 , y) is parallel to the X-axis.

[b] The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :

Sets of wages in L.E.	70–	80–	90–	100–	X–	120–	130–
Number of workers	10	13	k – 4	20	16	14	11

Find : 1 The value of each of X and k

2 The mode of wages in L.E. by using the histogram.

Important Questions

on Algebra and Statistics

Important questions on Unit One

First Multiple choice questions

1 $\sqrt[3]{-27}+3=$ ....

(a) zero (b) 3 (c) 6 (d) -24

2 The irrational number located between 2 and 3 is ....

(a) $\sqrt{3}$ (b) $\sqrt{-1}$ (c) $\sqrt{7}$ (d) $2\frac{1}{2}$

3 $\sqrt{8}-\sqrt{2}=$ ....

(a) $\sqrt{2}$ (b) $\sqrt{6}$ (c) $2\sqrt{2}$ (d) 2

4 $[3,7]-[2,5]=$ ....

(a) $[5,7]$ (b) $]5,7[$ (c) $\{5,7\}$ (d) $]5,7]$

5 The S.S. of the equation : $x^3=8$ in $\mathbb{Q}$ is ....

(a) $\{-2\}$ (b) $\{2\}$ (c) $\{2,-2\}$ (d) $\{64\}$

6 The multiplicative inverse of the number $\frac{\sqrt{3}}{6}$ is ....

(a) $\frac{-\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

7 A right circular cylinder of volume 90π cm^3. and height 10 cm. , then the diameter length of its base is ....

(a) 2 cm. (b) 4 cm. (c) 6 cm. (d) 3 cm.

8 If $\sqrt{9}=\sqrt[3]{x}$, then $x=$ ....

(a) 27 (b) 64 (c) -64 (d) -27

9 $[3,5]-\{5\}=$ ....

(a) $[3,4]$ (b) $[3,5[$ (c) $\{3,4\}$ (d) $]3,5]$

10 The volume of the cuboid whose dimensions are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. and $\sqrt{6}$ cm. is .... cm^3.

(a) 6 (b) 36 (c) $6\sqrt{6}$ (d) $18\sqrt{2}$

11 The S.S. of the inequality : $-X < 2$ in $\mathbb{R}$ is ....

(a) $]-\infty , 2[$ (b) $]-\infty , -2[$ (c) $]-2 , \infty[$ (d) $]2 , \infty[$

---

12 If $X < \sqrt[3]{36} < X + 1$, $X \in \mathbb{Z}$, then $X =$ ....

(a) 2 (b) 3 (c) 4 (d) 6

---

13 $\mathbb{R} =$ ....

(a) $[0 , \infty[$ (b) $]-\infty , \infty[$ (c) $]-\infty , 0[$ (d) $[1 , \infty[$

---

14 The volume of the sphere whose diameter length is 6 cm. equals .... cm^3.

(a) 9π (b) 12π (c) 36π (d) 228π

---

15 If the volume of a sphere is $\frac{500}{3}\pi$ cm^3, then its radius length is .... cm.

(a) 10 (b) $\frac{5}{3}$ (c) 5 (d) π

---

16 If three quarters of the volume of a sphere equals 8π cm^3, then the length of its radius equals .... cm.

(a) 64 (b) 8 (c) 4 (d) 2

---

17 The volume of a cube is 64 cm^3, then its lateral area is .... cm^2.

(a) 4 (b) 8 (c) 64 (d) 96

---

18 A right circular cylinder whose base radius length is r cm. and its height equals the length of its diameter , then its volume is .... cm^3.

(a) πr^3 (b) πr^2 (c) $2\pi r^3$ (d) $2 r^3$

---

19 $\mathbb{Q} \cap \grave{\mathbb{Q}} =$ ....

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{Q}$ (d) $\grave{\mathbb{Q}}$

---

20 The S.S. of the equation : $X^2 + 9 = 0$ in $\mathbb{R}$ is ....

(a) $\{-9\}$ (b) $\{-3\}$ (c) $\{3 , -3\}$ (d) $\varnothing$

---

21 The S.S. of the inequality : $-1 \le -X < 1$ in $\mathbb{R}$ is ....

(a) $]-1 , 1]$ (b) $[-1 , 1[$ (c) $]-1 , 1[$ (d) $[-1 , 1]$

---

22 The edge length of the cube whose volume is $2\sqrt{2}$ cm^3 equals .... cm.

(a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

23 The S.S. of the equation : $(x^2+4)(x^2-9)=0$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\{-3\}$ (c) $\{3, -3\}$ (d) $\varnothing$

---

24 A right circular cylinder whose radius length is 7 cm. and its height is 5 cm. , then its lateral area is .... cm^2.

(a) 50π (b) 70π (c) 9π (d) 35π

---

25 $\mathbb{R}_+ \cup \mathbb{R}_- =$ ....

(a) $\varnothing$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\mathbb{R}-\{0\}$

---

26 If $x>5$, then $\sqrt{(5-x)^2}=$ ....

(a) $5-x$ (b) $\sqrt{5}-\sqrt{x}$ (c) $x-5$ (d) $x+5$

Second Complete questions

1 $\mathbb{Q} \cup \grave{\mathbb{Q}} =$ ....

---

2 $\mathbb{Q} \cap \grave{\mathbb{Q}} =$ ....

---

3 If $\sqrt{a}=2\sqrt{3}$, then $a=$ ....

---

4 $\{-1, 0, 1\} \cap]-1, 1[=$ ....

---

5 $[3, 4]-\{3, 5\}=$ ....

---

6 The multiplicative inverse of the number $(\sqrt{3}+\sqrt{2})$ in the simplest form is ....

---

7 If $\frac{1}{x}=\sqrt{5}-2$, then the value of x in the simplest form is ....

---

8 $\mathbb{R}_+$ in an interval form is ....

---

9 $\sqrt{27}-\sqrt{3}=\sqrt{\text{..............}}$

---

10 The irrational number $\sqrt{10}$ lies between the two consecutive integers .... and ....

---

11 $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$, ... «in the same pattern»

---

12 The sum of lengths of all edges of a cube is 48 cm. , then its volume is ....

13 The volume of the sphere whose diameter length is 2 cm. is .... π cm^3.

14 If the volume of a sphere = $\frac{9}{2}\pi$ cm^3, then its diameter length = ... cm.

15 The rectangle whose dimensions are $(3-\sqrt{5})$ cm. and $(3+\sqrt{5})$ cm. its perimeter = ...

16 The circumference of a circle is $4\sqrt{5}\pi$ cm., then its area is .... cm^2.

17 $\sqrt{9+16} = 3 +$ ....

18 The additive inverse of the number $\sqrt[3]{-8}$ is ....

19 If $x = \sqrt{3}+2$, $y = \sqrt{3}-2$, then $(x y , x + y) = ($ .... , ... $)$

20 A right circular cylinder whose volume is 125π cm^3, its height = its radius length , then its radius length is .... cm.

21 $]-2 , 3] \cap \mathbb{R} =$ ....

22 $[-1 , 2[\cap \mathbb{Z} =$ ....

23 $\mathbb{R}_- \cap [-3 , 2[=$ ....

24 $\sqrt{a}+\sqrt{18} = 4\sqrt{2}$, if a = ...

Third Essay questions

1 If $X = [-3 , 2[$, $Y = [-1 , 5]$, **find using the number line :**

1 $X \cup Y$ 2 $X \cap Y$ 3 $X - Y$

2 If $A =]-\infty , 3[$, $B = [-2 , 5]$

, **find using the number line :** $B - A$, $A \cap B$, $A \cup B$ and $\grave{A}$

3 If $x = \sqrt{3}+1$, $y = \frac{2}{\sqrt{3}+1}$, **find the value of :** $\frac{x y}{x - y}$

4 If $x = \frac{1}{\sqrt{5}+2}$, $y = \sqrt{5}+2$, **prove that :** x and y are conjugate numbers

, **then find the value of :** $x^2 y^2$ in its simplest form.

5 If $x = \sqrt{7}-\sqrt{5}$, $y = \frac{2}{\sqrt{7}-\sqrt{5}}$, **find the value of :** $x^2 + 2 x y + y^2$

6 If $x = \sqrt{7} + \sqrt{5}$, $y = \frac{2}{x}$, **prove that :** x and y are conjugate numbers

, then find the value of : $x^2 + xy + y^2$

---

7 If $x = \sqrt{4 + \sqrt{7}}$, $y = \sqrt{4 - \sqrt{7}}$, **find :** $(x + y)^2$ in the simplest form.

---

8 If $x = \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - \sqrt{5}}$, **prove that :** $x + \frac{1}{x} = 22$

---

9 **Find in $\mathbb{R}$ the S.S. of the inequality :** $5x - 3 < 2x + 9$

---

10 **Find the solution set of the inequality in $\mathbb{R}$, then represent it on the number line :** $5 - 3x > 11$

---

11 **Find the solution set in $\mathbb{R}$, then represent it on the number line :** $1 < 2x + 3 \leq 9$

---

12 **Find the solution set in $\mathbb{R}$, then represent it on the number line :** $\frac{x}{3} + 2 > 3$

---

13 **Find in $\mathbb{R}$ the solution set of the inequality :** $4x + 3 > 5x + 2 > 4x$, then represent it on the number line.

---

14 **Simplify to the simplest form :** $\sqrt[3]{2}\left(\sqrt[3]{4} + 2\right) - 2\left(\sqrt[3]{2} + 1\right)$

---

15 **Simplify to the simplest form :** $\sqrt{75} - 2\sqrt{27} + 3\sqrt{\frac{1}{3}}$

---

16 **Simplify to the simplest form :** $\sqrt[3]{81} + \sqrt[3]{24} - 3\sqrt[3]{\frac{1}{9}}$

---

17 **Find in the simplest form :** $\sqrt{12} + \sqrt[3]{54} - 2\sqrt{3} - \sqrt[3]{16}$

---

18 **Find in the simplest form :** $\sqrt{50} + \sqrt[3]{54} - 10\sqrt{\frac{1}{2}} - \sqrt[3]{16}$

---

19 **Find in $\mathbb{R}$ the solution set of the equation :** $(x - 2)^3 = 125$

---

20 A sphere with volume 36π cm^3 , **find :**

1. The radius length of the sphere.
2. The area of the sphere in terms of π

---

21 Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 27π cm^3

22 A right circular cylinder whose base radius length is $4\sqrt{2}$ cm. and its height is 9 cm. Find its volume in terms of π.

23 A right circular cylinder whose height is 10 cm. and its base radius length is 7 cm. Calculate its volume and its lateral area $\left(\pi = \frac{22}{7}\right)$

24 A metallic sphere , with diameter length 6 cm. has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height.

25 A metallic right circular cylinder whose base radius length is 3 cm. and its height is 4 cm. has got melt and changed into a sphere. Find the radius length of this sphere.

26 Find the volume of a right circular cylinder of lateral area 440 cm^2. and height 10 cm. $\left(\pi = \frac{22}{7}\right)$

Important questions on Unit Two

First Multiple choice questions

1 Which of the following ordered pairs satisfies the relation : $2X + y = 5$?

(a) $(-1, 3)$ (b) $(1, 3)$ (c) $(3, 1)$ (d) $(2, 2)$

2 The ordered pair which does not satisfy the relation : $2X + y = 5$ is ....

(a) $(1, 3)$ (b) $(-1, 7)$ (c) $(3, 1)$ (d) $(4, -3)$

3 If $(5, 2)$ satisfies the relation : $X + 2y = c$, then $c =$ ....

(a) 8 (b) 9 (c) 7 (d) 6

4 If the point $(k, 2k)$ satisfies the relation : $y + 2X = 8$, then $k =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

5 If the straight line which passes through the two points $(3, k)$ and $(4, 5)$ is parallel to the X-axis , then $k =$ ....

(a) 3 (b) 4 (c) 5 (d) -5

6 The slope of the straight line which passes through the two points $(4, 5)$ and $(4, 8)$ is ....

(a) zero. (b) undefined. (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

7 If $A(-3, 1)$ and $B(1, 3)$, then the slope of $\overleftrightarrow{AB} =$ ....

(a) -1 (b) 2 (c) $\frac{1}{2}$ (d) 1

8 The relation : $2X + 7y = 14$ is represented by a straight line intersecting the X-axis at ....

(a) $(2, 0)$ (b) $(0, 2)$ (c) $(7, 0)$ (d) $(0, 7)$

9 The slope of the horizontal straight line is ....

(a) zero. (b) 1 (c) 2 (d) undefined.

10 Which of the following relations is represented by the X-axis ?

(a) $X = 0$ (b) $y = 0$ (c) $X = y$ (d) $y = -X$

Second Complete questions

1 The slope of the straight line passing through the two points $(-1, 4)$, $(0, 5)$ is ....

2 The slope of the straight line parallel to y-axis is ....

3 The slope of the straight line perpendicular to y-axis is ....

4 If the straight line passing through the two points $(3, 4)$ and $(2, y)$ is parallel to X-axis , then $y =$ ....

5 The relation : $y = 5$ is represented by a straight line parallel to .... axis and its slope is ....

6 If $(-3, 2)$ satisfies the relation : $5X - ky = 7$, then $k =$ ....

7 If the points A , B and C are collinear , then the slope of $\overleftrightarrow{AB}$ = the slope of ....

8 If the two ordered pairs $(a, 2)$ and $(3, b)$ satisfy the relation : $X + 2y = 3$, then $a =$ .... and $b =$ ....

9 If $A = (3, y)$, $B = (6, 5)$ and the slope of $\overleftrightarrow{AB}$ equals $\frac{2}{3}$, then $y =$ ....

10 The relation : $8X + 3y = 24$ is represented graphically by a straight line intersecting y-axis at the point ....

Third Essay questions

1 Represent the relation : $y = X + 2$ graphically.

2 Represent the relation : $X + y = 3$ graphically , then find the point of intersection with X-axis.

3 Graph the relation : $y = 2 - X$, then find the slope of the straight line.

4 Find the two points of intersection of the straight line : $2X + 3y = 12$ with the coordinate axes.

5 **Prove that :** The points A $(-1, 3)$, B $(2, 4)$ and C $(5, 5)$ are collinear.

6 Are the points A $(2 , -1)$, B $(-1 , 3)$ and C $(2 , 3)$ collinear ?

---

7 If the slope of the straight line which passes through the two points $(4 , 17)$ and $(6 , k)$ is 4 , find the value of k

---

8 If $(k , 2k)$ satisfies the relation : $3x + y = 30$, find the value of k

---

9 Find the slope of $\overleftrightarrow{AB}$, where A $(-1 , 3)$ and B $(2 , 5)$, is the point C $(8 , 1) \in \overleftrightarrow{AB}$?

---

10 If A = $(3 , 3)$ and B = $(3 , 5)$, **prove that :** $\overleftrightarrow{AB}$ // y-axis.

Important questions on Unit Three

First Multiple choice questions

1 The mode of the values : 3 , 4 , 10 , 4 is ....

(a) 3 (b) 4 (c) 20 (d) 10

2 The arithmetic mean of the numbers : 10 , 12 , 8 is ....

(a) 5 (b) 6 (c) 9 (d) 10

3 The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is ....

(a) 22 (b) 23 (c) 24 (d) 25

4 The arithmetic mean of the values : $5 - X$, 5 , $5 + X$ is ....

(a) 5 (b) 81 (c) 13 (d) 3

5 If the arithmetic mean of the values : 5 , 7 , 8 , X is 6 , then $X =$ ....

(a) 8 (b) 5 (c) 7 (d) 4

6 If the order of the median of a set of values is the fourth , then the number of these values is ....

(a) 5 (b) 6 (c) 7 (d) 8

7 If the mode of the set of values : 15 , 11 , 5 X , 4 is 15 , then $X =$ ....

(a) 3 (b) 4 (c) 5 (d) 15

8 If the mode of the set of values : 12 , 7 , $X + 1$, 7 , 12 is 7 , then $X =$ ....

(a) 4 (b) 6 (c) 8 (d) 11

9 The centre of the first set of the sets : 7 – , 13 – , 19 – , 25 – is ....

(a) 6 (b) 7 (c) 10 (d) 13

10 If the lower limit of a set is 4 and the upper limit of the same set is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

11 If the lower limit of a set is 10 , its upper limit is X and its centre is 15 , then $X =$ ....

(a) 10 (b) 5 (c) 20 (d) 8

12 If the arithmetic mean of five numbers is 8 , then the sum of these numbers is ....

(a) 13 (b) 16 (c) 40 (d) 64

---

13 The point of intersection of the ascending and descending curves determines .... on the set-axis.

(a) the arithmetic mean (b) the median

(c) the mode (d) the frequencies

---

14 If the point (16 , 30) is the point of intersection of the ascending and descending curves , then the median is ....

(a) 16 (b) 23 (c) 60 (d) 30

---

15 If the order of the median of a frequency distribution is 50 , then the sum of frequencies is ....

(a) 50 (b) 25 (c) 100 (d) 5

Second Complete questions

1 The median of the values : 5 , 3 , 11 , 7 , 2 is ....

---

2 The mode of the values : 3 , 5 , 7 , 3 , 8 is ....

---

3 If the arithmetic mean of the values : 18 , 23 , 29 , 2 k – 1 , k is 18 , then k = ...

---

4 The order of the median of the values : 7 , 6 , 5 , 8 , 4 is ....

---

5 The arithmetic mean is one of the measures of ....

---

6 If the mode of the values : 4 , 11 , 8 , 2 X is 8 , then X = ...

---

7 If the point of intersection of the ascending and descending curves is (35 , 20) , then the sum of frequencies is ....

---

8 The arithmetic mean of the values : 2 – X , 4 , 5 , 3 + X , 1 is ....

---

9 If the order of the median of a set of values is the fifth and sixth , then the number of these values is ....

---

10 The point of intersection of the ascending and descending curves determines .... on the set-axis.

Third Essay questions

1 **The following frequency distribution shows the marks of 20 students in mathematics :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

Calculate the mean.

---

2 **The following table shows the frequency distribution of bonus of 100 workers :**

Bonus	20 –	30 –	40 –	50 –	n –	70 –
Number of workers	10	k	22	25	20	8

Find : 1 The value of each of n and k

2 The mean.

---

3 **The following frequency distribution shows the marks of 35 students in mathematics :**

Sets	2 –	4 –	6 –	8 –	10 – 12	Total
Frequency	5	8	10	8	4	35

Graph the histogram of that distribution and from the graph find the mode mark.

---

4 **The following table shows the frequency distribution of the weights of 20 children in kg. :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	4	7	4	2	20

Using the ascending or descending cumulative frequency curve , find the median of this distribution.

Final Revision

of Algebra and Statistics

Revision for the important rules of Algebra and Statistics

First Real numbers

Remember that

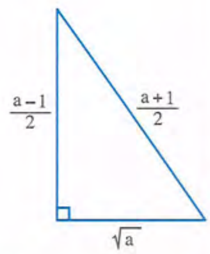
- $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$
- $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$
- $\mathbb{R} - \mathbb{Q} = \grave{\mathbb{Q}}$
- $\mathbb{R} - \grave{\mathbb{Q}} = \mathbb{Q}$
- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- $\pi \in \grave{\mathbb{Q}}$
- $\mathbb{R}^* = \mathbb{R} - \{0\}$

Remember The representing of the irrational number on the number line

Each irrational number can be represented by a point on the number line.

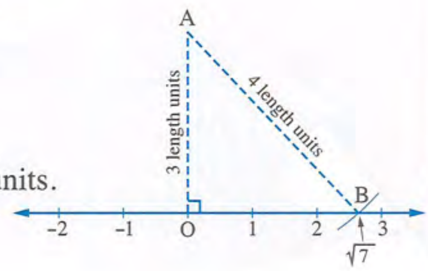
and to draw a line segment with length $= \sqrt{a}$ length unit where $a > 1$

Draw a right-angled triangle in which :



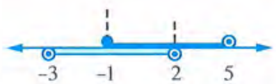
- The length of one side of the right-angle $= \frac{a-1}{2}$ length unit.
- The length of the hypotenuse $= \frac{a+1}{2}$ length unit.

and we can apply this to represent the irrational number $\sqrt{7}$ on the number line as the following :



- From the point which represents the number zero on the number line , we draw a perpendicular line segment as $\overline{OA}$ where $OA = \frac{7-1}{2} = 3$ length units.
- Using the compasses with a distance $= \frac{7+1}{2} = 4$ length units. and centre at A , draw an arc to cut the number line on the right side of the point O at the point B , then B is the point which represents $\sqrt{7}$ as in the figure.
- Notice that : To represent the number $\left(-\sqrt{7}\right)$, we draw the arc which cuts the number line on its left side , not on its right side.
- Notice that : To represent the number $\left(1 + \sqrt{7}\right)$, we follow the same previous steps but we draw the perpendicular line segment $\overline{OA}$ from the point which represents the number 1 , not the number 0

Remember **The operations on intervals**

Intervals	Intersection	Union	Difference	Complement
$X = [-1 , 5[$, $Y =]-3 , 2[$	 $X \cap Y = [-1 , 2[$	 $X \cup Y =]-3 , 5[$	 $X - Y = [2 , 5[$, $Y - X =]-3 , -1[$	 $\grave{X} = \mathbb{R} - [-1 , 5[$ $=]-\infty , -1[\cup [5 , \infty[$
$X =]-\infty , 1]$, $Y = [-2 , 1[$	 $X \cap Y = [-2 , 1[$	 $X \cup Y =]-\infty , 1]$	 $X - Y$ $=]-\infty , -2[\cup \{1\}$, $Y - X = \varnothing$	 $\grave{X} =]1 , \infty[$
$X = [-1 , 5]$, $Y =]-1 , 5[$	 $X \cap Y =]-1 , 5[$	 $X \cup Y = [-1 , 5]$	 $X - Y = \{-1 , 5\}$, $Y - X = \varnothing$	 $\grave{Y} = \mathbb{R} -]-1 , 5[$ $=]-\infty , -1] \cup [5 , \infty[$
$X =]-3 , 4]$, $Y = \{-3 , 4\}$	 $X \cap Y = \{4\}$	 $X \cup Y = [-3 , 4]$	 $X - Y =]-3 , 4[$, $Y - X = \{-3\}$	 $\grave{Y} = \mathbb{R} - \{-3 , 4\}$

Remember **The operations on the square roots and the cube roots**

The square roots

① $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ *For Example :* $\sqrt{3}\times\sqrt{12}=\sqrt{3\times 12}=\sqrt{36}=6$

② $\dfrac{\sqrt{a}}{\sqrt{b}}=\sqrt{\dfrac{a}{b}}$ (where $b\neq 0$) *For Example :* $\dfrac{\sqrt{8}}{\sqrt{2}}=\sqrt{\dfrac{8}{2}}=\sqrt{4}=2$

③ $\dfrac{\sqrt{a}}{\sqrt{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}\times\dfrac{\sqrt{b}}{\sqrt{b}}=\dfrac{\sqrt{ab}}{b}$ (where $b\neq 0$) *For Example :* $\dfrac{\sqrt{2}}{\sqrt{5}}=\dfrac{\sqrt{2}}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{\sqrt{10}}{5}$

The cube roots

① $\sqrt[3]{a}\times\sqrt[3]{b}=\sqrt[3]{ab}$ *For Example :* $\sqrt[3]{3}\times\sqrt[3]{9}=\sqrt[3]{3\times 9}=\sqrt[3]{27}=3$

② $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}}=\sqrt[3]{\dfrac{a}{b}}$ (where $b\neq 0$) *For Example :* $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}}=\sqrt[3]{\dfrac{32}{4}}=\sqrt[3]{8}=2$

Example **Simplify to the simplest form :**

1. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}$
2. $\sqrt{18}-\dfrac{\sqrt{12}}{\sqrt{6}}$
3. $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)$
4. $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}$
5. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

Solution

1. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}=\sqrt{2\times 16}-\sqrt{2\times 36}+3\times 2\sqrt{\frac{1}{2}}$
$=4\sqrt{2}-6\sqrt{2}+3\sqrt{\frac{1}{2}\times 4}=4\sqrt{2}-6\sqrt{2}+3\sqrt{2}=\sqrt{2}$

2. $\sqrt{18}-\dfrac{\sqrt{12}}{\sqrt{6}}=\sqrt{2\times 9}-\sqrt{2}=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

3. $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)=5\sqrt{2}\times 2\sqrt{2}+5\sqrt{2}\times\sqrt{12}=10\sqrt{4}+5\sqrt{24}=10\times 2+5\sqrt{4\times 6}$
$=20+5\times 2\sqrt{6}=20+10\sqrt{6}$

4. $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{2\times 27}+6\sqrt[3]{8\times 2}-3\times 2\sqrt[3]{\frac{1}{4}}$
$=3\sqrt[3]{2}+6\times 2\times\sqrt[3]{2}-3\times\sqrt[3]{8\times\frac{1}{4}}=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

5. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}=\sqrt[3]{8\times 9}+\sqrt[3]{\frac{1}{3}\times\frac{9}{9}}-\sqrt[3]{9}$
$=2\sqrt[3]{9}+\sqrt[3]{\frac{9}{27}}-\sqrt[3]{9}=2\sqrt[3]{9}+\frac{1}{3}\sqrt[3]{9}-\sqrt[3]{9}=\frac{4}{3}\sqrt[3]{9}$

Remember — The two conjugate numbers

If a and b are two positive rational numbers :
then each of the two numbers $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$
is conjugate to the other one and we find that :

- Their sum $=\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)=2\sqrt{a}=$ twice the first term
- Their product $=\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2=a-b$
  = The square of the first term – the square of the second term

For example : The number $\left(\sqrt{3}-\sqrt{2}\right)$ its conjugate is $\left(\sqrt{3}+\sqrt{2}\right)$, then we find that :

- Their sum $=2\sqrt{3}$
- Their product $=3-2=1$

Remark

If we have a real number whose denominator is written in the form $\left(\sqrt{a}+\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

For example :

For writting the number $\frac{12}{\sqrt{6}-\sqrt{2}}$ in the simplest form , we multiply the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{6}+\sqrt{2}\right)$

$$\therefore \frac{12}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\frac{12\left(\sqrt{6}+\sqrt{2}\right)}{6-2}=3\left(\sqrt{6}+\sqrt{2}\right)=3\sqrt{6}+3\sqrt{2}$$

Important remarks from multiplying by inspection

- **We know that :** $(x-y)(x+y)=x^2-y^2$
- **And we know also :**

$(x+y)^2=x^2+2xy+y^2$

Then

- $x^2+xy+y^2=(x+y)^2-xy$
- $x^2+y^2=(x+y)^2-2xy$

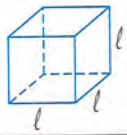
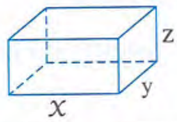
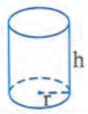
$(x-y)^2=x^2-2xy+y^2$

Then

- $x^2-xy+y^2=(x-y)^2+xy$

or $x^2+y^2=(x-y)^2+2xy$

Summary of rules of areas and volumes of some solids

The solid		The lateral area	The total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi r h$	$2\pi r h + 2\pi r^2$ $= 2\pi r(h+r)$	$\pi r^2 h$
The sphere			$4\pi r^2$	$\frac{4}{3}\pi r^3$

Remember that : The circumference of the circle $= 2\pi r$, the area of the circle $= \pi r^2$

Remember | Solving an equation of the first degree in one unknown in ℝ

- Solving the equation of the first degree in one unknown in ℝ means finding the real number which satisfies this equation.

And the following example shows how to solve an equation of the first degree in one unknown.

Example

Find in ℝ the solution set of each of the following equations , then represent the solution on the number line :

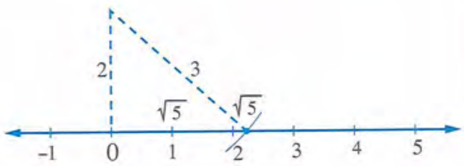
1 $\sqrt{5}\,x - 1 = 4$ **2** $x - \sqrt{3} = 2$

Solution

1 $\because \sqrt{5}\,x - 1 = 4$ $\therefore \sqrt{5}\,x = 4 + 1 = 5$

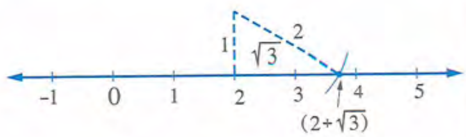
$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5}\}$



2 $\because x - \sqrt{3} = 2$ $\therefore x = 2 + \sqrt{3}$

$\therefore$ The S.S. $= \{2 + \sqrt{3}\}$



Remember **Solving an inequality of the first degree in one unknown in $\mathbb{R}$**

- Solving the inequality means finding all values of the unknown which satisfy this inequality.
- The solution set of the inequality in $\mathbb{R}$ will be written as an interval.

And the following example shows how to solve an inequality of the first degree in one unknown in $\mathbb{R}$

Example

Find in $\mathbb{R}$ the solution set of each of the following inequalities , then represent the solution on the number line :

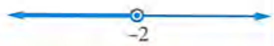
1. $2x + 6 < 2$
2. $5 - 4x \le -3$
3. $3 < 3 - 5x < 13$
4. $x - 2 \ge 3x - 5$

Solution

1 $\because 2x + 6 < 2$ $\quad\therefore 2x < 2 - 6$

$\therefore 2x < -4$ $\quad\therefore x < \frac{-4}{2}$

$\therefore x < -2$ $\quad\therefore$ The S.S. $=]-\infty , -2[$



2 $\because 5 - 4x \le -3$ $\quad\therefore -4x \le -8$

$\therefore x \ge \frac{-8}{-4}$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

$\therefore x \ge 2$ $\quad\therefore$ The S.S. $= [2 , \infty[$

3 $\because 3 < 3 - 5x < 13$ (adding -3 to all sides)

$\therefore 0 < -5x < 10$ (dividing all sides by -5) $\quad\therefore 0 > x > -2$

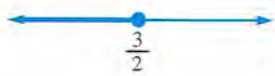
(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

$\therefore$ The S.S. $=]-2 , 0[$

4 $\because x - 2 \ge 3x - 5$ $\quad\therefore x - 3x \ge -5 + 2$

$\therefore -2x \ge -3$ $\quad\therefore x \le \frac{3}{2}$

$\therefore$ The S.S. $=]-\infty , \frac{3}{2}]$



Second | Relation between two variables

Remember | The linear relation

It is a relation of the first degree between two variables x and y , it is in the form :

$a x + b y = c$, where a , b and c are real numbers , a and b $\neq 0$ together.

And there is an infinite number of ordered pairs which satisfy this relation and it is enough to get three ordered pairs satisfying the relation at the graphical representation.

Example 1

Find three ordered pairs satisfying the relation : $3x - 2y = 6$

Solution

$\because 3x - 2y = 6$ $\quad \therefore -2y = 6 - 3x$ $\quad \therefore y = \frac{3x - 6}{2}$

- Putting $x = 0$ $\quad \therefore y = -3$ $\quad \therefore (0 , -3)$ satisfies the relation.
- Putting $x = 1$ $\quad \therefore y = -\frac{3}{2}$ $\quad \therefore \left(1 , -\frac{3}{2}\right)$ satisfies the relation.
- Putting $x = 2$ $\quad \therefore y = 0$ $\quad \therefore (2 , 0)$ satisfies the relation.

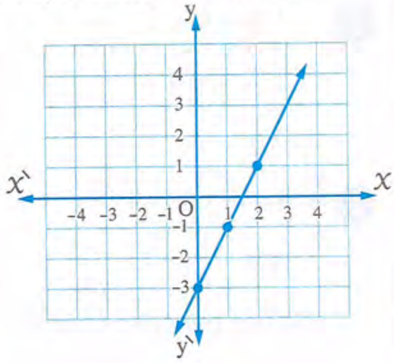
Example 2

Represent graphically the relation : $2x - y = 3$

Solution

$\because 2x - y = 3$ $\quad \therefore y = 2x - 3$

x	0	1	2
y	– 3	– 1	1



Remember | The slope of the straight line

The slope of the straight line = $\frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

i.e. $S = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

For example : The slope of the straight line passing through the two points (2 , 3) , (– 5 , 2) is :

$S = \frac{2 - 3}{-5 - 2} = \frac{-1}{-7} = \frac{1}{7}$

Notice that :

- The slope of the straight line parallel to x-axis = 0
- The slope of the straight line parallel to y-axis is undefined.

Third Statistics

Remember The tables and cumulative frequency curves

The following frequency table shows the weekly wages in pounds of 50 workers in a factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (frequency)	5	12	22	7	4	50

1 Forming the ascending cumulative frequency table and graphing the curve

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

Less than 54 = 0

Less than 58 = 5 + 0 = 5

Less than 62 = 5 + 12 = 17

Less than 66 = 5 + 12 + 22 = 39

Less than 70 = 5 + 12 + 22 + 7 = 46

Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

"The ascending cumulative frequency table"

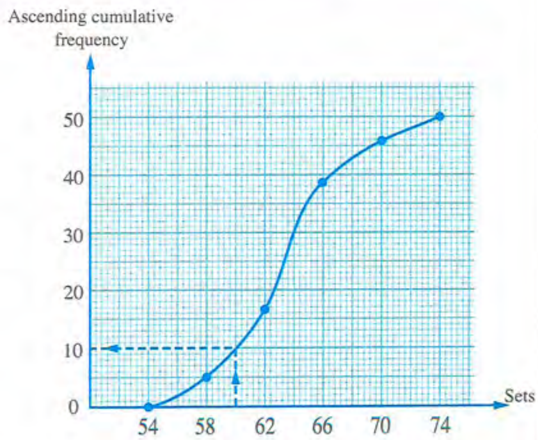
Notice that :

The ascending cumulative frequency begins with zero and ends at the total frequency.

- From the opposite graph , we can find the number of individuals which is less than a certain value.

For Example :

The number of workers whose wages are less than 60 pounds is 10 workers.



The ascending cumulative frequency curve

2 Forming the descending cumulative frequency table and graphing the curve

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

54 and more = $5 + 12 + 22 + 7 + 4 = 50$

58 and more = $12 + 22 + 7 + 4 = 45$

62 and more = $22 + 7 + 4 = 33$

66 and more = $7 + 4 = 11$

70 and more = 4

74 and more = 0

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

"The descending cumulative frequency table"

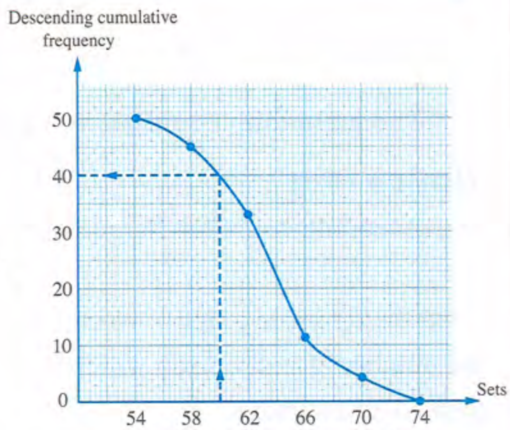
Notice that :

The descending cumulative frequency begins with the total frequency and ends with zero.

- From the opposite graph , we can find the number of individuals which is more than or equal to a certain value.

For Example :

The number of workers whose wages are 60 pounds or more is 40 workers.



The descending cumulative frequency curve.

Remember **The measures of the central tendency**

(1) The mean. (2) The median. (3) The mode.

1 The mean

[a] The mean of a set of values (simple frequency distribution)

The mean of a set of values $= \dfrac{\text{The total of values}}{\text{Number of values}}$

For example : The mean of the numbers : 5 , 3 , 7 , 9 $= \dfrac{5+3+7+9}{4} = 6$

[b] The mean of a frequency distribution with sets

Example

The following table shows the distribution of the marks of 50 pupils in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	**50**

Find the mean of these marks.

Solution

1. Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit + the upper limit}}{2}$$

$\therefore$ The centre of the first set $= \dfrac{10+20}{2} = 15$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \dfrac{50+60}{2} = 55$

2. Form the following table :

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

The mean $= \dfrac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \dfrac{1700}{50} = 34$ marks.

2 The median

[a] The median of a set of values

The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

If the values number is even , then

The median $= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :

If the values are

42 , 23 , 17 , 30 and 20

We arrange them ascendingly as follows

17 , 20 , 23 , 30 , 42

The median = 23

For example :

If the values are

27 , 13 , 23 , 24 , 13 , 21

We arrange them ascendingly as follows

13 , 13 , $\boxed{21 , 23}$, 24 , 27

The median $= \frac{21 + 23}{2} = 22$

[b] Finding the median of a frequency distribution with sets graphically

For finding the median of a frequency distribution with sets graphically , do the following steps :

1. Form the ascending or the descending cumulative frequency table , then draw the cumulative frequency curve of it.
2. Find the order of the median $= \frac{\text{The total of frequency}}{2}$
3. Determine the point which represents the order of the median on the vertical axis, from this point , draw a horizontal straight line to cut the curve at a point, then from this point , draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.

   The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

Example

The following table shows the frequency distribution of marks of 50 students in math exam :

Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	**Total**
Number of students	2	5	8	19	14	2	**50**

Find the median mark of the student.

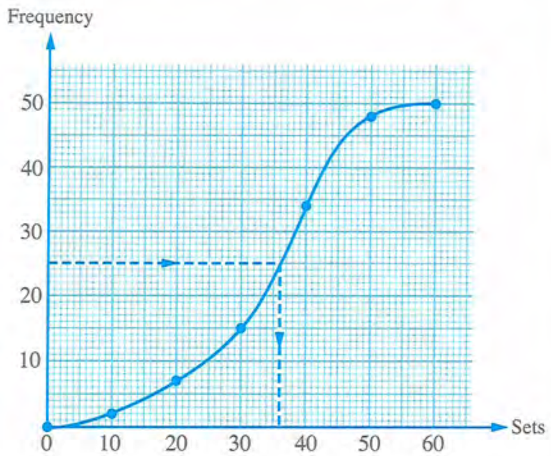
Solution

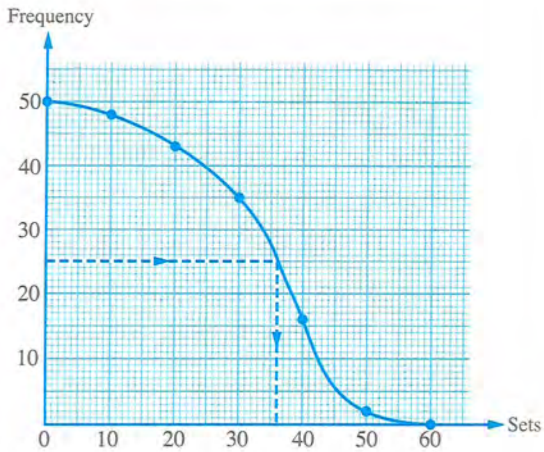
Using the ascending cumulative frequency curve :

The upper boundaries of sets	**Frequency**
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

Using the descending cumulative frequency curve :

The lower boundaries of sets	**Frequency**
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0





$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the two previous graphs , the median = 36 approximately

3 The mode

[a] The mode of a set of values

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example : The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

[b] The mode of a frequency distribution with sets

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

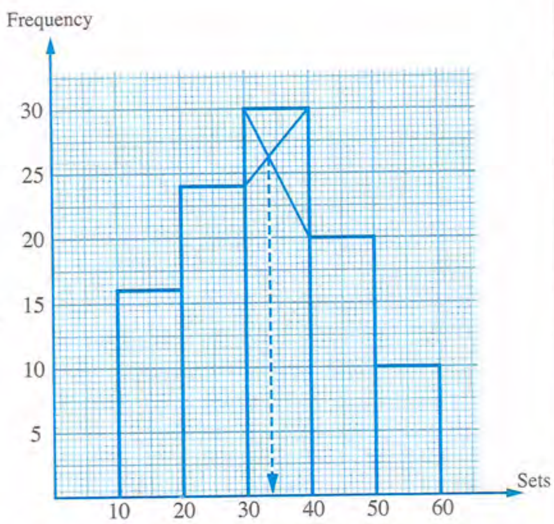
Sets of marks	10 –	20 –	30 –	40 –	50 –	Total
Number of pupils	16	24	30	20	10	**100**

Find the mode mark for these pupils.

Solution

You can find the mode of that distribution graphically using the histogram as follows :

1. Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.
2. Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
3. Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
4. Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)
5. Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)



6. Repeat drawing the remained adjacent rectangles till the last set (50 –)
7. Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.

   From this point , draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.

   i.e. The mode mark is 34 approximately.

Final Examinations

on Algebra and Statistics

Model 1

Answer the following questions :

1 Complete the following :

1. The S.S. of the equation : $(x^2 + 3)(x^3 + 1) = 0$ is .... , $x \in \mathbb{R}$
2. If the lower boundary of a set is 10 and the upper boundary is x and its centre is 15 , then $x =$ ....
3. $]-2 , 2] \cup \{-2 , 0\} =$ ....
4. The cube whose volume is 8 cm^3 , then the sum of all its edge lengths is .... cm.
5. The multiplicative inverse of the number $(\sqrt{3} + \sqrt{2})$ is .... in the simplest form.

---

2 Choose the correct answer from the given ones :

1. If the radius length of a sphere is 6 cm. , then its volume is ....

   (a) 6π cm^3 (b) 36π cm^3 (c) 72π cm^3 (d) 288π cm^3

2. If the point (a , 1) satisfies the relation $x + y = 5$, then a = ...

   (a) 1 (b) -4 (c) 4 (d) 5

3. $(2\sqrt[3]{2})^3 =$ ....

   (a) 4 (b) 8 (c) 16 (d) 40

4. The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is ....

   (a) 22 (b) 23 (c) 24 (d) 25

5. If the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 , k is 14 , then k = ...

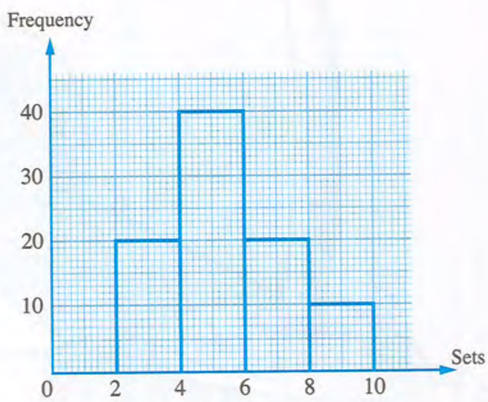
   (a) 3 (b) 6 (c) 27 (d) 84

6. **In the opposite figure :**

   The value of the mode = ...

   (a) 4 (b) 5

   (c) 6 (d) 40



3 **[a] Find the value of :** $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$

[b] If $X = \frac{3}{\sqrt{5} - \sqrt{2}}$ and $y = \sqrt{5} - \sqrt{2}$

, **prove that :** X and y are two conjugate numbers.

---

4 **[a]** The area of a square is 1089 cm^2. Find the length of its diagonal.

[b] Find the S.S. of the inequality : $\frac{3X+1}{6} < X + 1 < \frac{X+4}{2}$ in $\mathbb{R}$

, then represent it on the number line.

---

5 **[a]** The radius length of the base of a right circular cylinder is $4\sqrt{2}$ cm. and its height is 9 cm. Find its volume in terms of π and if its volume equals the volume of a sphere , find the radius length of the sphere.

[b] Find the arithmetic mean of the following frequency distribution :

The sets	5 –	15 –	25 –	35 –	45 –	**Total**
Frequency	7	10	12	13	8	**50**

Model 2

Answer the following questions :

1 **Complete the following :**

1. The additive inverse of the number : $-\sqrt{3} - \sqrt{5}$ is ....
2. $(\sqrt{8} + \sqrt{2})(\sqrt{8} - \sqrt{2}) =$ ....
3. The conjugate of the number $\frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$ is ....
4. If the volume of a sphere is $\frac{9}{2}\pi$ cm^3 , then its diameter length is .... cm.
5. $[3 , 4] - \{3 , 5\} =$ ....

---

2 **Choose the correct answer from the given ones :**

1. If the volume of a cube is 27 cm^3 , then the area of one of its faces is ....

   (a) 3 cm^2 (b) 9 cm^2 (c) 36 cm^2 (d) 54 cm^2

2. If the mode of the values 4 , 11 , 8 , 2 X is 4 , then $X =$ ....

   (a) 2 (b) 4 (c) 6 (d) 8

3 If the arithmetic mean of the values 18 , 23 , 29 , 2 k – 1 , k is 18 , then k = ...

(a) 1 (b) 7 (c) 29 (d) 90

4 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

5 A right circular cylinder the radius length of its base is r cm. and its height equals its diameter length , then its volume = ... cm³.

(a) πr^3 (b) πr^2 (c) $2 \pi r^3$ (d) $2 r^3$

6 The solution set of the equation : $x(x^2 - 1) = 0$, $x \in \mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{0 , -1 , 1\}$

---

3 [a] **Reduce to the simplest form :** $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

[b] **Prove that :** $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

---

4 [a] **Find the S.S. of the inequality :** $-2 < 3x + 7 \leq 10$ in $\mathbb{R}$, then represent the interval of solution on the number line.

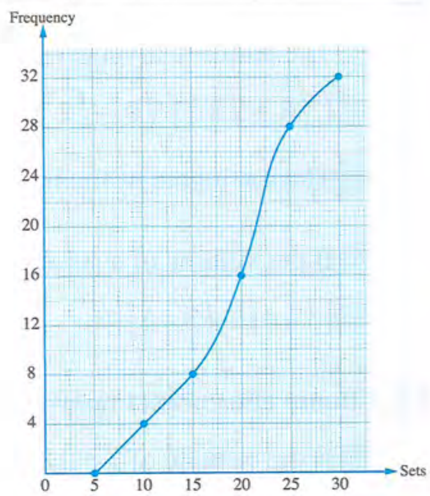
[b] If $x = \sqrt{2} + \sqrt{3}$, **find the value of :** $x^4 - 2x^2 + 1$

---

5 [a] The opposite graph represents the marks of 32 pupils in an exam.

Complete :

The median mark = ...



[b] **Find the arithmetic mean of the following frequency distribution :**

The sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

Model for the merge students

Answer the following questions :

1 Complete each of the following :

1 The conjugate of the number $\sqrt{3}+\sqrt{2}$ is ....

2 $\sqrt{18}+\sqrt{54}-3\sqrt{2}=$ ....

3 The mode for the numbers : 3 , 5 , 3 , 4 , 3 is ....

4 The median of the values : 2 , 3 , 5 , 7 , 9 is ....

5 The solution set of the equation : $x^2+9=0$ in $\mathbb{R}$ is ....

2 Choose the correct answer from those given :

1 The arithmetic mean for the values : 9 , 6 , 5 , 14 , 1 is ....

(a) 7 (b) 3 (c) 5 (d) 9

2 The simplest form of the expression : $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ is ....

(a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$

3 The additive inverse of the number $-\sqrt{5}$ is ....

(a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) -5

4 $[3 , 5]-\{3 , 5\}=$ ....

(a) $]3 , 5[$ (b) $[3 , 5[$ (c) $\varnothing$ (d) $]3 , 5]$

5 A cube is of volume 64 cm^3 , then its edge length is .... cm.

(a) 4 (b) 8 (c) 16 (d) 64

3 Match from the column (A) to the suitable one from the column (B) :

(A)	(B)
1 The S.S. of the equation : $x^2-25=0$ in $\mathbb{R}$ is	$[0 , 2]$
2 $[-3 , 2]\cap[0 , 2]=$	7
3 If the order of the median is fourth , then the number of values is	$\{5 , -5\}$
4 $\sqrt{3}$ is an .... number.	
5 The S.S. of the inequality : $3\le x\le 7$ on the number line is	irrational

4 **Put (✔) for the correct statements and (✘) for the incorrect ones :**

1. The arithmetic mean of a set of values = sum of values ÷ its number. ( )
2. If $x = \sqrt{13} - \sqrt{7}$, $y = \sqrt{13} + \sqrt{7}$, then x , y are two conjugate numbers. ( )
3. The irrational number $\sqrt{7}$ lies between 2 and 3 ( )
4. $\sqrt{75} - 2\sqrt{27} = 7\sqrt{3}$ ( )
5. The simplest form of the number $\frac{1}{\sqrt{5}}$ is $\frac{\sqrt{5}}{5}$ ( )

---

5 **[a] Complete :** If the lower limit of a set is 4 and the upper limit is 8

, then its centre = $\frac{\dots\dots + \dots\dots}{2}$ = ...

[b] Complete the following table to obtain the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Sets	The centre of the set « x »	Frequency « f »	$x \times f$
5 –	10	7	10 × 7 = 70
15 –	20	10	20 × 10 = ...
25 –	....	....	.... × 12 = ...
35 –	....	....	.... × 13 = ...
45 –	....	....	.... × 8 = ...
Total		50	....

The arithmetic mean = $\frac{\Sigma (x \times f)}{\Sigma (f)} = \frac{\dots\dots}{\dots\dots}$ = ...

1 Cairo Governorate

El-Waily Educational Admin.
St. Joseph Maronite Language Schools

Answer the following questions :

1 Choose the correct answer :

1 Let A (3 , 5) and B (5 , – 1) , then the slope of $\overleftrightarrow{AB}$ = ...

(a) $\frac{-1}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) – 3

2 The solution set of the equation : $x^2 - 9 = 0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{3 , -3\}$ (c) $\{3\}$ (d) $\{-3\}$

3 4.274 ≃ .... (to the nearest 0.1)

(a) 4 (b) 4.2 (c) 4.3 (d) 4.27

4 The mode for the values : 3 , 5 , 3 , 4 , 3 is ....

(a) 4 (b) 3 (c) 5 (d) 18

5 If the point (k , 2) satisfies the equation : $x + y - 6 = 0$, then k = ...

(a) 4 (b) – 4 (c) 6 (d) – 6

6 The lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

2 Complete :

1 The conjugate of the number $\frac{4}{\sqrt{6}-\sqrt{2}}$ is ....

2 $[-3 , 2] \cup]1 , 4[$ = ...

3 The additive inverse of the number $-5 + \sqrt{3}$ is ....

4 A sphere , its diameter length is 6 cm. , then its volume is .... cm^3

5 $[-1 , 5] -]-1 , 5[$ = ...

3 [a] Prove that : $\sqrt{18} + \sqrt{32} - 3\sqrt{2} - \frac{1}{2}\sqrt{8} = 3\sqrt{2}$

[b] If $x = \sqrt{5} - \sqrt{2}$, $y = \frac{3}{\sqrt{5}-\sqrt{2}}$

, **prove that :** x and y are conjugate to each other , **then find the value of :**

1 $x + y$ 2 xy

4 [a] Represent graphically the linear relation : $y = 2 - x$

[b] Find the solution set of the inequality :

$-2 < 3x + 7 < 10$ in $\mathbb{R}$, then represent the S.S. on the number line.

5 **[a]** Find the volume of a right circular cylinder of height 10 cm. and its base radius length is 7 cm.

[b] Find the arithmetic mean of the following frequency distribution :

Sets	0 –	10 –	20 –	30 –	40 –	Total
Frequency	4	5	6	3	2	**20**

2 Cairo Governorate

Maadi Zone
H. El-Maadi Official Lang. School

Answer the following questions :

1 **Choose the correct answer :**

1. The solution set of the equation : $x(x^2 - 1) = 0$ in $\mathbb{R}$ is ....
   (a) $\{0\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{0, -1, 1\}$
2. If the arithmetic mean of the values : 18 , 23 , 29 , k is 18 , then k = ...
   (a) 4 (b) 7 (c) 2 (d) 9
3. If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....
   (a) 2 (b) 4 (c) 6 (d) 8
4. If the radius length of a sphere is 6 cm. , then its volume is ....
   (a) 6π cm.^{3} (b) 36π cm.^{3} (c) 72π cm.^{3} (d) 288π cm.^{3}
5. If the point (a , 1) satisfies the relation : $x + y = 5$, then a = ...
   (a) 1 (b) – 4 (c) 4 (d) 5
6. $[3, 5] - \{3, 5\}$ = ...
   (a) $]3, 5]$ (b) $[3, 5[$ (c) $\varnothing$ (d) $]3, 5[$

2 **Complete each of the following :**

1. The slope of the straight line parallel to x-axis is ....
2. $\sqrt[3]{64} = \sqrt{\text{..............}}$
3. If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then x = ...
4. $[-2, 5[\cap \mathbb{R}^+$ = ...
5. The multiplicative inverse of the number $\sqrt{10} - 3$ is ....

3 **[a]** If X = $]-1, 4]$ and Y = $[3, \infty[$ **, using the number line find each of the following :**

1. X ∪ Y 2. X – Y 3. X ∩ Y

[b] If $x = \sqrt{3} + 1$ and $y = \frac{2}{\sqrt{3}+1}$

1 **Prove that :** x and y are conjugate.

2 **Find the value of :** $\frac{x+y}{xy}$ in the simplest form.

4 [a] A right circular cylinder of base radius length 4 cm. and its height is 9 cm. Find its volume in terms of π

[b] Find the slope of $\overleftrightarrow{AB}$ where A (– 1 , 5) and B (2 , 6)

5 [a] **Graph the relation :** $y = 2x$

[b] **From the following frequency distribution :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	7	10	8	6	9	40

Find the mean.

3 Cairo Governorate

Hadayek El-Qobba Educational Zone
Language School

Answer the following questions :

1 **Choose the correct answer :**

1 $[2 , 5] - \{2\} =$ ....

(a) $[2 , 5]$ (b) $]2 , 5]$ (c) $[2 , 5[$ (d) $]2 , 5[$

2 The multiplicative inverse of the number $\frac{\sqrt{3}}{6}$ is ....

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{6}$

3 If the volume of a cube is 27 cm^3 , then its edge length is .... cm.

(a) 3 (b) 9 (c) $3\sqrt{3}$ (d) 54

4 The slope of the straight line parallel to x-axis is ....

(a) 1 (b) undefined. (c) zero (d) – 1

5 If (2 , k) lies on the straight line $y = 3x + 1$, then k = ...

(a) – 1 (b) 5 (c) 6 (d) 7

6 The arithmetic mean of the values : 13 , 15 , 16 , 14 and 17 is ....

(a) 15 (b) 13 (c) 14 (d) 17

2 Complete the following :

1 If $\sqrt{a} = \sqrt[3]{27}$, then a = ...

2 The additive inverse of the number $\sqrt{7} - \sqrt{3}$ is ....

3 If A = (5 , – 3) and B = (6 , 2) , then the slope of $\overleftrightarrow{AB}$ = ...

4 If the circumference of a circle is 44 cm. , then its diameter length is .... cm. $\left(\pi = \frac{22}{7}\right)$

5 The S.S. of the equation : $x^2 - 5 = 0$ in $\mathbb{R}$ is ....

---

3 [a] Prove that : $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

[b] A right circular cylinder , if the radius length of its base is 6 cm. and its height is 10 cm. , find the lateral area and the volume of it ($\pi = 3.14$)

---

4 [a] If $x = \sqrt{5} + \sqrt{3}$, $y = \frac{2}{\sqrt{5} + \sqrt{3}}$ **, find the value of :** $(x + y)^2$

[b] Find the S.S. of the following inequality in $\mathbb{R}$ and represent it on the number line :

$3x + 7 \leq 10$

---

5 [a] If X =]– ∞ , 3] , Y = [1 , 5[**, find by using the number line :**

1 $X \cap Y$

2 $X \cup Y$

[b] Find the mode of the following frequency distribution :

Sets	0 –	4 –	8 –	12 –	16 –	Total
Frequency	4	6	12	10	8	40

4 Giza Governorate

North Giza Educational Directorate
Orman Language School

Answer the following questions :

1 Choose the correct answer :

1 The simplest form of $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ is ....

(a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $\sqrt[3]{3}$

2 The volume of a cube is 64 cm^3 , then its edge length is .... cm.

(a) 4 (b) 8 (c) 16 (d) 64

3 The mean of the values : 34 , 23 , 25 , 40 , 22 , 12 is ....

(a) 22 (b) 23 (c) 24 (d) 26

4 If the point (k , 1) satisfies the relation : $X + y = 5$, then k = ...

(a) 1 (b) -4 (c) 4 (d) 5

5 $\left(2\sqrt[3]{2}\right)^3 =$ ....

(a) 4 (b) 8 (c) 16 (d) 40

6 If the mode of the values : 4 , 11 , 8 , $2X$ is 4 , then $X =$ ....

(a) 2 (b) 4 (c) 6 (d) 8

2 Complete the following :

1 The multiplicative inverse of the number $\sqrt{10} - 3$ is ....

2 $[3 , 5] -]3 , 5[=$ ....

3 The median of the numbers : 41 , 19 , 15 , 30 , 20 is ....

4 $\sqrt{18} - \sqrt{2} =$ ....

5 If the slope of the straight line passing through (2 , k) , (3 , − 1) is 2 , then k = ...

3 [a] Find in the simplest form : $\sqrt{18} + \sqrt{32} - 3\sqrt{2} - \frac{1}{2}\sqrt{8}$

[b] If $X = \sqrt{5} - \sqrt{2}$, $y = \frac{3}{\sqrt{5} - \sqrt{2}}$, **prove that :** X and y are two conjugate numbers.

4 [a] Represent graphically the linear relation : $y = 2 - X$

[b] Find the solution set of the inequality :

$-2 < 3X + 7 \le 10$ in $\mathbb{R}$, then represent the S.S. on the number line.

5 [a] A right circular cylinder of base radius length 4 cm. and its height is 9 cm. Find its volume in terms of π

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

5 Giza Governorate

Abo El-Nomros Zone
Mathematics Department

Answer the following questions :

1 Choose the correct answer :

1 The solution set of the inequality : $X + 2 \le 0$ in $\mathbb{R}$ is ....

(a) $\{-2\}$ (b) $[2 , \infty[$ (c) $]-\infty , -2]$ (d) $\varnothing$

2 $4 \in$ ....

(a) $]4, 6[$ (b) $]0, 4]$ (c) $\{1, 5\}$ (d) $]1, 4[$

3 The slope of any straight line parallel to y-axis is ....

(a) 2 (b) zero (c) 1 (d) undefined.

4 A cube of volume 216 cm^3 has a total area equal to .... cm^2

(a) 6 (b) 36 (c) 144 (d) 216

5 The mode of the values : 2 , 8 , 6 , 4 , 6 is ....

(a) 2 (b) 4 (c) 5 (d) 6

6 $\sqrt{25} = \sqrt[3]{\ldots\ldots}$

(a) 5 (b) 15 (c) 125 (d) – 5

---

2 Complete the following :

1 $]-2, 2] \cup \{-2, 0\} =$ ....

2 If (k , 2) satisfies the relation : $x + 2y = 5$, then k = ...

3 If the radius length of a sphere is 6 cm. , then its volume is .... π cm^3

4 $\mathbb{R} - \mathbb{Q} =$ ....

5 If the order of the median of some values is seven , then the number of these values is ....

---

3 [a] Find the solution set of the following equation in $\mathbb{R}$: $(2x + 3)(5x^2 - 10) = 0$

[b] Find in the simplest form : $\sqrt{72} + 2\sqrt{32} - 3\sqrt{2}$

[c] If $A =]-2, 1]$ and $B = [0, \infty[$ **, use the number line to find :**

1 $A \cup B$ 2 $A \cap B$

---

4 [a] Find the volume of the cuboid of dimensions 5 cm. , 3 cm. , 2 cm.

[b] If $x = \sqrt{5} + \sqrt{2}$, $y = \sqrt{5} - \sqrt{2}$ **, then find in the simplest form the value of :** $\frac{x + y}{xy - 1}$

[c] Find the slope of the straight line which passes through the points (2 , 5) and (3 , 7)

---

5 [a] Graph the linear relation : $y - x = 2$

[b] Find the mean of the following frequency distribution :

Sets	8 –	12 –	16 –	20 –	24 –	Total
Frequency	4	10	16	12	8	50

6 Alexandria Governorate

Eastern Educational Zone
Taymour English School

Answer the following questions :

1 Choose the correct answer :

1. $\sqrt{50}-\sqrt{18}-\sqrt{2}=$ ....
   (a) $\sqrt{30}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$
2. $[2 , 7]-\{2 , 7\}=$ ....
   (a) $[1 , 6]$ (b) $]2 , 7[$ (c) $\varnothing$ (d) $\{0\}$
3. The irrational number lying between 2 and 3 is ....
   (a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$
4. The multiplicative inverse of the number $\frac{\sqrt{3}}{6}$ is ....
   (a) $-\frac{\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $\sqrt{2}$
5. If the volume of a sphere is $562.5\,\pi$ cm^3 , then its radius length is .... cm.
   (a) 3 (b) 5.5 (c) 7.5 (d) 15
6. If A $(-1 , 2)$ and B $(3 , 2)$, then the slope of $\overleftrightarrow{AB}=$ ....
   (a) zero (b) $-\frac{1}{2}$ (c) $\frac{2}{3}$ (d) -2

---

2 Complete the following :

1. If the lower limit of a set is 8 and the upper limit is 14 , the its centre is ....
2. If $(-1 , 5)$ satisfies the relation : $3x+ky=7$, then $k=$ ....
3. If $\frac{1}{x}=\sqrt{5}-2$, then $x=$ ....
4. $(\sqrt{3}+\sqrt{2})^2+(\sqrt{3}-\sqrt{2})^2=$ ....
5. The solution set of the equation : $\sqrt{2}\,x-1=3$ in $\mathbb{R}$ is ....

---

3 **[a] Find in $\mathbb{R}$ the S.S. of :** $-1<2x+1\leq 5$

[b] If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$, $y=\sqrt{7}-\sqrt{3}$, **then find the value of :** $(x-y)^2$

---

4 **[a] Simplify :** $\sqrt{128}+\sqrt{16}-2\sqrt{32}$

[b] Draw the graph of the relation : $x+y=5$

---

5 **[a]** If $X=[-2 , 3]$, $Y=[1 , 5[$, **find :** 1. $X\cap Y$ 2. $X\cup Y$

[b] Below is the frequency distribution of the weekly bonus of 100 workers in a factory :

Bonus	20 –	30 –	40 –	50 –	60 –	70 –	Total
No. of workers	10	k	22	26	20	8	100

1 Calculate the value of k

2 Calculate the mean.

7 Alexandria Governorate

Borj El-Arab Ed. Admin.
Mathematics Orientation

Answer the following questions :

1 Choose the correct answer :

1 $\sqrt{8}-\sqrt{2}=$ ....

(a) $\sqrt{2}$ (b) $\sqrt{6}$ (c) $\sqrt{8}$ (d) $3\sqrt{2}$

2 The S.S. of the equation : $x^2-9=0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{3, -3\}$

3 The arithmetic mean of the values : 7 , 5 , 3 is ....

(a) 5 (b) 3 (c) 7 (d) 15

4 The slope of any line parallel to x-axis is ....

(a) 1 (b) 0 (c) – 1 (d) undefined.

5 The additive inverse of the number 0 is ....

(a) – 1 (b) 5 (c) 0 (d) 1

6 The range of the values : 7 , 19 , 5 , 2 , 14 is ....

(a) 17 (b) 5 (c) 7 (d) 49

2 Complete :

1 $4x \times 2x=$ ....

2 The volume of the sphere whose radius length is 3 cm. equals .... π cm^3.

3 The mode of the values : 3 , 6 , 4 , 7 , 3 is ....

4 The slope of the straight line passing through the two points (3 , 2) , (4 , 4) is ....

5 The median of the values : 6 , 4 , 7 , 9 , 8 is ....

3 [a] Find the S.S. of the equation : $2x^3-5=11$ in $\mathbb{R}$

[b] If $x=\sqrt{7}-\sqrt{3}$, $y=\sqrt{7}+\sqrt{3}$, find the value of : $\frac{y+x}{xy}$

4 **[a]** If X = $[-2 , 3]$, Y = $[1 , 5]$ **, find by using the number line :**

1 $X \cup Y$ 2 $X \cap Y$

[b] A right circular cylinder whose base radius length is 5 cm. and its height is 7 cm. Find its volume. $\left(\pi = \frac{22}{7}\right)$

---

5 **[a] Graph the straight line that represents the relation :** $x + y = 5$

[b] Find the arithmetic mean of the following distribution :

The sets	1 –	3 –	5 –	7 –	9 –	Total
Frequency	4	6	8	7	5	30

8 El-Kalyoubia Governorate — Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The edge length of a cube vessel with capacity one litre is ....

(a) 10 cm^3 (b) 1 cm. (c) 10 cm. (d) 1 dm^3

2 $]-7 , 1] \cap [2 , 5] =$ ....

(a) $[1 , 2]$ (b) $]-7 , 5]$ (c) $[1 , 2[$ (d) $\varnothing$

3 The volume of the sphere whose radius length is 6 cm. is .... cm^3

(a) 6π (b) 36π (c) 72π (d) 288π

4 The median of the values : 7 , 8 , 9 , 6 and 5 is ....

(a) 5 (b) 7 (c) 6 (d) 9

5 Fifth of 5^{20} = ...

(a) 5^4 (b) 5^{10} (c) 5^{19} (d) 5^{21}

6 If (2 k , k) satisfies $2y + x = 16$, then k = ...

(a) 4 (b) 6 (c) 8 (d) 12

---

2 **Complete each of the following :**

1 If $\sqrt[3]{x} = 8$, then x = ...

2 The mode of the values : 7 , 3 , 8 , 2 , 3 , 4 , 3 and 7 is ....

3 The slope of the straight line parallel to y-axis is ....

4 The number which has no multiplicative inverse is ....

5 The degree of the expression : $3x^3 - 5x^2 + 7xy - 4$ is ....

3 **[a] Find in $\mathbb{R}$ the S.S. of : $(x-2)^3 = 125$**

[b] Find the volume of the right circular cylinder whose base circumference is 10π cm. and its height is 6 cm.

[c] Find in $\mathbb{R}$ the S.S. of the inequality : $5 \le 3x - 1 < 14$, then represent the solution set on the number line.

4 **[a] Simplify to the simplest form : $\sqrt[3]{54} + 5\sqrt[3]{16} - 2\sqrt[3]{250}$**

[b] If $X =]-\infty , 3]$ and $Y =]-1 , \infty[$ **, find :**

1 $X \cup Y$ 2 $X - Y$ 3 $\grave{X}$

[c] If $x = \frac{3}{\sqrt{6}-\sqrt{3}}$ and $y = \sqrt{6}-\sqrt{3}$ **, then find :** $(x-y)(x+y)$

5 **[a] Graph the relation :** $y = 2x + 1$ and find the slope using the graph.

[b] Find the arithmetic mean of the following distribution :

The sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

9 El-Sharkia Governorate

Minya Al-Qamh Educational Admin.
Minya Al-Qamh Language School – (A)

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $\sqrt{16} - \sqrt[3]{-64} =$ ....

(a) 8 (b) 0 (c) – 8 (d) 4

2 The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is ....

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{6}$ (d) $3\sqrt{2}$

3 The solution set of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is ....

(a) $\{-32\}$ (b) $\{2\}$ (c) $\varnothing$ (d) $\{4\}$

4 The arithmetic mean of the values : 6 , 12 , 18 , 4 is ....

(a) 9 (b) 10 (c) 15 (d) 40

5 The volume of a cube is 64 cm^3 , then the perimeter of one face is .... cm.

(a) 4 (b) 8 (c) 16 (d) 64

6 If the lower limit of a set is 10 and the upper limit is 20 , then its centre is ....

(a) 5 (b) 10 (c) 15 (d) 20

2 **Complete each of the following to be true :**

1. The slope of the straight line passing through the points (1 , 2) and (3 , 5) is ....
2. If the ordered pair $(\ell , 3\ell)$ satisfies the relation : $x + y = 12$, then $\ell =$ ....
3. If the mode of the values : 5 , 9 , 5 , x , 9 is 9 , then $x =$ ....
4. If three times of a number is 60 , then $\frac{1}{4}$ of this number is ....
5. The conjugate of the number $\left(\sqrt{3}-\sqrt{2}\right)$ is ....

---

3 **[a] Find in its simplest form :** $\sqrt{18}+\sqrt{50}-2\sqrt{8}$

[b] If $X =]-\infty , 5]$, $Y =]1 , 7]$, **find using the number line :**

1. $X \cap Y$
2. $Y - X$

---

4 **[a] Find in $\mathbb{R}$ the S.S. of the inequality :** $-8 < 3x + 1 \le 4$

[b] If $x = \sqrt{3}+\sqrt{2}$, $y = \frac{1}{\sqrt{3}+\sqrt{2}}$, **find the value of :** $\frac{x+y}{xy}$

---

5 **[a]** If (– 4 , a) satisfies the relation : $y = x + 2$, find the value of a

[b] Find the arithmetic mean of the following distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

10 El-Monofia Governorate

Shebein El-Koum Zone
Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. $\sqrt{9}-\sqrt[3]{-27} =$ ....

   (a) 6 (b) – 3 (c) zero (d) 9

2. The solution set of : $-x < 2$ in $\mathbb{R}$ is ....

   (a) $]-\infty , -2[$ (b) $]-2 , \infty[$ (c) $]2 , \infty[$ (d) $[2 , \infty[$

3. The degree of the algebraic term : $-3xy^3$ is ....

   (a) – 3 (b) 3 (c) 4 (d) 2

4. If (– 3 , 2) satisfies the relation : $ky = 7 - x$, then k = ...

   (a) – 3 (b) 2 (c) 7 (d) 5

5 The number of ordered pairs satisfying the relation : $2y = 3x + 4$ is ....

(a) 2 (b) 3 (c) 4 (d) infinite.

6 If $5x = 35$, $xy = 1$, then y = ...

(a) $\frac{1}{35}$ (b) 35 (c) $\frac{1}{7}$ (d) 7

2 Complete :

1 The upper limit of a set is 5 and the lower limit is 3 , then the centre is ....

2 The slope of the straight line representing y-axis is ....

3 The mode of the values 3 , 6 , 1 , 3 , 9 is ....

4 The perimeter of a square is 8 cm. , then the length of its side is .... cm.

5 The sum of degrees of 5 pupils in history is 30 , then the mean is ....

3 **[a]** A right circular cylinder , the diameter length of its base is 6 cm. and its height is 9 cm.

Find :

1 The volume in terms of π 2 The lateral area in terms of π

[b] Find in $\mathbb{R}$ the solution set of : $5 < 2x + 3 < 9$

4 **[a]** If $X =]-3 , 2[$, $Y =]-1 , 3[$ **, use the number line to find :**

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

[b] Find in the simplest form : $4\sqrt{12} + 2\sqrt{48} - \frac{2}{5}\sqrt{75}$

5 **[a]** Prove that the point $C(1 , 2) \in \overleftrightarrow{AB}$ where $A(3 , 6)$ and $B(4 , 8)$

[b] The following table represents the marks of 20 pupils in science :

Marks	5 –	10 –	15 –	20 –	25 –	Total
Frequency	3	4	7	4	2	20

1 Draw the descending cumulative frequency curve.

2 Find the median.

11 El-Dakahlia Governorate — Maths Supervision

Answer the following questions :

1 Complete each of the following :

1 $[3 , 8] - [3 , 8[$ = ...

2 The slope of the straight line passing through (2 , 3) and (4 , – 9) is ....

3 If the lower boundary of a set is 8 and the upper boundary is 12 , then its centre is ....

4 The square with side length $\sqrt{7}$ cm. , its area = ... cm^2.

5 If the mode of the values : 4 , 5 , 6 , $X + 3$ is 5 , then X = ...

2 Choose the correct answer from those given :

1 The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is ....

(a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{2}}{2}$

2 The ordered pair satisfying the relation : $2X + y = 5$ is ....

(a) $(-1 , 3)$ (b) $(3 , 1)$ (c) $(2 , 2)$ (d) $(1 , 3)$

3 $\sqrt{16 + 9} = 4 +$ ....

(a) 25 (b) 3 (c) 1 (d) 0

4 If the volume of a cube is 64 cm^3 , then its lateral area is .... cm^2.

(a) 4 (b) 16 (c) 64 (d) 96

5 If three times a number is 18 , then half this number is ....

(a) 18 (b) 9 (c) 6 (d) 3

6 The intersection point of the ascending and descending cumulative curves determines the .... on the sets axis.

(a) order of the median (b) median

(c) mean (d) mode

3 [a] Find the S.S. of the inequality : $3X + 8 < 14$ in $\mathbb{R}$, then represent the interval of solution on the number line.

[b] Find three ordered pairs satisfying the relation : $y = 3 - 2X$, then represent graphically.

4 [a] If $X = \frac{3}{\sqrt{5} - \sqrt{2}}$, $y = \sqrt{5} - \sqrt{2}$

1 **Prove that :** X and y are two conjugate numbers. 2 **Find the value of :** $\frac{X + y}{X y}$

[b] A right circular cylinder , its volume is 250π cm^3 , its height is 10 cm. **Find :**

1 The length of its base radius. 2 Its lateral area in terms of π

5 [a] Simplify to the simplest form : $\sqrt{125} + 2\sqrt[3]{81} - \sqrt{20} - \sqrt[3]{24}$

[b] Find the arithmetic mean of the following distribution :

Sets	2 –	4 –	6 –	8 –	10 –	Total
Frequency	2	1	3	3	1	10

12 Ismailia Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1 The slope of y-axis is ....

(a) 0 (b) undefined. (c) – 1 (d) 1

2 If X is the additive inverse of y , then $X + y =$ ....

(a) – 1 (b) 1 (c) 2 (d) 0

3 The mode of : 6 , 3 , 8 , 6 , 4 , 6 , 9 is ....

(a) 6 (b) 3 (c) 8 (d) 9

4 $\varnothing$ .... $\{0\}$

(a) = (b) $\not\subset$ (c) $\subset$ (d) $\in$

5 The mean of : 24 , 10 , 16 , 20 , 30 is ....

(a) 16 (b) 10 (c) 20 (d) 25

6 If (3 , k) satisfies the relation : $y = 2X + 1$, then k = ...

(a) 7 (b) 6 (c) 24 (d) 9

---

2 Complete :

1 The median of : 20 , 7 , 8 , 16 , 9 is ....

2 If $X = 3 + \sqrt{5}$, $y = 3 - \sqrt{5}$, then $X - y =$ ....

3 The lower limit of a set is 10 and its upper limit is 14 , then the centre of this set is ....

4 The edge length of a cube is 5 cm. , then its volume is .... cm^3

5 The age of Ali now is X years and the age of his friend now is y years , then the sum of their ages after 5 years is $X + y +$ .... years.

---

3 [a] Graph the relation : $X = -3$

[b] If A (3 , 5) , B (5 , 12) , **find :** The slope of $\overleftrightarrow{AB}$

[c] Simplify : $\sqrt{27} + 2\sqrt{12} - 2\sqrt{3}$

---

4 [a] The volume of a sphere is 38808 cm^3. Find its diameter length. $\left(\pi = \frac{22}{7}\right)$

[b] If X = [2 , ∞[, Y = [– 3 , 5[, **using the number line find :**

1 $X \cup Y$ 2 $X \cap Y$ 3 $X - Y$

5 **[a] Find in $\mathbb{R}$ the S.S. and represent it on the number line :** $8 < 3x - 1 \leq 14$

[b] From the following table :

Sets	4 –	8 –	12 –	16 –	20 –	Total
Frequency	4	7	6	5	8	30

Find the mean.

13 El-Beheira Governorate

Maths Supervision
Official Language Schools

Answer the following questions :

1 **Choose the correct answer :**

1. The multiplicative inverse of $\sqrt{3}$ is ....
   (a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{3}{\sqrt{3}}$
2. The S.S. of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....
   (a) $\varnothing$ (b) $\{3 , -3\}$ (c) $\{3\}$ (d) $\{-3\}$
3. If (k , 3) satisfies the relation : $y = 2x + 5$, then k = ...
   (a) 1 (b) – 1 (c) 2 (d) 3
4. The volume of a cube is 27 cm^3 , then its lateral area is .... cm^2
   (a) 12 (b) 54 (c) 36 (d) 27
5. If $2x + 1 = 7$, then $3x =$ ....
   (a) 6 (b) 9 (c) 12 (d) – 12
6. The mean of the values : 3 , 2 , 4 , 7 is ....
   (a) 2 (b) 3 (c) 7 (d) 4

2 **Complete :**

1. $3a^2b \times$ .... $= 12a^4b^2$
2. If the mode of the values : 6 , 9 , $x - 2$, 10 is 6 , then $x =$ ....
3. $[2 , 7] - \{7\} =$ ....
4. The slope of the straight line parallel to x-axis is ....
5. The median of : 24 , 20 , 11 , 36 , 40 is ....

3 **[a]** If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$ **, find the value of :** $\frac{x + y}{xy}$

[b] If the slope of the straight line passing through the two points A (4 , k) , B (3 , 2) is 5 , find the value of k

4 **[a] Find in $\mathbb{R}$ the S.S. of the inequality :**

$-1 \leq 2X + 3 < 5$ and represent the S.S. on the number line.

[b] Simplify : $\sqrt{50} + 2\sqrt{18} - \sqrt{32} - 8\sqrt{\frac{1}{2}}$

---

5 **[a]** If the volume of a sphere is $\frac{500}{3}\pi$ cm^3 , find the length of its diameter.

[b] Find the mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

14 El-Menia Governorate

Menia Directorate of Education
Matay G.L.S.

Answer the following questions :

1 **Complete :**

1. The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is ....
2. The S.S. of the equation : $X^2 + 9 = 0$ in $\mathbb{R}$ is ....
3. $(\sqrt{7} + 2)(\sqrt{7} - 2) =$ ....
4. Let A $(-3 , 1)$ and B $(2 , -5)$, then the slope of $\overleftrightarrow{AB} =$ ....
5. $]3 , 6[\cup \{4 , 5 , 6\} =$ ....

---

2 **Choose the correct answer :**

1. The volume of a cube is 125 cm^3 , then its lateral area is .... cm^2

   (a) 5 (b) 10 (c) 100 (d) 125

2. $\sqrt{2} + \sqrt{8} =$ ....

   (a) 2 (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

3. If $(-2 , 1)$ satisfies $2X + k\,y = 1$, then k = ...

   (a) 2 (b) 3 (c) 4 (d) 5

4. If $\sqrt{6} \in]X , X + 1[$, then $X =$ ....

   (a) 1 (b) 2 (c) 3 (d) 4

5. If the mode of the values : 11 , 7 , 9 , $X + 2$, 5 is 7 , then $X =$ ....

   (a) 3 (b) 5 (c) 7 (d) 9

6. If the lower limit of a set is 40 and the upper limit is 80 , then its centre is ....

   (a) 20 (b) 40 (c) 60 (d) 80

3 **[a] Find in the simplest form :** $\sqrt{48} - 2\sqrt{12} + 6\sqrt{\frac{1}{3}}$

[b] A sphere , its volume is 36 π cm^3. Find the length of its diameter.

[c] If $a = \sqrt{5} - \sqrt{2}$, $b = \sqrt{5} + \sqrt{2}$, **find the value of :** $(a + b)^2$

---

4 **[a]** If $A =]-2 , 3]$, $B = [1 , 5]$, **find in the form of an interval , using the number line :**

1. $A \cap B$
2. $A \cup B$

[b] Prove that : $\sqrt{2}$ lies between 1.4 and 1.5

---

5 **[a] Find the S.S. of each of the following in $\mathbb{Q}$:**

1. $8 x^3 - 20 = 7$
2. $-2 < 3 x + 7 \le 10$

[b] Find the arithmetic mean of the following frequency distribution :

The sets	0 –	4 –	8 –	12 –	16 –	**Total**
Frequency	2	10	8	7	3	**30**

15 Qena Governorate

Qena Directorate of Education
Math Supervision

محافظة قنا

Answer the following questions :

1 **Choose the correct answer :**

1. The slope of any line parallel to x-axis is ....
   (a) 1 (b) undefined. (c) – 1 (d) zero
2. $[1 , 3] - \{1 , 3\} =$ ....
   (a) $]1 , 3[$ (b) $[1 , 3[$ (c) $]-1 , -3[$ (d) $]-1 , 3[$
3. The irrational number lying between 2 and 1 is ....
   (a) – 3 (b) 1 (c) $\sqrt{3}$ (d) $\sqrt{5}$
4. The volume of a cuboid whose dimensions are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. and $\sqrt{6}$ cm. is ....
   (a) 6 $cm.^3$ (b) 36 cm. (c) $6\sqrt{6}$ cm. (d) $18\sqrt{2}$ cm.
5. If the lower limit of a set is 6 and the upper limit is 10 , then its centre is ....
   (a) 4 (b) 6 (c) 10 (d) 8
6. $\left(2\sqrt[3]{2}\right)^3 =$ ....
   (a) 4 (b) 8 (c) 16 (d) 40

2 Complete the following :

1. $\sqrt[3]{125} = \sqrt{\cdots\cdots\cdots}$
2. If $(-1, 4)$ satisfies the relation : $x + y = k$, then $k = \cdots\cdots$
3. The edge length of a cube is 3 cm. , then the area of any one of its faces is $\cdots\cdots$ cm^2
4. $[2, 7[\cup \{2, 7\} = \cdots\cdots$
5. The conjugate of $\sqrt{5} + \sqrt{2}$ is $\cdots\cdots$

---

3 [a] **Find in the simplest form :** $\sqrt{18} + \sqrt{50} - \sqrt{54}$

[b] If $x = \dfrac{4}{3 - \sqrt{5}}$ and $y = 3 - \sqrt{5}$, **prove that :** x and y are conjugate numbers

, **then find the value of :** $(x + y)^2$

---

4 [a] **Find the S.S. of the inequality :**

$-2 < 3x + 7 < 10$ in $\mathbb{R}$, then represent it on the number line.

[b] If $X = [-1, 4]$ and $Y = [2, 7]$, **then find each of :**

1. $X \cup Y$
2. $X \cap Y$

---

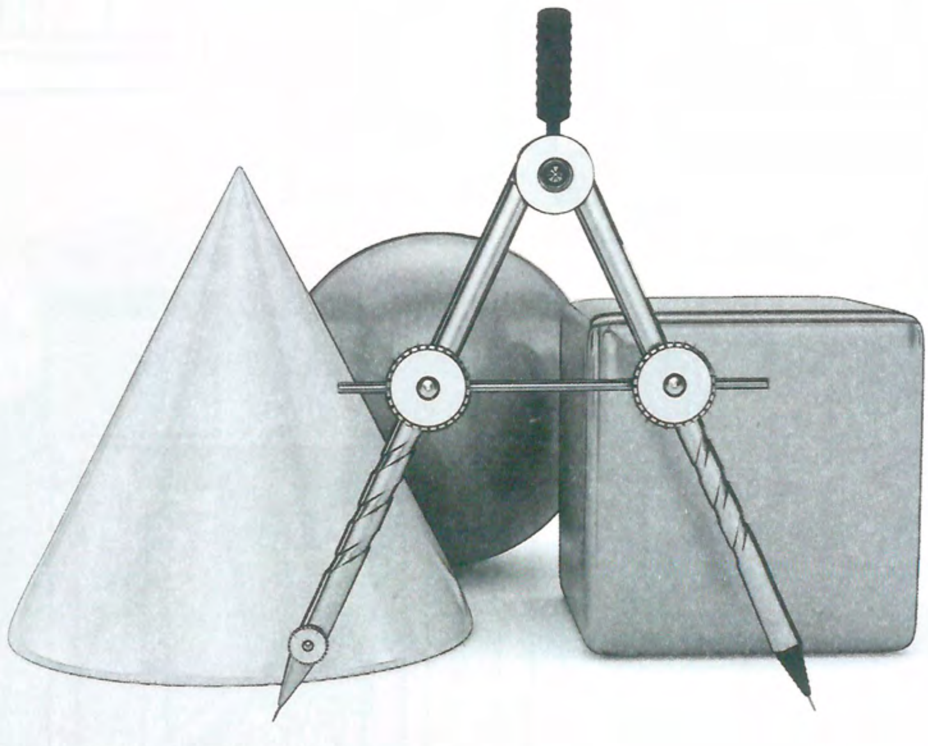
5 [a] If $(3m, 2m)$ satisfies the relation : $y = 2x - 8$, find the value of m

[b] **The following table shows the frequency distribution of marks of 40 pupils in an exam :**

Sets of marks	30 –	40 –	50 –	60 –	70 –	80 –	Total
Frequency	3	4	12	8	7	6	40

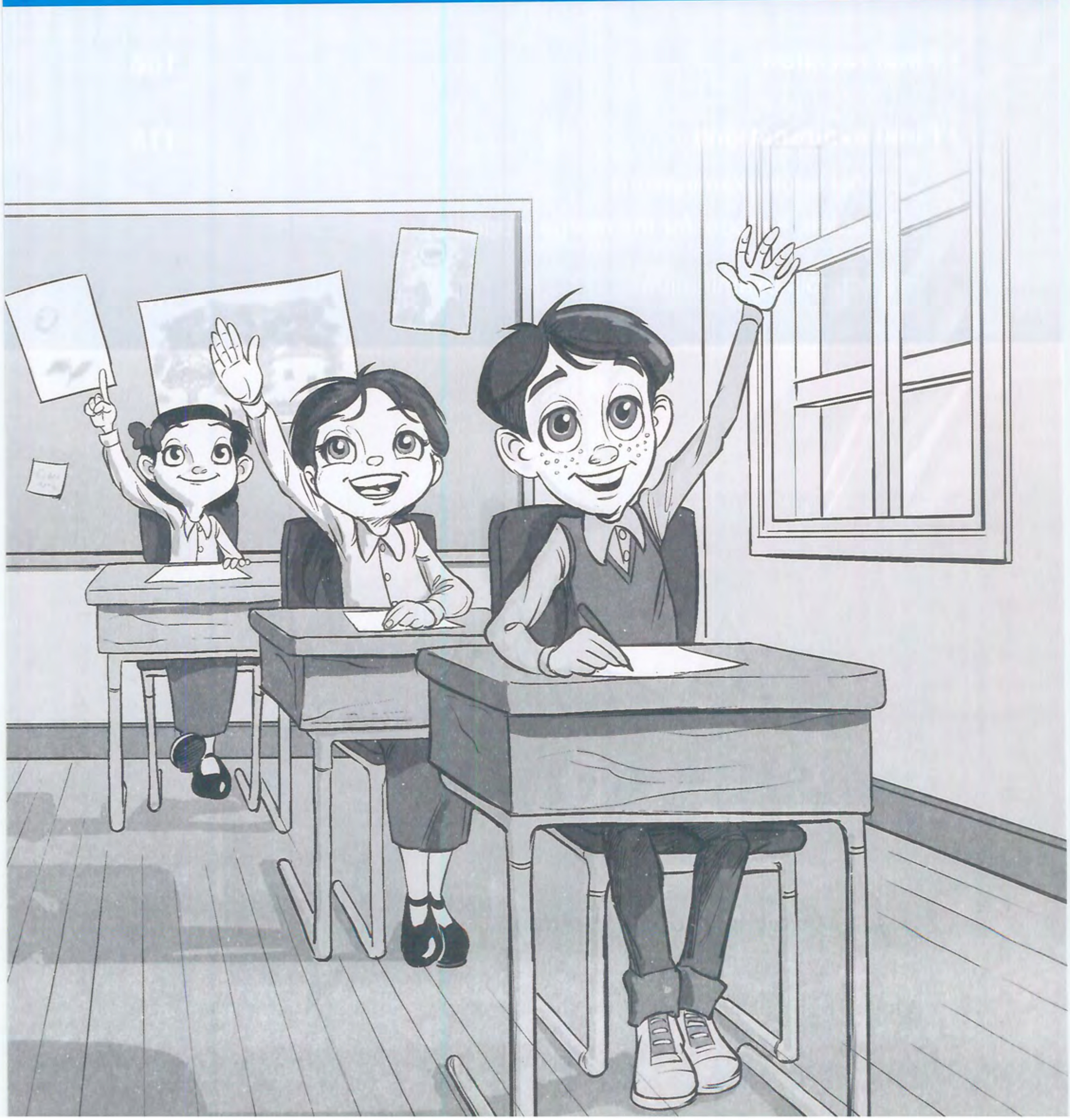
Find the mean mark.

Second Geometry



Accumulative Tests

on Geometry



1 **Choose the correct answer from the given ones :**

1. The number of medians of the triangle is ....

(a) 1 (b) 2 (c) 3 (d) 4

2. The point of concurrence of the medians of the triangle divides each median in the ratio .... from the vertex.

(a) 2 : 1 (b) 1 : 2 (c) 3 : 1 (d) 3 : 2

3. If M is the point of intersection of the medians of Δ ABC , $\overline{AD}$ is a median , then AD = ...

(a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

4. If $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of the medians , AM = 12 cm. , then AD = ... cm.

(a) 8 (b) 4 (c) 18 (d) 9

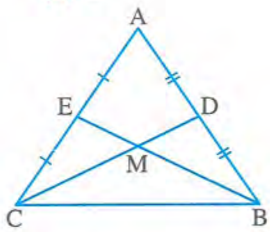
5. The point of intersection of medians of the triangle divides each of them in the ratio 4 : .... from the base.

(a) 2 (b) 8 (c) 1 (d) 4

6. In Δ XYZ , $\overline{XD}$ is a median , M is the point of intersection of the medians , then XM .... MD

(a) > (b) < (c) = (d) ≤

7. **In the opposite figure :**



BM = 6 cm. , then ME = ... cm.

(a) 3 (b) 6

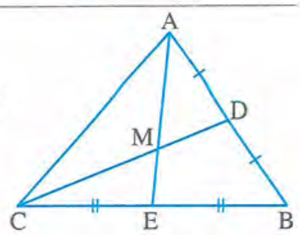
(c) 7 (d) 9

8. In Δ ABC , $\overline{AD}$ is a median , M is the point of intersection of its medians , then $(AM)^2$ = ... $(AD)^2$

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) $\frac{1}{2}$

---

2 **[a] In the opposite figure :**



ABC is a triangle in which D , E are the midpoints of $\overline{AB}$, $\overline{BC}$ respectively , $\overline{AE} \cap \overline{CD} = \{M\}$, if AM = 4 cm. , CD = 9 cm.

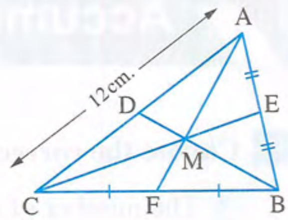
Find : The length of each of $\overline{AE}$, $\overline{MC}$

[b] In the opposite figure :

E is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{BC}$, AC = 12 cm.

Find with proof : The length of $\overline{AD}$



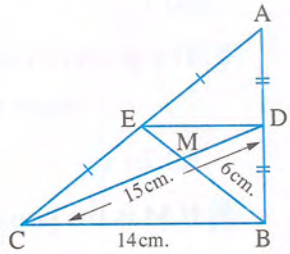
---

3 In the opposite figure :

M is the point of intersection of the medians of Δ ABC

, BM = 6 cm. , BC = 14 cm. , DC = 15 cm.

Find : The perimeter of Δ MDE



Accumulative test 2 till lesson 2 – unit 4

1 **Choose the correct answer from the given ones :**

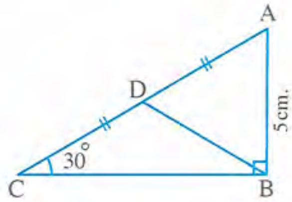
1. The length of the side opposite to the angle of measure 30° in the right-angled triangle = ... the length of the hypotenuse.

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

2. If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , then the length of the median drawn from B equals .... cm.

   (a) 10 (b) 5 (c) 3 (d) 4

3. **In the opposite figure :**

   ABC is a right-angled triangle at B

   , D is the midpoint of $\overline{AC}$, m (∠ ACB) = 30°

   , AB = 5 cm. , then BD = ... cm.

   (a) 5 (b) 10 (c) 2.5 (d) 15

4. The point of intersection of medians of the triangle divides each median in the ratio .... from the base.

   (a) 2 : 1 (b) 3 : 6 (c) 3 : 2 (d) 1 : 3

5. If $\overline{BD}$ is a median in Δ ABC , BD = $\frac{1}{2}$ AC , then ....

   (a) m (∠ ABC) = 90° (b) m (∠ BAC) = 90°

   (c) m (∠ ABC) = 30° (d) m (∠ ACB) = 90°

6. If M is the point of intersection of the medians of Δ ABC , D is the midpoint of $\overline{BC}$, then MD : AD = ...

   (a) 1 : 2 (b) 2 : 3 (c) 1 : 3 (d) 3 : 2

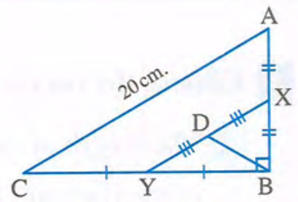
7. If M is the point of intersection of the medians of Δ ABC , $\overline{AD}$ is a median of length 9 cm. , then AM = ... cm.

   (a) 6 (b) 3 (c) 4 (d) 2

8. A rectangle , its diagonals intersect at M , the length of its diagonal is 6 cm. , then the length of the median $\overline{AM}$ is ....

   (a) 1 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

2 **[a] In the opposite figure :**



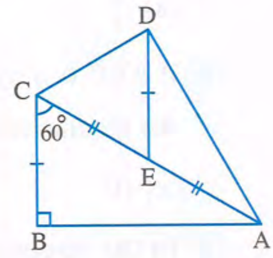
m (∠ ABC) = 90° , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

, D is the midpoint of $\overline{XY}$, AC = 20 cm.

Find : The length of $\overline{BD}$

[b] In the opposite figure :



ABC is a right-angled triangle at B

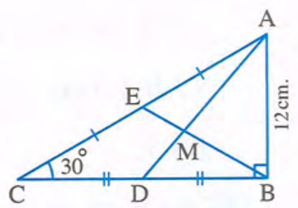
, m (∠ ACB) = 60°

, E is the midpoint of $\overline{AC}$, DE = BC

Prove that : m (∠ ADC) = 90°

---

3 **In the opposite figure :**



ABC is a right-angled triangle at B

, m (∠ C) = 30° , D is the midpoint of $\overline{BC}$

, E is the midpoint $\overline{AC}$, $\overline{AD} \cap \overline{BE} = \{M\}$

, if AB = 12 cm. , AD = 15 cm.

Find with proof :

1. The length of $\overline{AE}$
2. The length of $\overline{ME}$
3. The perimeter of Δ AME

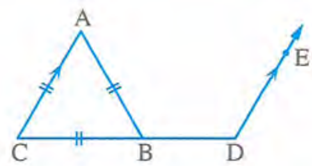
Accumulative test 3 till lesson 3 – unit 4

1 **Choose the correct answer from the given ones :**

1. The measure of the exterior angle of the equilateral triangle equals ....
   (a) 60°   (b) 90°   (c) 120°   (d) 180°

2. The two base angles of the isosceles triangle are ....
   (a) complementary.   (b) congruent.
   (c) supplementary.   (d) different.

3. **In the opposite figure :**
   ABC is an equilateral triangle
   , $\overrightarrow{DE} \parallel \overline{CA}$, then m (∠ D) = ...



   (a) 100°   (b) 60°
   (c) 120°   (d) 150°

4. The point of intersection of the medians of the triangle divides each median in the ratio .... from the base.
   (a) 1 : 2   (b) 2 : 1   (c) 3 : 1   (d) 1 : 3

5. ABC is a right-angled triangle at B , AC = 20 cm. , D is the midpoint of $\overline{AC}$, then BD = ... cm.
   (a) 10   (b) 8   (c) 6   (d) 5

6. XYZ is an isosceles triangle in which m (∠ Y) = 100° , then m (∠ Z) = ...
   (a) 100°   (b) 80°   (c) 60°   (d) 40°

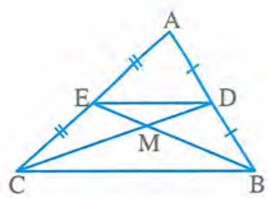
7. If Δ XYZ is right-angled at Y , m (∠ X) = 60° , XZ = 10 cm. , then XY = ... cm.
   (a) 10   (b) 6   (c) 8   (d) 5

8. In Δ ABC , if AB = AC , m (∠ A) = 2 m (∠ B) , then m (∠ C) = ...
   (a) 30°   (b) 45°   (c) 60°   (d) 90°

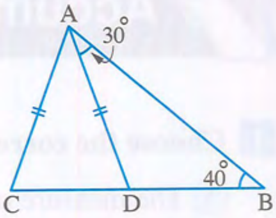
---

2 **[a] In the opposite figure :**

$\overline{BE}$, $\overline{CD}$ are two medians in Δ ABC intersect at point M
, the perimeter of Δ MDE = 12 cm.
Find : The perimeter of Δ MBC



[b] In the opposite figure :



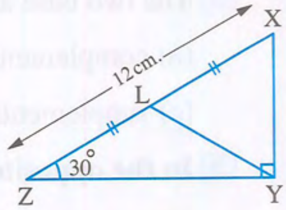
$AD = AC$, $B \in \overrightarrow{CD}$

, $m(\angle B) = 40°$

, $m(\angle BAD) = 30°$

Prove that : $AB = CB$

3 **[a] In the opposite figure :**



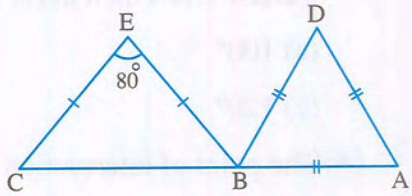
XYZ is a right-angled triangle at Y

, L is the midpoint of $\overline{XZ}$, $m(\angle Z) = 30°$

, $XZ = 12$ cm.

Find : The perimeter of ΔXLY

[b] In the opposite figure :



$B \in \overline{AC}$, ΔABD is equilateral

, $EB = EC$, $m(\angle E) = 80°$

Find : $m(\angle DBE)$

Accumulative test 4 till lesson 4 – unit 4

1 **Choose the correct answer from the given ones :**

1 If the measures of two angles in a triangle are 42° , 69° , then the triangle is ....

(a) isosceles. (b) scalene. (c) equilateral. (d) otherwise.

2 If the length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the hypotenuse.

(a) half (b) double (c) quarter (d) third

3 A right-angled triangle , the measure of one of its angles is 45° , then it is ....

(a) isosceles triangle. (b) scalene triangle.

(c) equilateral triangle. (d) otherwise.

4 **In the opposite figure :**

ABC is a triangle in which

m (∠ B) = m (∠ C) , then X = ...

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

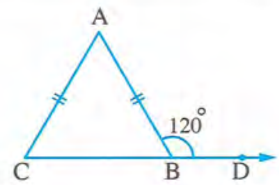
(c) 2 (d) 4

5 If ABC is a triangle , AB = BC , then ∠ C is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

---

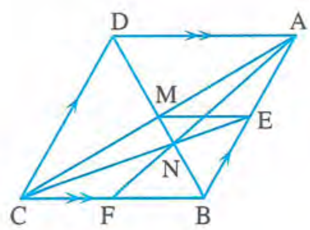
2 **[a] In the opposite figure :**



AB = AC , m (∠ ABD) = 120°

Prove that : Δ ABC is equilateral.

[b] In the opposite figure :

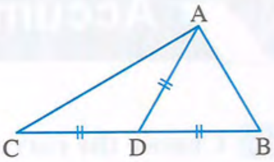


ABCD is a parallelogram its diagonals intersect at M , if N ∈ $\overline{BM}$

where BN = 2 NM , $\overrightarrow{CN} \cap \overline{AB} = \{E\}$

Prove that : EM = $\frac{1}{2}$ BC

3 **[a] In the opposite figure :**

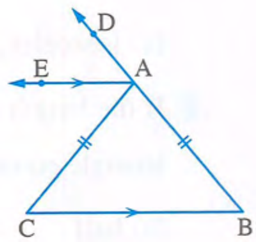


ABC is a triangle

, AD = BD = CD

Find : m (∠ BAC)

[b] In the opposite figure :



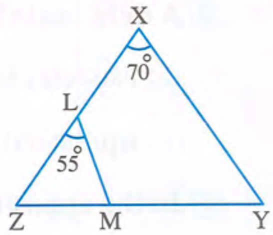
$D \in \overrightarrow{BA}$, AB = AC

, $\overrightarrow{AE} \parallel \overline{BC}$

Prove that : $\overrightarrow{AE}$ bisects ∠ DAC

---

4 **In the opposite figure :**



XZ = XY , m (∠ MLZ) = 55°

, m (∠ X) = 70°

Prove that : ML = MZ

Accumulative test 5 till lesson 5 – unit 4

1 Choose the correct answer from the given ones :

1. The median of the isosceles triangle from the vertex angle bisects it and .... to the base.

   (a) axis of symmetry (b) parallel (c) congruent (d) perpendicular

2. An isosceles triangle , the measure of one of its angles is 60° , then the number of its axes of symmetry is ....

   (a) 4 (b) 3 (c) 2 (d) 1

3. The length of the hypotenuse in the right-angled triangle equals .... the length of the side opposite to the angle of the measure 30°

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 3

4. The number of medians of the isosceles triangle is ....

   (a) zero (b) 1 (c) 2 (d) 3

5. If ABC is a triangle , AB = AC , m (∠ B) = 50° , then m (∠ A) = ...

   (a) 80° (b) 110° (c) 40° (d) 50°

6. The triangle which has no axes of symmetry is ....

   (a) the isosceles triangle. (b) the scalene triangle.

   (c) the equilateral triangle. (d) the right-angled triangle.

7. If $\overleftrightarrow{AB}$ is the axis of symmetry of $\overline{FD}$, then $\frac{AD}{AF}$ = ...

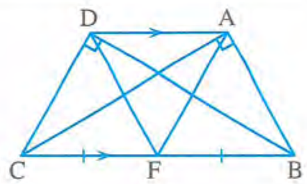
   (a) zero (b) 1 (c) $\frac{1}{2}$ (d) 2

8. ABC is an equilateral triangle , X is the point of intersection of its axes of symmetry , $\overrightarrow{AX}$ cuts $\overline{BC}$ at D , if DX = 5 cm. , then AX = ...

   (a) 10 cm. (b) 15 cm. (c) 2.5 cm. (d) 7.5 cm.

---

2 [a] In the opposite figure :



$\overline{AF}$, $\overline{DF}$ are two medians

Prove that : AF = FD

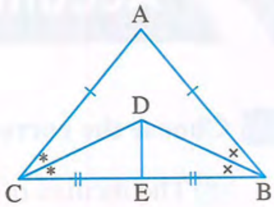
[b] In the opposite figure :

$AB = AC$, $\overrightarrow{BD}$ bisects $\angle ABC$

, $\overrightarrow{CD}$ bisects $\angle ACB$

, E is the midpoint of $\overline{BC}$

Prove that : $\overline{DE} \perp \overline{BC}$



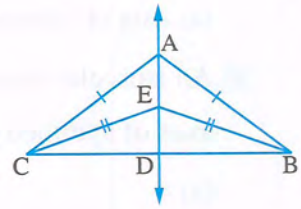
3 [a] In the opposite figure :

ABC is a triangle in which $AB = AC = 10$ cm.

, $BE = EC$, $BC = 16$ cm.

, $\overrightarrow{AE} \cap \overline{BC} = \{D\}$

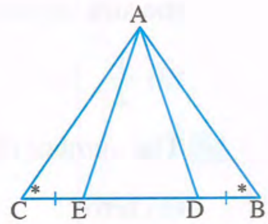
Find : The length of $\overline{AD}$



[b] In the opposite figure :

$BD = EC$, $m(\angle B) = m(\angle C)$

Prove that : $m(\angle ADE) = m(\angle AED)$



Accumulative test 6 till lesson 1 – unit 5

1 **Choose the correct answer from the given ones :**

1 If $X - z > y - z$, then X .... y

(a) = (b) > (c) < (d) ≤

2 If C ∈ the axis of symmetry of $\overline{AB}$, then AC – BC = ...

(a) zero (b) 1 (c) 3 (d) 2

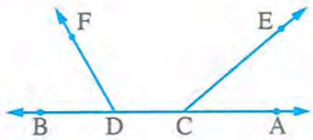
3 If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of one of its base angles is ....

(a) 45° (b) 40° (c) 50° (d) 100°

4 If Δ ABC is right-angled at B , AB = $\frac{1}{2}$ AC , then m (∠ A) = ...

(a) 45° (b) 30° (c) 90° (d) 60°

5 **In the opposite figure :**



C ∈ $\overleftrightarrow{AB}$, D ∈ $\overleftrightarrow{AB}$

, m (∠ ACE) < m (∠ BDF)

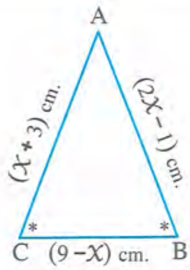
, then m (∠ ECD) .... m (∠ FDC)

(a) > (b) <

(c) = (d) ≤

---

2 **[a] In the opposite figure :**

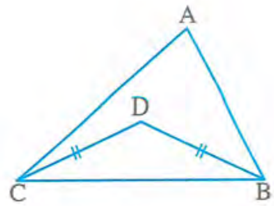


ABC is a triangle ,

m (∠ B) = m (∠ C)

Find : The perimeter of Δ ABC

[b] In the opposite figure :

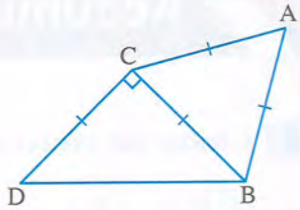


m (∠ ABC) > m (∠ ACB)

, BD = CD

Prove that : m (∠ ABD) > m (∠ ACD)

3 **[a] In the opposite figure :**

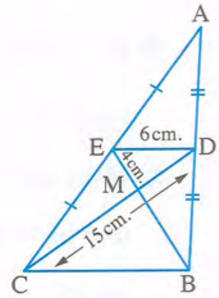


AB = BC = AC = CD

, m (∠ BCD) = 90°

Find : m (∠ ABD)

[b] In the opposite figure :



ABC is a triangle , D , E are the midpoint of $\overline{AB}$, $\overline{AC}$ respectively

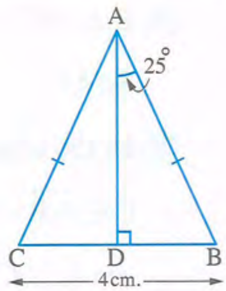
, $\overline{CD} \cap \overline{BE} = \{M\}$, if CD = 15 cm.

, EM = 4 cm. , DE = 6 cm.

Find : The perimeter of Δ MBC

---

4 **[a] In the opposite figure :**



ABC is a triangle , AB = AC , $\overline{AD} \perp \overline{BC}$

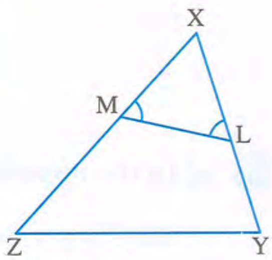
, m (∠ BAD) = 25° , BC = 4 cm.

Find :

1 m (∠ DAC)

2 The length of $\overline{DC}$

[b] In the opposite figure :



XZ > XY

, m (∠ XLM) = m (∠ XML)

Prove that : ZM > YL

1 **Choose the correct answer from the given ones :**

1 In Δ ABC , if AB > AC , then m ($\angle$ C) .... m ($\angle$ B)

(a) < (b) > (c) = (d) ≤

2 In Δ XYZ , $\overline{XY}$ is the shortest side , then the angle of the smallest measure is ....

(a) X (b) Y (c) Z (d) otherwise.

3 If M is the point of intersection of medians of Δ ABC , $\overline{AD}$ is a median , then AD : MD = ...

(a) 2 : 3 (b) 3 : 2 (c) 3 : 1 (d) 1 : 3

4 A triangle has 3 axes of symmetry , then the measure of the exterior angle at one of its vertices equals ....

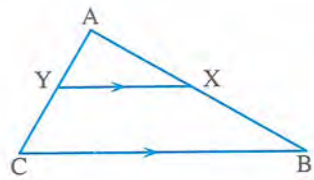
(a) 90° (b) 80° (c) 120° (d) 60°

5 In Δ ABC , AB = 7 cm. , BC = 5 cm. , AC = 6 cm. , then m ($\angle$ B) .... m ($\angle$ C)

(a) > (b) < (c) = (d) ≡

---

2 **[a] In the opposite figure :**

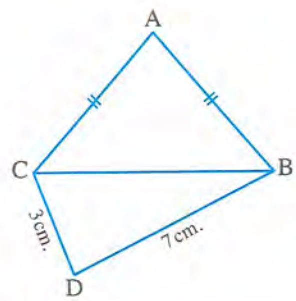


ABC is a triangle in which AB > AC

, $\overline{XY}$ // $\overline{BC}$

Prove that : m ($\angle$ AYX) > m ($\angle$ AXY)

[b] In the opposite figure :



AB = AC

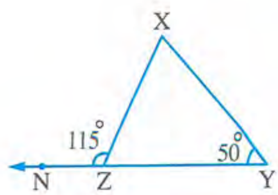
, BD = 7 cm.

, DC = 3 cm.

Prove that : m ($\angle$ ACD) > m ($\angle$ ABD)

---

3 **[a] In the opposite figure :**



XYZ is a triangle in which

m ($\angle$ Y) = 50° , N $\in \overrightarrow{YZ}$

, m ($\angle$ XZN) = 115°

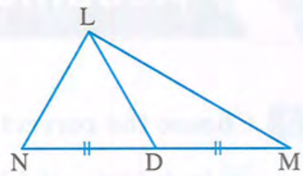
Prove that : Δ XYZ is isosceles

[b] In the opposite figure :

If the perimeter of Δ LMD > the perimeter of Δ LDN

, D is the midpoint of $\overline{MN}$

Prove that : m ($\angle$ N) > m ($\angle$ M)



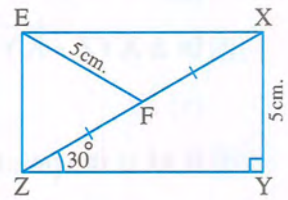
---

4 **[a] In the opposite figure :**

m ($\angle$ Y) = 90° , m ($\angle$ XZY) = 30°

, XY = EF = 5 cm. , F is the midpoint of $\overline{XZ}$

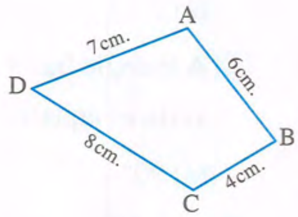
Prove that : m ($\angle$ XEZ) = 90°



[b] In the opposite figure :

From the data on the figure.

Prove that : m ($\angle$ ABC) > m ($\angle$ ADC)



Accumulative test 8 till lesson 3 – unit 5

1 **Choose the correct answer from the given ones :**

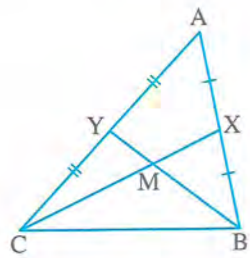
1. In Δ ABC , if m ($\angle$ B) > m ($\angle$ C) , then ....
   (a) AB > AC (b) BC > AC (c) AC > AB (d) AB > BC
2. In Δ XYZ , m ($\angle$ Y) = 130° , then the longest side is ....
   (a) $\overline{XZ}$ (b) $\overline{XY}$ (c) $\overline{YZ}$ (d) its median.
3. In Δ XYZ , m ($\angle$ Z) = 70° , m ($\angle$ Y) = 60° , then YZ .... XZ
   (a) < (b) > (c) = (d) twice
4. In Δ ABC , if AB > AC , then m ($\angle$ B) .... m ($\angle$ C)
   (a) > (b) < (c) = (d) ≥
5. If the measures of two angles in a triangle are 48° , 84° , then its type is ....
   (a) isosceles. (b) equilateral. (c) scalene. (d) right-angled.
6. If A lies on the axis of symmetry of $\overline{BC}$, then $\overline{AB}$ .... $\overline{AC}$
   (a) = (b) // (c) ⊥ (d) ≡
7. If ABC is an obtuse-angled triangle at C , then BC .... AB
   (a) > (b) < (c) = (d) ≥
8. The longest side in Δ XYZ where m ($\angle$ Y) = m ($\angle$ X) + m ($\angle$ Z) is ....
   (a) $\overline{XY}$ (b) $\overline{XZ}$ (c) $\overline{YZ}$ (d) otherwise.

---

2 **[a]** In Δ ABC , m ($\angle$ A) = $(5\,X + 2)°$, m ($\angle$ B) = $(6\,X - 10)°$, m ($\angle$ C) = $(X + 20)°$

Order the lengths of the sides of the triangle ascendingly

[b] In the opposite figure :



X , Y are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively , XM > YM

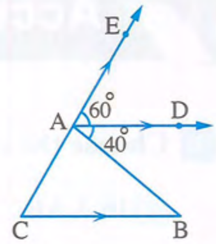
Prove that : m ($\angle$ MBC) > m ($\angle$ MCB)

3 **[a] In the opposite figure :**

$\overrightarrow{AD} \parallel \overline{BC}$, m ($\angle$ EAD) = 60°

, m ($\angle$ BAD) = 40°

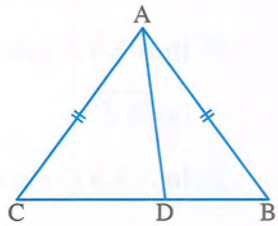
Prove that : AB > AC



[b] In the opposite figure :

AB = AC , D $\in \overline{BC}$

Prove that : AB > AD



1 **Choose the correct answer from the given ones :**

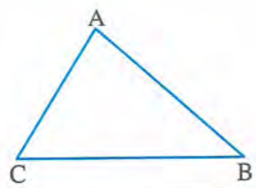
1. The sum of the lengths of any two sides in a triangle is .... the length of the third side.
   (a) greater than (b) equal to (c) smaller than (d) twice
2. Which of the following numbers can be the lengths of sides of a triangle ?
   (a) 5 , 3 , 2 (b) 6 , 3 , 2 (c) 6 , 3 , 3 (d) 3 , 3 , 3
3. If z , 12 , 2 z are the lengths of sides of a triangle , then the greatest value of z = ...
   (a) 12 (b) 11 (c) 4 (d) 3
4. The measure of the exterior angle of the equilateral triangle equals ....
   (a) 60° (b) 80° (c) 120° (d) 180°
5. If ABC is a right-angled triangle at B , then ....
   (a) AC < AB (b) AC < BC (c) AB < AC (d) BC > AB
6. ABC is a right-angled triangle at B , if AC = 20 cm. , then the length of the median drawn from B equals .... cm.
   (a) 10 (b) 8 (c) 6 (d) 5
7. A triangle has one axis of symmetry and the lengths of two sides in it are 3 cm. , 8 cm. , then its perimeter = ... cm.
   (a) 14 (b) 19 (c) 11 (d) 24
8. In Δ XYZ , XY = 8 cm. , YZ = 5 cm. , then its perimeter $\in$ ....
   (a) $]3 , 13[$ (b) $[16 , 26]$ (c) $]16 , 26[$ (d) $\{16 , 26\}$

---

2 **[a]** In Δ ABC , m ($\angle$ A) = 50° , m ($\angle$ B) = 70°

Order the lengths of the sides of the triangle descendingly.

[b] In the opposite figure :

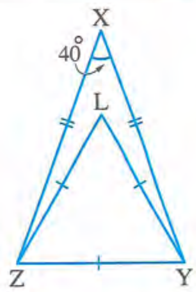


ABC is a triangle.

Prove that : AB < $\frac{1}{2}$ the perimeter of Δ ABC

---

3 **In the opposite figure :**



LYZ is an equilateral triangle

, XY = XZ , m ($\angle$ X) = 40°

Find : m ($\angle$ XZL)

Important Questions

on Geometry

Important questions on Unit Four

First Multiple choice questions

1 The number of medians of the right-angled triangle equals ....

(a) one. (b) two. (c) three. (d) four.

---

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{4}$ (d) 2

---

3 The length of the median drawn from the vertex of the right angle in the right-angled triangle equals .... the length of the hypotenuse.

(a) third (b) quarter (c) half (d) double

---

4 The measure of the exterior angle of the equilateral triangle equals ....°

(a) 60 (b) 90 (c) 120 (d) 180

---

5 If the measure of the vertex angle of an isosceles triangle is 50° , then the measure of one of its base angles is ....°

(a) 65 (b) 45 (c) 55 (d) 70

---

6 If the measure of one of the base angles of an isosceles triangle is 40° , then the measure of its vertex angle equals ....°

(a) 40 (b) 50 (c) 80 (d) 100

---

7 An isosceles triangle , the measure of one of its angles is 60° , then the number of its axes of symmetry is ....

(a) 1 (b) zero. (c) 3 (d) 2

---

8 If the measures of two angles of a triangle are 42° , 69° , then its type is ....

(a) scalene. (b) isosceles. (c) equilateral. (d) right-angled.

---

9 The point of concurrence of the medians of the triangle divides each of them in the ratio of .... from the base.

(a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

---

10 The point of concurrence of the medians of the triangle divides each median in the ratio of 5 : .... from the vertex.

(a) 2.5 (b) 5 (c) 6 (d) 10

11 If M is the point of intersection of the medians of Δ ABC , $\overline{AD}$ is a median , then AD = ... AM

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

12 In Δ ABC , $\overline{AD}$ is a median , M is the point of intersection of its medians. If AD = 9 cm. , then AM = ... cm.

(a) 3 (b) 6 (c) 9 (d) 12

13 The triangle in which the measures of two angles are 48° , 84° , then the number of its axes of symmetry is ....

(a) 1 (b) 2 (c) 3 (d) zero.

14 If X lies on the axis of symmetry of $\overline{AB}$, then XA .... XB

(a) // (b) $\perp$ (c) = (d) $\equiv$

15 Δ XYZ is an isosceles triangle in which m ($\angle$ X) = 100° , then m ($\angle$ Y) = ...°

(a) 100 (b) 80 (c) 60 (d) 40

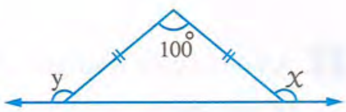
16 Δ ABC is right-angled at B , AC = 12 cm. , m ($\angle$ A) = 60° , then AB = ... cm.

(a) 12 (b) 6 (c) 4 (d) 3

17 A right-angled isosceles triangle , then the measure of one of its base angles is ....°

(a) 30 (b) 45 (c) 60 (d) 90

18 **In the opposite figure :**



$x + y =$ ....

(a) 100° (b) 140°

(c) 180° (d) 280°

Second Complete questions

1 The medians of a triangle intersect at ....

2 The base angles of the isosceles triangle are ....

3 The bisector of the vertex angle of an isosceles triangle is .... to the base and ....

4 The median drawn from the vertex angle of an isosceles triangle is .... and ....

5 The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of .... and ....

6 The straight line perpendicular to a line segment at its midpoint is ....

7 The isosceles triangle in which the measure of one of its angles equals 60° is ....

8 If the measure of one angle in a right-angled triangle is 45° , then the triangle is ....

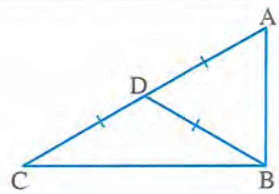
9 If $C \in$ the axis of symmetry of $\overline{AB}$, then AC – BC = ...

10 If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is ....

11 In Δ ABC , AB = AC , m (∠ A) = 3 m (∠ B) , then m (∠ C) = ...°

12 **In the opposite figure :**

m (∠ B) = ...°



Third Essay questions

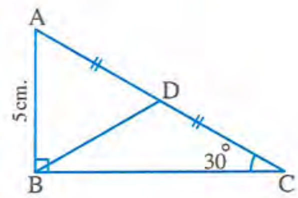
1 **In the opposite figure :**

m (∠ ABC) = 90°

, m (∠ C) = 30° , AD = DC

, AB = 5 cm.

Calculate : The length of each of $\overline{CA}$ and $\overline{BD}$



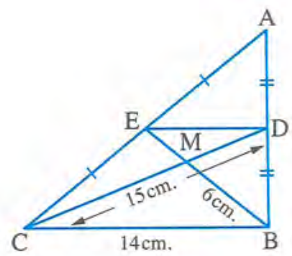
2 **In the opposite figure :**

M is the intersection point of

the medians of the triangle ABC

, BM = 6 cm. , BC = 14 cm. , DC = 15 cm.

Find : The perimeter of Δ MDE



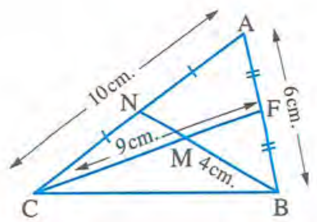
3 **In the opposite figure :**

F and N are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, AB = 6 cm. , AC = 10 cm.

, BM = 4 cm , CF = 9 cm.

Find : The perimeter of the figure AFMN



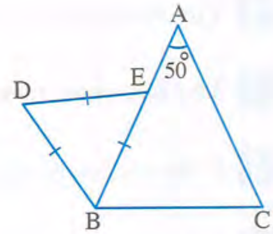
4 **In the opposite figure :**

AB = AC

, m (∠ A) = 50°

, BD = BE = DE

Find : m (∠ DBC)

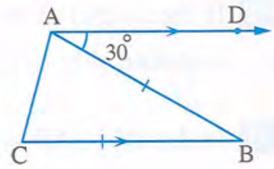


5 **In the opposite figure :**

$\overrightarrow{AD}$ // $\overline{CB}$, m (∠ DAB) = 30°

, BA = BC

Find : The measures of the angles of Δ ABC

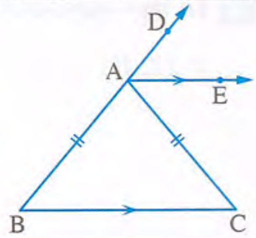


6 **In the opposite figure :**

AB = AC

, $\overrightarrow{AE}$ // $\overline{BC}$

Prove that : $\overrightarrow{AE}$ bisects ∠ DAC



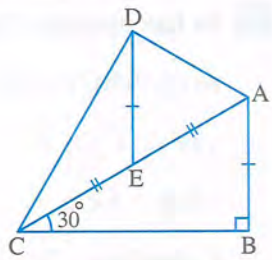
7 **In the opposite figure :**

AB = DE , E is the midpoint of $\overline{AC}$

, m (∠ B) = 90°

, m (∠ ACB) = 30°

Prove that : m (∠ ADC) = 90°



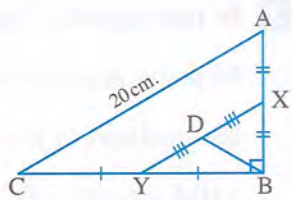
8 **In the opposite figure :**

m (∠ ABC) = 90° , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

, D is the midpoint of $\overline{XY}$, AC = 20 cm.

Find : The length of $\overline{BD}$



9 **In the opposite figure :**

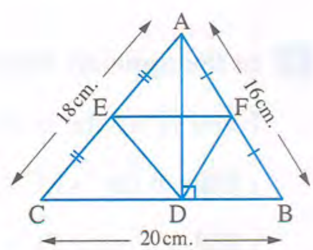
AB = 16 cm. , AC = 18 cm.

, BC = 20 cm.

, F and E are the midpoints of $\overline{AB}$ and $\overline{CA}$ respectively

, $\overline{AD} \perp \overline{CB}$

Find : The perimeter of Δ DEF



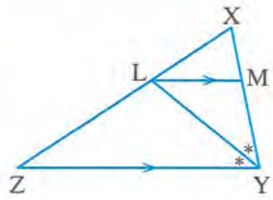
10 **In the opposite figure :**

XYZ is a triangle , $\overrightarrow{YL}$ bisects $\angle$ XYZ

and intersects $\overline{XZ}$ at L

, $\overline{LM}$ // $\overline{YZ}$ and intersects $\overline{XY}$ at M

Prove that : Δ LMY is an isosceles triangle.



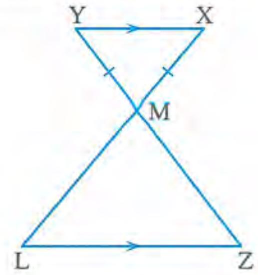
11 **In the opposite figure :**

XM = YM

, $\overline{XY}$ // $\overline{ZL}$

Prove that :

Δ MLZ is an isosceles triangle.



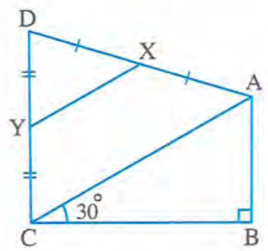
12 **In the opposite figure :**

m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

, X and Y are the midpoints

of $\overline{AD}$ and $\overline{CD}$ respectively.

Prove that : AB = XY

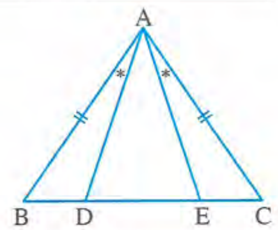


13 **In the opposite figure :**

AB = AC

, m ($\angle$ BAD) = m ($\angle$ CAE)

Prove that : AD = AE

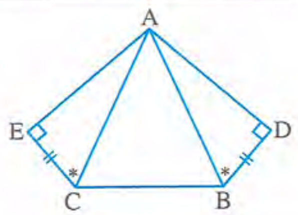


14 **In the opposite figure :**

BD = CE , m ($\angle$ D) = m ($\angle$ E) = 90°

, m ($\angle$ ABD) = m ($\angle$ ACE)

Prove that : m ($\angle$ ABC) = m ($\angle$ ACB)



15 **In the opposite figure :**

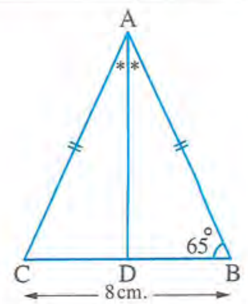
AB = AC , $\overrightarrow{AD}$ bisects $\angle$ BAC

, BC = 8 cm.

, m ($\angle$ B) = 65°

Prove that : $\overline{AD} \perp \overline{BC}$

and find : The length of $\overline{DC}$, m ($\angle$ DAC)

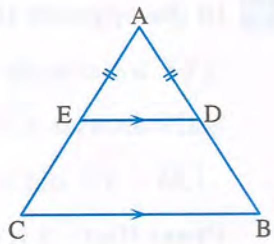


16 **In the opposite figure :**

$\overline{DE} \parallel \overline{BC}$

, AD = AE

Prove that : Δ ABC is an isosceles triangle.



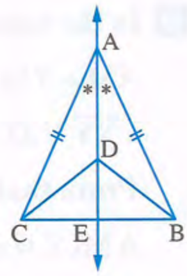
17 **In the opposite figure :**

AB = AC , $\overrightarrow{AE}$ bisects $\angle$ BAC

, $\overrightarrow{AE} \cap \overline{BC} = \{E\}$, $D \in \overline{AE}$

Prove that : **1** $BE = \frac{1}{2} BC$

2 BD = CD

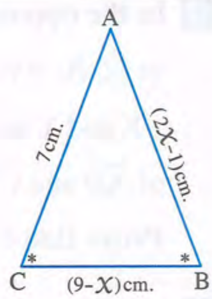


18 **In the opposite figure :**

m ($\angle$ B) = m ($\angle$ C)

Find : **1** The value of X

2 The perimeter of Δ ABC



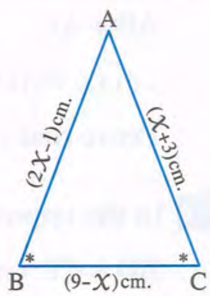
19 **In the opposite figure :**

AB = $(2X - 1)$ cm. , AC = $(X + 3)$ cm.

, BC = $(9 - X)$ cm.

, m ($\angle$ B) = m ($\angle$ C)

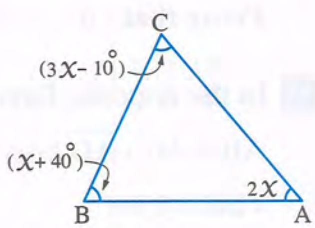
Find : The perimeter of Δ ABC



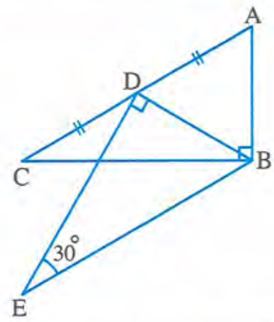
20 **In the opposite figure :**

Prove that :

Δ ABC is an isosceles triangle.



21 **In the opposite figure :**



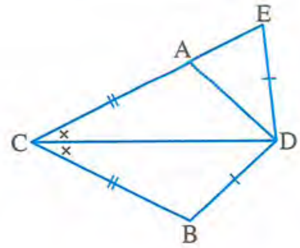
m (∠ ABC) = m (∠ BDE) = 90°

, D is the midpoint of $\overline{AC}$

, m (∠ BED) = 30°

Prove that : AC = BE

22 **In the opposite figure :**



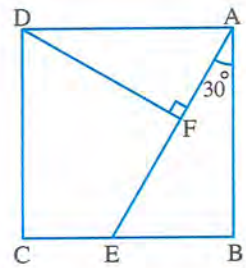
m (∠ ACD) = m (∠ BCD)

, AC = BC

, DB = DE

Prove that : m (∠ E) = m (∠ EAD)

23 **In the opposite figure :**



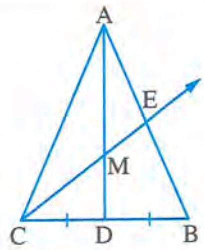
ABCD is a square

, m (∠ BAE) = 30°

, $\overline{DF} \perp \overline{AE}$, AF = 4 cm.

Calculate : The area of the square.

24 **In the opposite figure :**



ABC is a triangle

, D is the midpoint of $\overline{BC}$

, M $\in \overline{AD}$ where AM = 2 MD

, $\overrightarrow{CM} \cap \overline{AB} = \{E\}$, EC = 12 cm.

Find : The length of $\overline{EM}$

Important questions on Unit Five

First Multiple choice questions

1 In ΔABC , if $AB < AC$, then $m(\angle B)$ .... $m(\angle C)$

(a) $<$ (b) $\leq$ (c) $>$ (d) $=$

---

2 If the lengths of two sides of a triangle are 3 cm. , 6 cm. , then the length of the third side belongs to ....

(a) $[3 , 9[$ (b) $[3 , 9]$ (c) $]3 , 9]$ (d) $]3 , 9[$

---

3 ABC is a triangle in which $m(\angle B) = 70°$, $m(\angle C) = 50°$, then AC .... AB

(a) $<$ (b) $\leq$ (c) $>$ (d) $=$

---

4 In ΔABC , if $m(\angle A) > m(\angle B)$, then BC .... AC

(a) $>$ (b) $=$ (c) $<$ (d) $\geq$

---

5 The lengths 4 cm. , 9 cm. and .... cm. can be the lengths of sides of a triangle.

(a) 3 (b) 4 (c) 5 (d) 6

---

6 XYZ is a right-angled triangle at Y , then XZ .... YZ

(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

---

7 In ΔABC , if $m(\angle C) = 100°$, then its longest side is ....

(a) $\overline{AC}$ (b) $\overline{AB}$ (c) $\overline{BC}$

---

8 The numbers that can be the lengths of sides of a triangle are ....

(a) 7 , 7 , 4 (b) 3 , 4 , 9 (c) 4 , 5 , 12 (d) 5 , 5 , 15

---

9 In ΔABC , $AB + BC - AC >$ ....

(a) 2 AC (b) – 2 AC (c) 2 (d) zero.

---

10 If the lengths of two sides of an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side is .... cm.

(a) 2 (b) 5 (c) 4 (d) 3

---

11 The triangle whose side lengths are 2 cm. , $(X + 3)$ cm. and 7 cm. is an isosceles triangle if $X =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

---

12 The length of any side in a triangle .... the sum of the lengths of the other two sides.

(a) $>$ (b) $<$ (c) $=$ (d) $\leq$

Second Complete questions

1 In a triangle , if two angles are unequal in measure , then the greater angle in measure is ....

2 In a triangle , if two sides have unequal lengths , then the longer side is opposite to ....

3 In Δ ABC , if m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then the longest side is ....

4 In Δ ABC , if m ($\angle$ A) = 100° , then the longest side is ....

5 In Δ ABC , AB + BC .... AC

6 In a triangle , if the lengths of two sides are 5 cm. , 9 cm. , then the length of the third side $\in$].............. , ...[

7 The distance between any point and a given straight line is the length of .... drawn from this point to the given line.

8 In the right-angled triangle , ... is the longest side.

9 In Δ ABC , if m ($\angle$ A) = 2 m ($\angle$ B) = 80° , then AB > ....

10 If X > Y , A > B , then Y + B < ....

Third Essay questions

1 In Δ ABC , m ($\angle$ A) = 40° , m ($\angle$ B) = 80° arrange the lengths of the sides of Δ ABC descendingly.

2 ABC is a triangle in which : AB = 6 cm. , AC = 8 cm. and BC = 7 cm. Arrange the measures of the angles of the triangle ABC ascendingly.

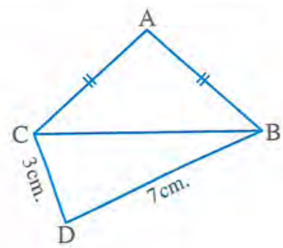
3 **In the opposite figure :**

AB = AC

, BD = 7 cm.

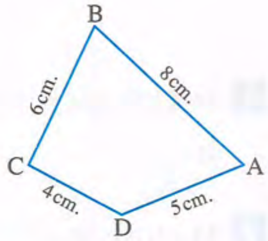
, DC = 3 cm.

Prove that : m ($\angle$ ACD) > m ($\angle$ ABD)



4 **In the opposite figure :**

Prove that : m (∠ BCD) > m (∠ BAD)

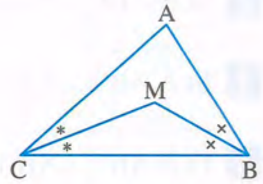


5 **In the opposite figure :**

ABC is a triangle in which : AB < AC

, $\overrightarrow{BM}$ bisects ∠ ABC , $\overrightarrow{CM}$ bisects ∠ ACB

Prove that : BM < CM

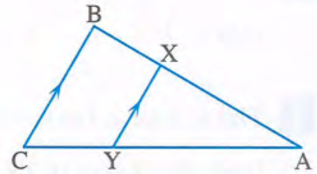


6 **In the opposite figure :**

ABC is a triangle in which :

AB > BC , $\overline{XY}$ // $\overline{BC}$

Prove that : AX > XY

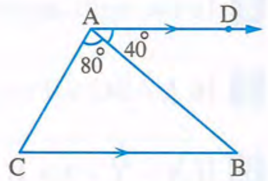


7 **In the opposite figure :**

$\overrightarrow{AD}$ // $\overline{CB}$, m (∠ DAB) = 40°

, m (∠ BAC) = 80°

Prove that : AB > AC



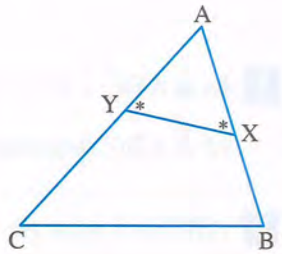
8 **In the opposite figure :**

ABC is a triangle in which :

AC > AB

, m (∠ AXY) = m (∠ AYX)

Prove that : YC > XB



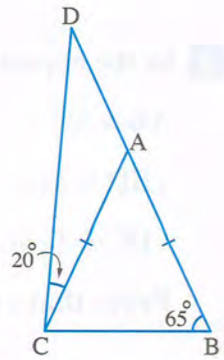
9 **In the opposite figure :**

AB = AC

, m (∠ ABC) = 65°

, m (∠ ACD) = 20°

Prove that : AB > AD

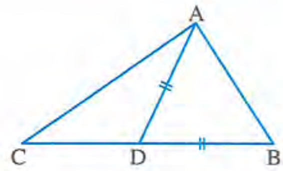


10 **In the opposite figure :**

ABC is a triangle , D $\in \overline{BC}$

, AD = BD

Prove that : BC > AC

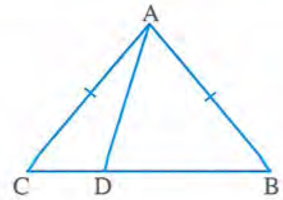


11 **In the opposite figure :**

ABC is a triangle in which :

AB = AC , D $\in \overline{BC}$

Prove that : AC > AD



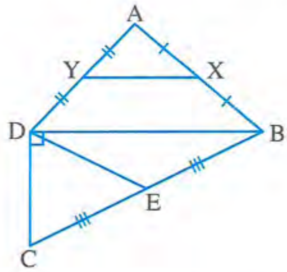
12 **In the opposite figure :**

ABCD is a quadrilateral in which : m (∠ BDC) = 90°

, X , Y and E are the midpoints of $\overline{AB}$, $\overline{AD}$

and $\overline{BC}$ respectively.

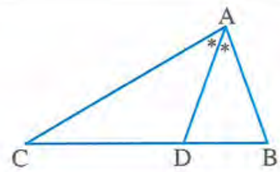
Prove that : XY < DE



13 **In the opposite figure :**

$\overrightarrow{AD}$ bisects ∠ BAC

Prove that : AC > DC

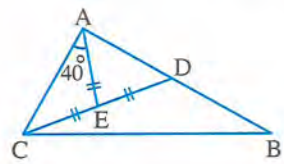


14 **In the opposite figure :**

DE = EC = AE , m (∠ CAE) = 40°

Prove that : 1 AC > AE

2 BC > AC



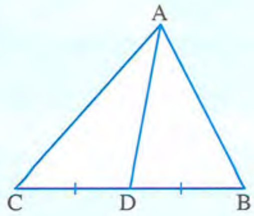
Final Revision

of Geometry

Revision for the important theorems , corollaries and rules of Geometry

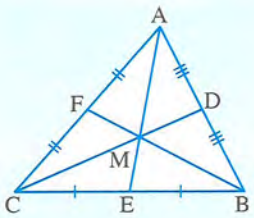
Medians of triangle

The median of the triangle is the line segment drawn from any vertex of the triangle vertices to the midpoint of the opposite side of this vertex.



If D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is a median in Δ ABC

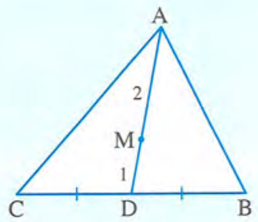
The medians of a triangle are concurrent.



If $\overline{CD}$, $\overline{BF}$ and $\overline{AE}$ are the medians of Δ ABC where $\overline{CD} \cap \overline{BF} \cap \overline{AE} = \{M\}$, then M is the intersection point of the medians of Δ ABC

The point of concurrence of the medians of the triangle divides each median in the ratio of :

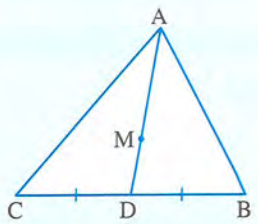
- 1 : 2 from the base.
- 2 : 1 from the vertex.



If M is the intersection point of the medians of Δ ABC , then :

- DM = $\frac{1}{2}$ AM
- AM = 2 DM
- DM = $\frac{1}{3}$ AD
- AM = $\frac{2}{3}$ AD

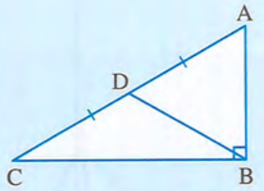
The point which divides the median of a triangle by the ratio 1 : 2 from the base is the point of the intersection of the medians of the triangle.



If DM : MA = 1 : 2 , then M is the intersection point of the medians of Δ ABC

Right-angled triangle

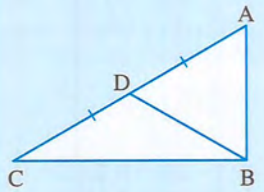
The length of the median from the vertex of the right angle equals half the length of the hypotenuse.



If Δ ABC is right-angled at B , $\overline{BD}$ is a median in it , then

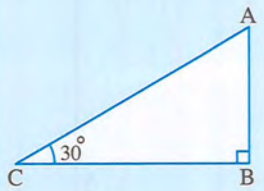
$BD = \frac{1}{2} AC$

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.



If $\overline{BD}$ is a median of Δ ABC , $BD = \frac{1}{2} AC$, then $m(\angle ABC) = 90°$

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

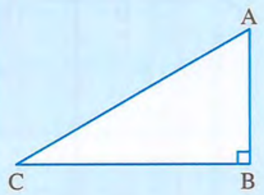


If Δ ABC is right-angled at B in which :

$m(\angle C) = 30°$

, then $AB = \frac{1}{2} AC$

In the right-angled triangle, the hypotenuse is the longest side of the triangle.

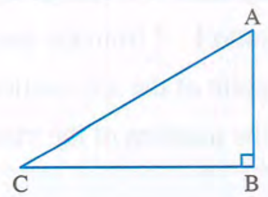


If Δ ABC is right-angled at B , then

AC > AB , AC > BC

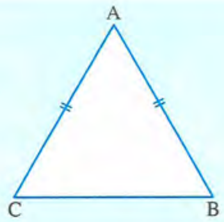
If Δ ABC is right-angled at B , then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



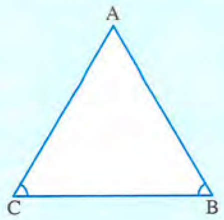
The isosceles triangle

The base angles of the isosceles triangle are congruent.



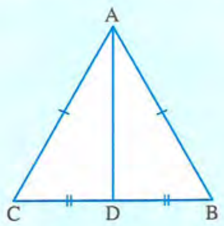
If Δ ABC in which :
AB = AC , then
m ($\angle$ B) = m ($\angle$ C)

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



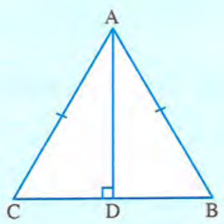
If Δ ABC in which :
m ($\angle$ B) = m ($\angle$ C)
, then AB = AC

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.



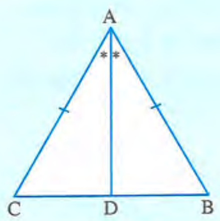
If Δ ABC in which :
AB = AC , $\overline{AD}$ is a median
, then $\overrightarrow{AD}$ bisects $\angle$ BAC
, $\overline{AD} \perp \overline{BC}$

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



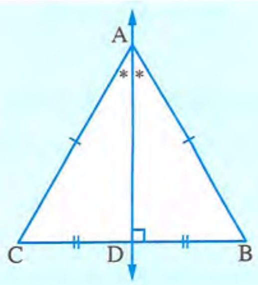
If Δ ABC in which :
AB = AC , $\overline{AD} \perp \overline{BC}$
, then D is the midpoint of $\overline{BC}$,
$\overrightarrow{AD}$ bisects $\angle$ BAC

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.



If Δ ABC in which :
AB = AC , $\overrightarrow{AD}$ bisects $\angle$ BAC , then D is the midpoint of $\overline{BC}$, $\overline{AD} \perp \overline{BC}$

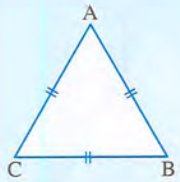
The number of axes of symmetry of the isosceles triangle equals 1



If Δ ABC in which :
AB = AC , $\overleftrightarrow{AD} \perp \overline{BC}$ and intersect it at D
, then $\overleftrightarrow{AD}$ is the axis of symmetry of the triangle ABC

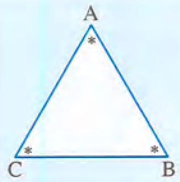
The equilateral triangle

If the triangle is an equilateral , then it is equiangular where each angle measure is 60°



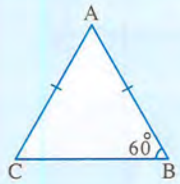
If Δ ABC in which :
AB = BC = CA , then
m ($\angle$ A) = m ($\angle$ B) = m ($\angle$ C) = 60°

If the angles of a triangle are congruent , then the triangle is equilateral.



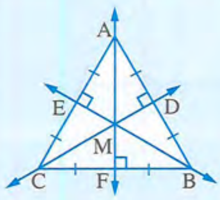
If Δ ABC in which :
m ($\angle$ A) = m ($\angle$ B) = m ($\angle$ C)
, then AB = BC = CA

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.



If Δ ABC in which :
AB = AC , m ($\angle$ B) = 60°
, then Δ ABC is an equilateral triangle.

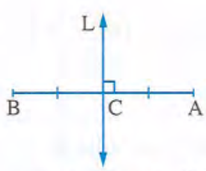
The equilateral triangle has three axes of symmetry.



If Δ ABC is an equilateral triangle
, $\overleftrightarrow{AF} \perp \overline{BC}$, $\overleftrightarrow{CD} \perp \overline{AB}$, $\overleftrightarrow{BE} \perp \overline{AC}$
, then $\overleftrightarrow{AF}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{BE}$ are the axes of symmetry of the triangle ABC

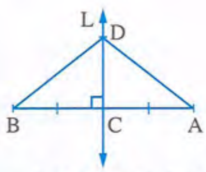
The axis of symmetry

The axis of symmetry of a line segment is the straight line perpendicular to it from its middle.



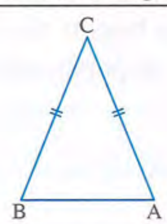
If the straight line L $\perp$ $\overline{AB}$, C $\in$ $\overline{AB}$ where CA = CB
, C $\in$ the straight line L
, then L is the axis of $\overline{AB}$

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).



If the straight line L is the axis of $\overline{AB}$, D $\in$ the straight line L , then DA = DB

If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

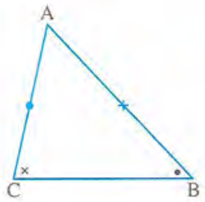


If CA = CB , then
C lies on the axis of $\overline{AB}$

Inequality relations in the triangle

Comparing the measures of angles in a triangle

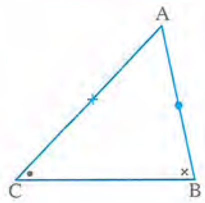
If two sides have unequal lengths , the longer is opposite to the angle of the greater measure



If AB > AC , then m (∠ C) > m (∠ B)

Comparing the lengths of sides in a triangle

If two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



If m (∠ B) > m (∠ C) , then AC > AB

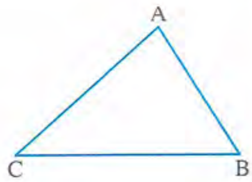
Triangle inequality

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

AB + BC > AC

, BC + CA > AB

, CA + AB > BC

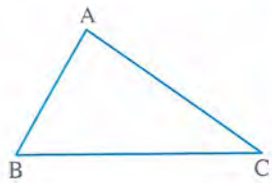


Notice that :

- The length of any side of a triangle is greater than the difference between the lengths of the two other sides and less than their sum.

  In Δ ABC :

  AC − AB < BC < AC + AB

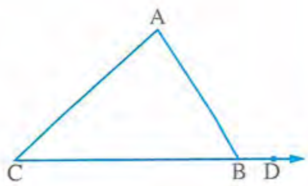


- The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

  In Δ ABC :

  m (∠ ABD) > m (∠ A)

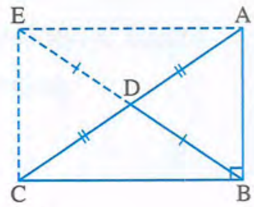
  , m (∠ ABD) > m (∠ C)



Proofs of the important theorems

Theorem

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.



Given ABC is a triangle in which m $(\angle ABC) = 90°$,

$\overline{BD}$ is a median in the triangle ABC

R.T.P. $BD = \frac{1}{2} AC$

Construction Draw $\overrightarrow{BD}$ and take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof In the figure ABCE : $\because$ $\overline{AC}$ and $\overline{BE}$ bisect each other

$\therefore$ The figure ABCE is a parallelogram.

$\because$ m $(\angle ABC) = 90°$

$\therefore$ The figure ABCE is a rectangle.

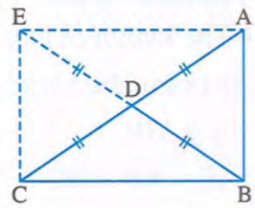
$\therefore BE = AC$

, $\because BD = \frac{1}{2} BE$

$\therefore BD = \frac{1}{2} AC$ (Q.E.D.)

Theorem

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.



Given In Δ ABC , $\overline{BD}$ is a median and DA = DB = DC

R.T.P. m $(\angle ABC) = 90°$

Construction Draw $\overrightarrow{BD}$, then take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof $\because BD = \frac{1}{2} BE = \frac{1}{2} AC$ $\therefore BE = AC$

$\therefore$ In the figure ABCE :

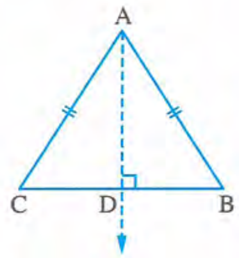
$\overline{AC}$ and $\overline{BE}$ are equal in length and bisect each other.

$\therefore$ The figure ABCE is a rectangle.

$\therefore$ m $(\angle ABC) = 90°$ (Q.E.D.)

Theorem

The base angles of the isosceles triangle are congruent.



Given ABC is a triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P. $\angle B \equiv \angle C$

Construction Draw $\overrightarrow{AD} \perp \overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$

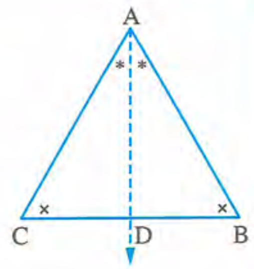
Proof $\because \Delta\Delta$ ADB, ADC in which :

$$\begin{cases} m(\angle ADB) = m(\angle ADC) = 90^\circ & \text{(const.)} \\ \overline{AB} \equiv \overline{AC} & \text{(given)} \\ \overline{AD} \text{ is a common side} & \end{cases}$$

$\therefore \Delta ADB \equiv \Delta ADC$, then we deduce that $\angle B \equiv \angle C$ (Q.E.D.)

Theorem

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



Given ABC is a triangle in which $\angle B \equiv \angle C$

R.T.P. $\overline{AB} \equiv \overline{AC}$

Construction bisect $\angle$ BAC by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D

Proof $\because \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\because \overrightarrow{AD}$ bisects $\angle$ BAC $\therefore m(\angle BAD) = m(\angle CAD)$

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore m(\angle ADB) = m(\angle ADC)$

$\therefore$ In $\Delta\Delta$ ABD and ACD :

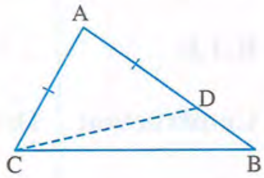
$$\begin{cases} \overline{AD} \text{ is a common side} & \\ m(\angle BAD) = m(\angle CAD) & \text{(const.)} \\ m(\angle ADB) = m(\angle ADC) & \text{(by proof)} \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that

$\overline{AB} \equiv \overline{AC}$, then Δ ABC is an isosceles triangle. (Q.E.D.)

Theorem

In a triangle , if two sides have unequal lengths , the longer is opposite to the angle of the greater measure.



Given ABC is a triangle in which AB > AC

R.T.P. m (∠ ACB) > m (∠ ABC)

Construction Take D ∈ $\overline{AB}$ such that AD = AC

Proof In Δ ACD : ∵ AD = AC ∴ m (∠ ADC) = m (∠ ACD) (1)

∵ ∠ ADC is an exterior angle of Δ DBC

∴ m (∠ ADC) > m (∠ B) (2)

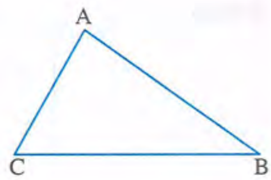
From (1) and (2) : ∴ m (∠ ACD) > m (∠ B)

, ∵ m (∠ ACB) > m (∠ ACD)

∴ m (∠ ACB) > m (∠ ABC) (Q.E.D.)

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



Given ABC is a triangle in which m (∠ C) > m (∠ B)

R.T.P. AB > AC

Proof ∵ $\overline{AB}$ and $\overline{AC}$ are two line segments.

∴ One of the following cases should be verified.

1 AB > AC 2 AB = AC 3 AB < AC

Unless AB > AC , then either AB = AC **or** AB < AC

- If : AB = AC , then m (∠ C) = m (∠ B) and this contradicts the given where m (∠ C) > m (∠ B)
- If : AB < AC , then m (∠ C) < m (∠ B) according to the preceding theorem. Again this contradicts the given , where m (∠ C) > m (∠ B)

∴ It should be that AB > AC (Q.E.D.)

Final Examinations

on Geometry

Model 1

Answer the following questions :

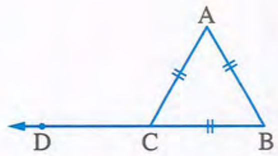
1 Complete the following :

1. The longest side in the right-angled triangle is ....
2. If the lengths of two sides in a triangle are 2 cm. and 7 cm. , then .... < the length of the third side < ....
3. If the measures of two angles in a triangle are different , then the greater in measure of them is opposite to ....
4. If the length of the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length , then ....
5. If the measure of an angle in the isosceles triangle equals 60° , then the triangle is ....

---

2 Choose the correct answer from those given :

1. **In the opposite figure :**



Δ ABC is equilateral , then m ($\angle$ ACD) = ...

(a) 45° (b) 60° (c) 120° (d) 135°

2. In Δ ABC which is right-angled at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals ....

(a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

3. XYZ is a triangle in which : m ($\angle$ Z) = 70° and m ($\angle$ Y) = 60° , then YZ .... XY

(a) > (b) < (c) = (d) twice

4. The lengths which can be lengths of sides of a triangle are ....

(a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

5. The triangle in which the measures of two angles of it are 42° and 69° is ....

(a) an isosceles triangle. (b) an equilateral triangle.
(c) a scalene triangle. (d) a right-angled triangle.

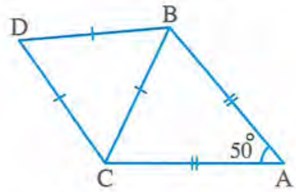
6. **In the opposite figure :**

m ($\angle$ C) = 2 m ($\angle$ A)
, BC = 6 cm.
, then AC = ... cm.

(a) 3 (b) 6
(c) 9 (d) 12

3 **[a] Complete :** ABC is a triangle in which AB > AC , then m (∠ C) .... m (∠ B)

[b] In the opposite figure :

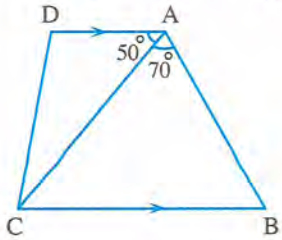


m (∠ A) = 50° , AB = AC

and Δ DBC is equilateral

Find : m (∠ ABD)

[c] In the opposite figure :



$\overline{AD}$ // $\overline{BC}$

, m (∠ BAC) = 70°

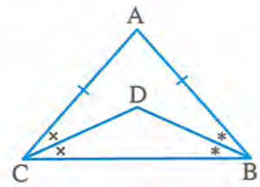
and m (∠ DAC) = 50°

Prove that : BC > AC

---

4 **[a] Prove that :** The two base angles of the isosceles triangle are congruent.

[b] In the opposite figure :



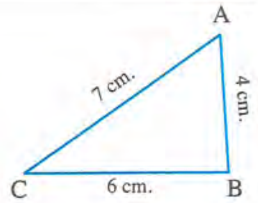
AB = AC , $\overrightarrow{BD}$ bisects ∠ B

and $\overrightarrow{CD}$ bisects ∠ C

Prove that : Δ DBC is isosceles.

---

5 **[a] In the opposite figure :**

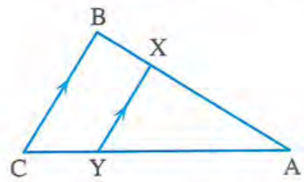


Arrange the angles

of Δ ABC descendingly

due to their measures.

[b] In the opposite figure :



AB > BC , $\overline{XY}$ // $\overline{BC}$

Prove that : AX > XY

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The triangle which has three axes of symmetry is ....

(a) scalene. (b) isosceles. (c) right-angled. (d) equilateral.

2 The sum of lengths of two sides in a triangle is .... the length of the third side.

(a) greater than (b) smaller than (c) equal to (d) twice

3 If the lengths of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is .... cm.

(a) 4 (b) 8 (c) 3 (d) 12

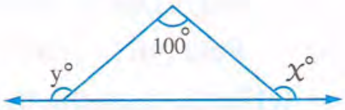
4 In Δ ABC , if m ($\angle$ B) = 130° , then the longest side of it is ....

(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

5 Δ XYZ is an isosceles triangle in which : m ($\angle$ X) = 100° , then m ($\angle$ Y) = ...

(a) 100° (b) 80° (c) 60° (d) 40°

6 **In the opposite figure :**



$x + y =$ ....

(a) 100° (b) 140°

(c) 180° (d) 280°

---

2 Complete the following :

1 If the measure of an angle in a right-angled triangle is 45° , then the triangle is ....

2 The length of any side in a triangle .... the sum of lengths of the two other sides.

3 If $\overline{AB} \equiv \overline{XY}$, then AB = ...

4 In Δ ABC , if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then BC = ... AC

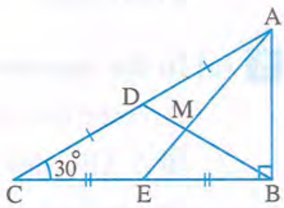
5 The axis of symmetry of a line segment is the straight line which .... at its midpoint.

---

3 [a] In Δ ABC , AB = 7 cm. , BC = 5 cm. and AC = 6 cm.

Arrange its angles ascendingly due to their measures.

[b] **In the opposite figure :**



Δ ABC is right-angled at B

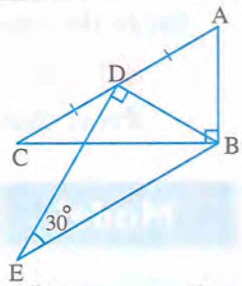
, m ($\angle$ C) = 30° , D is the midpoint of $\overline{AC}$

, E is the midpoint of $\overline{BC}$, AC = 9 cm.

Find : The length of each of $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$

---

4 [a] **In the opposite figure :**



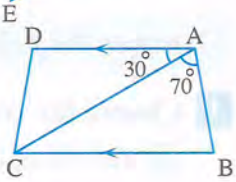
m ($\angle$ ABC) = m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE

[b] **In the opposite figure :**



$\overleftrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 70°

, m ($\angle$ DAC) = 30°

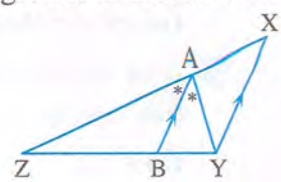
Prove that : AC > BC

---

5 [a] **Complete :**

If the measures of two angles of a triangle are different , then the greater in measure is opposite to ....

[b] **In the opposite figure :**



$\overleftrightarrow{AB}$ // $\overline{XY}$ and $\overrightarrow{AB}$ bisects $\angle$ YAZ

Prove that : XZ > YZ

Model for the merge students

Answer the following questions :

1 Complete each of the following :

1 The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.

2 In the right-angled triangle , the length of the median drawn from the vertex of the right angle equals ....

3 The base angles of the isosceles triangle are ....

4 In Δ ABC , if m ($\angle$ B) = 70° , m ($\angle$ C) = 50° , then AC .... AB

5 The median of the isosceles triangle from the vertex angle .... , ...

---

2 Choose the correct answer from those given :

1 If ABC is an equilateral triangle , then m ($\angle$ B) = ...

(a) 30° (b) 60° (c) 70° (d) 90°

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 2

3 If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of one of the base angles equals ....

(a) 60° (b) 40° (c) 30° (d) 50°

4 The number of axes of symmetry of the isosceles triangle is ....

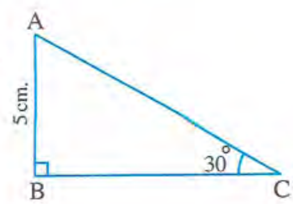
(a) 1 (b) 2 (c) 3 (d) zero

5 In Δ ABC , if m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then the longest side is ....

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$

---

3 In the opposite figure :



Δ ABC is a right-angled triangle at B , m ($\angle$ C) = 30° , AB = 5 cm.

Find : The length of $\overline{AC}$

$\because$ m ($\angle$ B) = ...° , m ($\angle$ C) = ...°

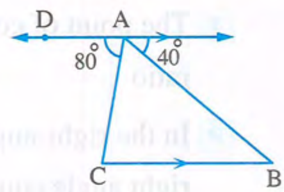
$\therefore$ AB = $\frac{1}{2}$ × .... $\therefore$ AC = ... cm.

4 [a] In ΔABC , $m(\angle A) = 40°$, $m(\angle B) = 75°$, $m(\angle C) = 65°$

Arrange the lengths of the sides of the triangle descendingly.

The order is : .... , ... , ...

[b] In the opposite figure :



$\overleftrightarrow{AD} \parallel \overline{BC}$

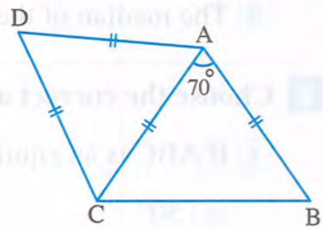
Complete :

1 $m(\angle B) =$ ....°

2 The side .... is the longest side of ΔABC

---

5 In the opposite figure :



$AB = AC = CD = AD = 10$ cm.

, $m(\angle BAC) = 70°$

Put (✔) or (✘) :

1 $m(\angle B) = 55°$ ( )

2 $m(\angle D) = 70°$ ( )

3 $m(\angle DCB) = 120°$ ( )

4 $AB + AD = 20$ cm. ( )

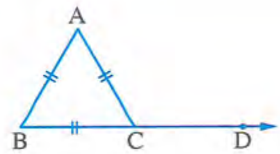
5 $AB + BC = BC + CD$ ( )

1 Cairo Governorate

Nozha Directorate of Education
Mathematics Supervisors

Answer the following questions :

1 Choose the correct answer :



1 **In the opposite figure :**

ABC is an equilateral triangle , then m (∠ ACD) = ...°

(a) 45 (b) 60 (c) 120 (d) 135

2 The number of axes of symmetry of a scalene triangle equals ....

(a) 3 (b) 2 (c) 1 (d) 0

3 If the triangle XYZ is right-angled at X , then YZ .... XZ

(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

4 The point M is the point of intersection of the medians of Δ ABC , $\overline{AD}$ is a median of length 12 cm. , then MD = ... cm.

(a) 12 (b) 6 (c) 4 (d) 3

5 The set of numbers which can be lengths of sides of a triangle is ....

(a) $\{4 , 6 , 10\}$ (b) $\{4 , 6 , 8\}$ (c) $\{2 , 3 , 6\}$ (d) $\{4 , 5 , 10\}$

6 If the measures of two angles in a triangle are 50° , 80° , then the triangle is ....

(a) scalene. (b) an isosceles triangle.

(c) an equilateral triangle. (d) a right-angled triangle.

---

2 Complete :

1 The longest side of the right-angled triangle is ....

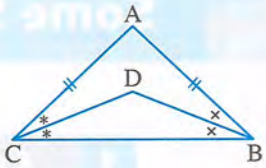
2 The angles of the equilateral triangle are .... in measure and the measure of each of them is ....°

3 In Δ DEH , m (∠ E) = 110° , then the longest side in this triangle is ....

4 If $\overline{AB} \equiv \overline{XY}$, then AB – XY = ...

5 In Δ ABC , if m (∠ A) = 67° and m (∠ B) = 33° , then AB > .... > ....

3 **[a] In the opposite figure :**

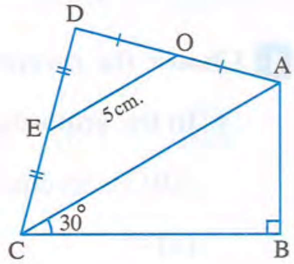


$AB = AC$, $\overrightarrow{BD}$ bisects $\angle ABC$

, $\overrightarrow{CD}$ bisects $\angle ACB$

, **prove that :** Δ DBC is an isosceles triangle.

[b] In the opposite figure :



ABCD is a quadrilateral in which

$m(\angle B) = 90°$, $m(\angle ACB) = 30°$

, E and O are the midpoints of $\overline{CD}$ and $\overline{AD}$, $EO = 5$ cm.

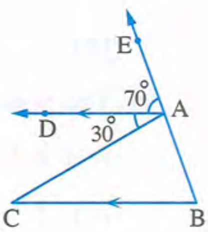
Find : The length of $\overline{BA}$

---

4 **[a]** In ΔABC , $AB = 6$ cm. , $AC = 7$ cm. , $BC = 8$ cm.

Arrange its angles ascendingly due to their measures.

[b] In the opposite figure :



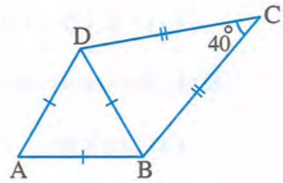
$\overrightarrow{AD} \,//\, \overline{BC}$, $m(\angle EAD) = 70°$

, $m(\angle DAC) = 30°$

Prove that : $AC > AB$

---

5 **[a] In the opposite figure :**

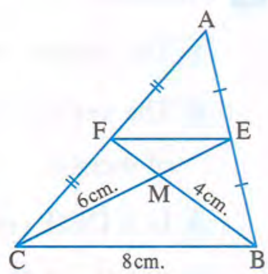


ADB is an equilateral triangle

, $DC = BC$, $m(\angle C) = 40°$

Find : $m(\angle ADC)$

[b] In the opposite figure :



ABC is a triangle in which

E , F are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively

, $\overline{BF} \cap \overline{CE} = \{M\}$, $BC = 8$ cm.

, $BM = 4$ cm. and $CM = 6$ cm.

Find : The perimeter of Δ MEF

2 Cairo Governorate

El-Zeiton Zone
Talaea Gaber El-Ansary Language School

Answer the following questions :

1 Choose the correct answer :

1 If Δ ABC is right-angled at A and AB = AC , then m ($\angle$ B) = ...°

(a) 30 (b) 45 (c) 60 (d) 90

2 In Δ ABC , if m ($\angle$ C) = 65° and m ($\angle$ A) = 75° , then ....

(a) AB > BC (b) AB > AC (c) BC < AB (d) AB = AC

3 In Δ ABC , if AB = AC and m ($\angle$ A) = 60° , then the number of axes of symmetry of the triangle ABC is ....

(a) 0 (b) 1 (c) 2 (d) 3

4 If the lengths of two sides of a triangle are 3 cm. and 7 cm. , then the length of the third side may be .... cm.

(a) 10 (b) 4 (c) 3 (d) 5

5 In Δ ABC , if AB = AC and m ($\angle$ A) = 80° , then m ($\angle$ B) = ...°

(a) 60 (b) 80 (c) 40 (d) 50

6 The point of concurrence of the medians of the triangle divides each median in the ratio .... from the vertex.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

2 Complete :

1 In Δ ABC , if D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is called ....

2 The number of medians in the right-angled triangle is ....

3 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

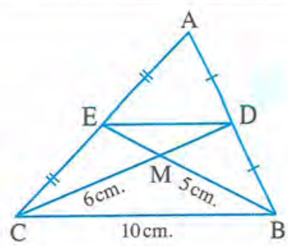
4 If the lengths of two sides in a triangle are 2 cm. and 7 cm. , then the length of the third side $\in$] .... , ... [

5 In Δ ABC , if m ($\angle$ A) = 67° and m ($\angle$ B) = 33° , then AB > .... > ....

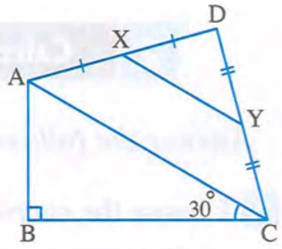
3 [a] In the opposite figure :

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overline{BE} \cap \overline{CD} = \{M\}$, BC = 10 cm. , MB = 5 cm. , MC = 6 cm.

Find : The perimeter of Δ MDE



[b] In the opposite figure :



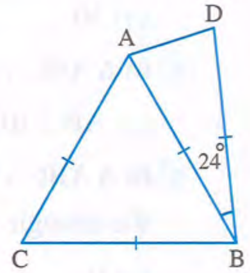
$m(\angle ABC) = 90°$, $m(\angle ACB) = 30°$

, X and Y are the midpoints of $\overline{AD}$ and $\overline{CD}$ respectively.

Prove that : $XY = AB$

---

4 **[a] In the opposite figure :**

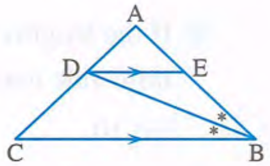


ACBD is a quadrilateral in which $AB = BC = CA = BD$

, $m(\angle ABD) = 24°$

Find : $m(\angle CAD)$

[b] In the opposite figure :

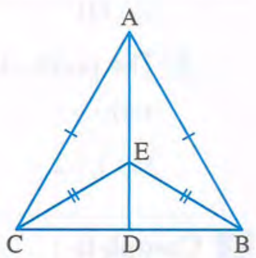


ABC is a triangle , $\overrightarrow{BD}$ bisects $\angle ABC$ and $\overline{ED} \,/\!/\, \overline{BC}$ where $E \in \overline{AB}$

Prove that : ΔEBD is an isosceles triangle.

---

5 **[a] In the opposite figure :**



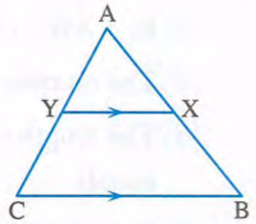
If $AB = AC$, $EB = EC$

, $\overrightarrow{AD} \cap \overline{BC} = \{D\}$ and $E \in \overrightarrow{AD}$

, **prove that :** 1 $\overrightarrow{AD} \perp \overline{BC}$

2 $BD = \frac{1}{2} BC$

[b] In the opposite figure :



ABC is a triangle

, $AB > AC$ and $\overline{XY} \,/\!/\, \overline{BC}$

Prove that : $m(\angle AYX) > m(\angle AXY)$

3 Cairo Governorate

Directory Rod El-Farag
Mousa Bin Nousir (G.L.S.)

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer :**

1 In ΔABC , $m(\angle B) = 90°$, $m(\angle A) = 30°$ and $AC = 10$ cm. , then $BC =$ .... cm.

(a) 20 (b) 10 (c) 5 (d) 25

2 The measure of the exterior angle of the equilateral triangle is ....°

(a) 30 (b) 60 (c) 180 (d) 120

3 The sum of lengths of two sides in a triangle is .... the length of the third side.

(a) > (b) < (c) ≤ (d) =

4 ABC is an isosceles triangle. If m ($\angle$ A) = 100° , then m ($\angle$ B) = ...°

(a) 80 (b) 60 (c) 40 (d) 90

5 In ΔABC , if m ($\angle$ B) = 130° , then the longest side of it is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) otherwise.

6 The point of intersection of the medians of the triangle divides each of them in the ratio .... from the base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 2 : 3

---

2 Complete each of the following :

1 The longest side in the right-angled triangle is ....

2 In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the ....

3 The base angles of the isosceles triangle are ....

4 If the lengths of two sides in a triangle are 2 cm. and 7 cm. , then the length of the third side $\in$] .... , ... [

5 The median of the isosceles triangle from the vertex angle .... , ...

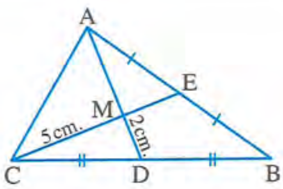
---

3 [a] In the opposite figure :

E is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{BC}$

, $\overline{AD} \cap \overline{CE} = \{M\}$, MC = 5 cm. and MD = 2 cm.

Find : The length of each of $\overline{AD}$ and $\overline{ME}$

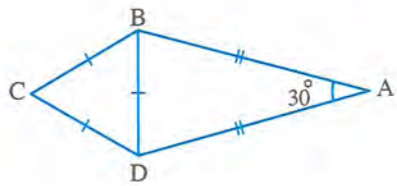


[b] In the opposite figure :

AB = AD , m ($\angle$ A) = 30°

, CB = BD = CD

Find : m ($\angle$ CBA)



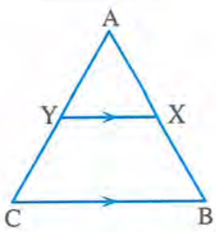
---

4 [a] In the opposite figure :

ABC is a triangle in which AB = AC , X $\in \overline{AB}$

, Y $\in \overline{AC}$ and $\overline{XY}$ // $\overline{BC}$

Prove that : The triangle AXY is an isosceles triangle.



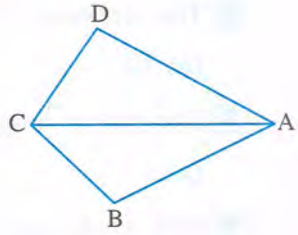
[b] In the opposite figure :

AD > DC

and AB > BC

Prove that :

m (∠ BCD) > m (∠ BAD)



---

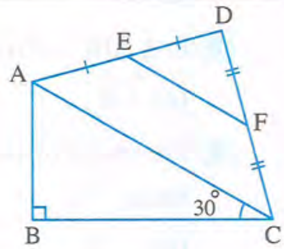
5 **[a] In the opposite figure :**

m (∠ B) = 90°

, m (∠ ACB) = 30°

, E , F are the midpoints of $\overline{AD}$, $\overline{DC}$

Prove that : AB = EF

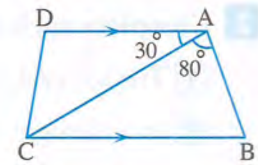


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 80°

and m (∠ DAC) = 30°

Prove that : BC > AB



4 Giza Governorate — El-Dokki Educational Administration

Answer the following questions :

1 **Complete :**

1. The number of axes of symmetry of an isosceles triangle equals ....
2. ABC is a triangle , AC = AB , m (∠ A) = 60° , then m (reflex ∠ B) = ...°
3. The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.
4. ABC is a triangle , m (∠ A) = 50° , m (∠ B) = 60° , then the smallest side is ....
5. The angle of measure 80° complements an angle of measure ....°

---

2 **Choose the correct answer :**

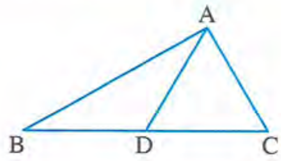
1. If the lengths of two sides of an isosceles triangle are 10 cm. and 20 cm. , then the perimeter of the triangle equals .... cm.

   (a) 30 (b) 40 (c) 50 (d) 60

2. ABC is a triangle , m (∠ A) = 90° , AB = 5 cm. , BC = 10 cm. , then m (∠ B) = ...°

   (a) 90 (b) 60 (c) 45 (d) 30

3 **In the opposite figure :**



AB + AD > ……………

(a) AC (b) DC

(c) DB (d) BC

4 The medians of the triangle intersect at …………… point.

(a) 1 (b) 2 (c) 3 (d) 4

5 The numbers that can be lengths of sides of a triangle are ……………

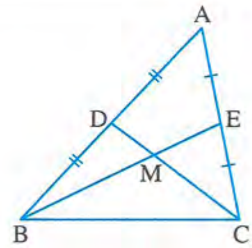
(a) 4 , 3 , 1 (b) 3 , 2 , 1 (c) 2 , 3 , 4 (d) 2 , 3 , 6

6 If $\overline{AB} \equiv \overline{BC}$, then AB – BC = ……………

(a) 0 (b) AC (c) AB (d) BC

---

3 [a] **In the opposite figure :**



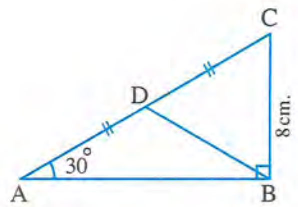
$\overline{BE}$, $\overline{DC}$ are two medians in Δ ABC

, $\overline{DC} \cap \overline{BE} = \{M\}$

, BE = 15 cm. , MD = 4 cm.

Find : The length of each of $\overline{BM}$, $\overline{DC}$

[b] **In the opposite figure :**



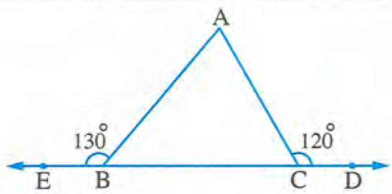
m ($\angle$ ABC) = 90° , m ($\angle$ A) = 30°

, BC = 8 cm. , $\overline{BD}$ is a median in Δ ABC

Find : 1 The length of $\overline{BD}$ 2 m ($\angle$ ABD)

---

4 [a] **In the opposite figure :**

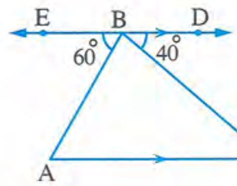


m ($\angle$ ACD) = 120° , m ($\angle$ ABE) = 130°

, B and C belong to $\overleftrightarrow{ED}$

Prove that : AB > AC

[b] **In the opposite figure :**



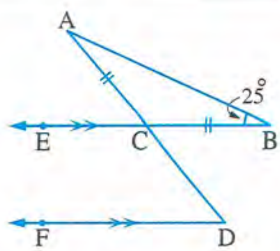
$\overleftrightarrow{ED}$ // $\overline{AC}$, m ($\angle$ ABE) = 60°

, m ($\angle$ CBD) = 40° , B $\in \overleftrightarrow{ED}$

Prove that : BC < AC

---

5 [a] **In the opposite figure :**



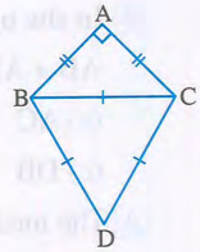
BC = AC

, $\overrightarrow{BE}$ // $\overrightarrow{DF}$

, m ($\angle$ B) = 25°

Find : m ($\angle$ D)

[b] In the opposite figure :



m (∠ A) = 90°

, AB = AC

, DC = BC = BD

Find : m (∠ ABD)

5 Giza Governorate

Awseem Educational Directorate
Mathematics Inspection

Answer the following questions :

1 Choose the correct answer :

1 In Δ ABC , if AC = 4 cm. , BC = 3 cm. , then m (∠ B) .... m (∠ A)

(a) > (b) < (c) = (d) ≤

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) twice (c) third (d) quarter

3 In Δ ABC , if m (∠ A) = 100° and AB = AC , then m (∠ ABC) = ...°

(a) 80 (b) 60 (c) 40 (d) 30

4 The point of intersection of the medians of the triangle divides each of them in the ratio .... from the base.

(a) 1 : 3 (b) 3 : 1 (c) 1 : 2 (d) 2 : 1

5 If Δ ABD is obtuse-angled at B and C is the midpoint of $\overline{BD}$, then the longest side is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{BD}$

6 The triangle whose side lengths are 2 cm. , (X + 3) cm. and 5 cm. , becomes an isosceles triangle when X = ...

(a) 1 (b) 2 (c) 3 (d) 4

2 Complete :

1 The total area of a cuboid = 120 cm^2 and its lateral area = 96 cm^2 , then the area of its base equals .... cm^2

2 The base angles of the isosceles triangle are ....

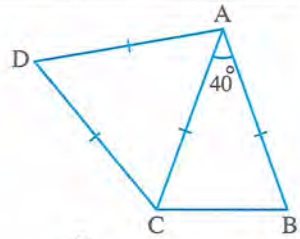
3 ABC is a right-angled triangle at B , m (∠ C) = 30° , AB = 5 cm.

, then AC = ... cm.

4 In Δ ABC , if m ($\angle$ C) = 30° , m ($\angle$ A) = 70° , then the smallest side in length is ....

5 In any triangle , if the lengths of two sides are not equal , then the greater side in length is opposite to ....

3 [a] **In the opposite figure :**

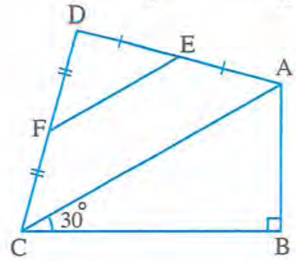


AB = AC = AD = CD

, m ($\angle$ BAC) = 40°

Find : m ($\angle$ BCD)

[b] **In the opposite figure :**



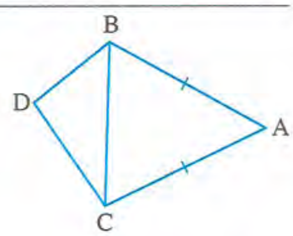
m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

, E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

Prove that : AB = EF

4 [a] **In the opposite figure :**

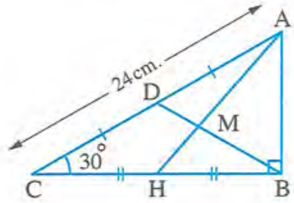


AB = AC , BD < CD

Prove that :

m ($\angle$ ABD) > m ($\angle$ ACD)

[b] **In the opposite figure :**



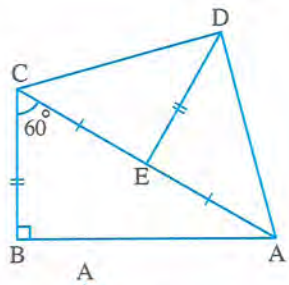
Δ ABC is right-angled at B

, $\overline{AH}$, $\overline{BD}$ are two medians in it , $\overline{AH} \cap \overline{BD} = \{M\}$

, m ($\angle$ C) = 30° , AC = 24 cm.

Find : The length of each of $\overline{AB}$, $\overline{BD}$, $\overline{BM}$

5 [a] **In the opposite figure :**



ABC is a right-angled triangle at B

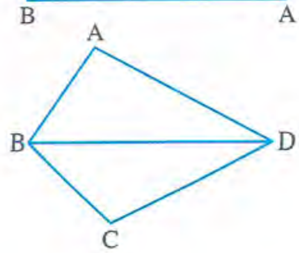
, m ($\angle$ ACB) = 60°

, E is the midpoint of $\overline{AC}$

, DE = BC

Prove that : m ($\angle$ ADC) = 90°

[b] **In the opposite figure :**



AB < AD and BC < CD

Prove that :

m ($\angle$ ABC) > m ($\angle$ ADC)

6 Alexandria Governorate

El-Montaza Educational Zone
Mathematics Directing – Model (C)

Answer the following questions :

1 Choose the correct answer :

1 The measure of the exterior angle of the equilateral triangle equals ....°

(a) 60 (b) 90 (c) 120 (d) 180

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) twice (c) third (d) quarter

3 Δ ABC is an isosceles triangle in which m ($\angle$ B) = 130° , then m ($\angle$ A) = ...°

(a) 65 (b) 50 (c) 25 (d) 130

4 The angle whose measure is 40° complements an angle of measure ....°

(a) 140 (b) 90 (c) 40 (d) 50

5 The lengths of two sides in a triangle are 4 cm. and 9 cm. and it has one axis of symmetry , then the length of the third side is .... cm.

(a) 13 (b) 9 (c) 5 (d) 4

6 The point of concurrence of the medians of the triangle divides each median in the ratio .... from the vertex.

(a) 1 : 2 (b) 1 : 3 (c) 3 : 1 (d) 2 : 1

2 Complete each of the following :

1 The sum of lengths of two sides in a triangle is .... the length of the third side.

2 In Δ ABC , if AB = AC and $\overline{AD} \perp \overline{BC}$, then DC .... BD

3 In Δ XYZ , if XY = 3 cm. and YZ = 5 cm. , then XZ $\in$] .... , ... [

4 If L_1 and L_2 are two straight lines in the same plane , $L_1 \cap L_2 = \varnothing$, then L_1 and L_2 are ....

5 If Δ ABC $\equiv$ Δ XYZ , then AC = ...

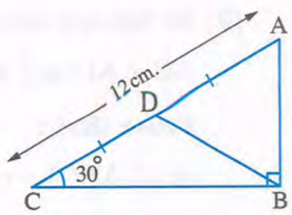
3 **[a]** Arrange ascendingly the lengths of the sides of Δ XYZ , if m ($\angle$ X) = 55° and m ($\angle$ Z) = 65°

[b] In the opposite figure :

ABC is a right-angled triangle at B , m ($\angle$ C) = 30°

, AC = 12 cm. and D is the midpoint of $\overline{AC}$

Find by proof : The perimeter of Δ ABD

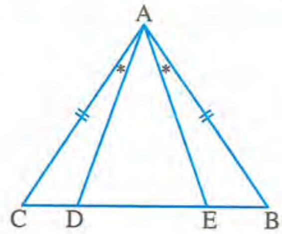


4 **[a] In the opposite figure :**

m (∠ CAD) = m (∠ BAE)

and AB = AC

Prove that : AD = AE

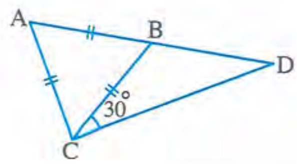


[b] In the opposite figure :

B ∈ $\overline{AD}$, AB = BC = AC

and m (∠ BCD) = 30°

Prove that : Δ CBD is an isosceles triangle.



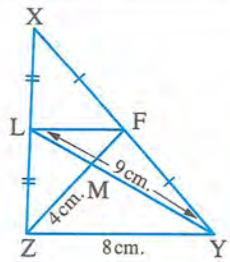
5 **[a] In the opposite figure :**

XYZ is a triangle in which : F is the midpoint of $\overline{XY}$

, L is the midpoint of $\overline{XZ}$, $\overline{YL} \cap \overline{ZF} = \{M\}$

, YZ = 8 cm. , YL = 9 cm. , MZ = 4 cm.

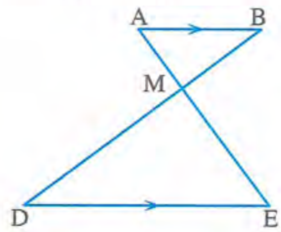
Find by proof : The perimeter of Δ MFL



[b] In the opposite figure :

$\overline{AB}$ // $\overline{DE}$ and MB > MA

Prove that : MD > ME



7 Alexandria Governorate

Agami Educational Zone
Mathematics Supervisor

Answer the following questions :

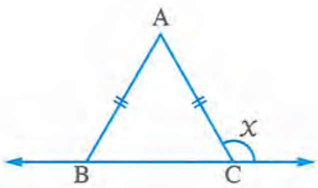
1 **Choose the correct answer :**

1 **In the opposite figure :**

ABC is an isosceles triangle , then x .... 90°

(a) > (b) <

(c) = (d) ≤



2 In Δ XYZ , if m (∠ Y) = 115° , then the longest side is ....

(a) $\overline{XY}$ (b) $\overline{ZX}$

(c) $\overline{YZ}$ (d) the median of the triangle.

3 In Δ ABC , AB = AC , m (∠ B) = 6 x° , m (∠ A) = 3 x° , then x = ...

(a) 12 (b) 30 (c) 60 (d) 90

4 If A is a point lying on the axis of symmetry of $\overline{BC}$, then AB .... AC

(a) $>$ (b) $<$ (c) $=$ (d) otherwise.

5 In Δ ABC , AB = 5 cm. , BC = 3 cm. , then AC $\in$ ....

(a) $[2 , 8]$ (b) $]2 , 8[$ (c) $[2 , 8[$ (d) $]2 , 8]$

6 In Δ ABC , AB + BC − AC 0

(a) $>$ (b) $<$ (c) $=$ (d) $\leq$

2 Complete the following :

1 If $L_1 // L_2$, then $L_1 \cap L_2 =$ ....

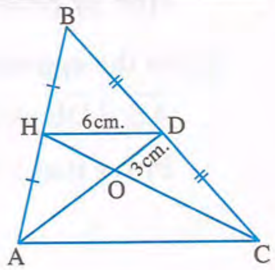
2 The bisector of the vertex angle of an isosceles triangle .... and ....

3 The least number of acute angles in any triangle is ....

4 The number of axes of symmetry of the isosceles triangle equals ....

5 The intersection point of the medians of the triangle divides each median by the ratio 2 : .... from the base.

3 [a] In the opposite figure :



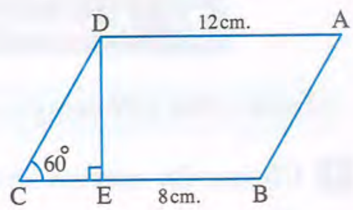
HD = 6 cm. , HC = 12 cm.

, H is the midpoint of $\overline{AB}$ and D is the midpoint of $\overline{BC}$

, $\overline{AD} \cap \overline{CH} = \{O\}$, DO = 3 cm.

Calculate : The perimeter of the triangle AOC

[b] In the opposite figure :

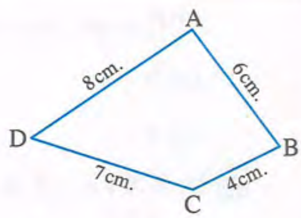


ABCD is a parallelogram , m ($\angle$ C) = 60°

, $\overline{DE} \perp \overline{BC}$, AD = 12 cm. , BE = 8 cm.

Find : The length of $\overline{AB}$

4 [a] In the opposite figure :

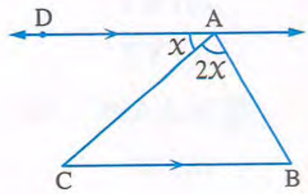


ABCD is a quadrilateral in which AB = 6 cm.

, BC = 4 cm. , CD = 7 cm. , DA = 8 cm.

Prove that : m ($\angle$ BCD) > m ($\angle$ BAD)

[b] In the opposite figure :

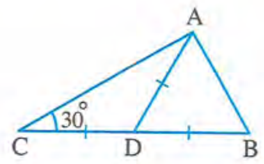


$\overleftrightarrow{AD} // \overline{BC}$

, m ($\angle$ BAD) = 120°

Prove that : AB < BC

5 **[a] In the opposite figure :**



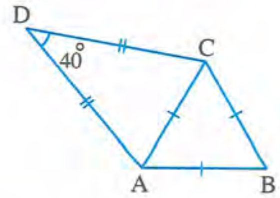
$D \in \overline{BC}$ such that $DA = DB = DC$

and $m(\angle C) = 30°$

Prove that : 1 ΔADB is an equilateral triangle.

2 ΔBAC is a right-angled triangle.

[b] In the opposite figure :



$m(\angle D) = 40°$, $DC = DA$

and ΔABC is an equilateral triangle.

Find : $m(\angle DCB)$

8 El-Kalyoubia Governorate — Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The lengths 9 cm. , 4 cm. and .... cm. may be the side lengths of a triangle.

(a) 4 (b) 5 (c) 8 (d) 13

2 XYZ is a triangle , $m(\angle X) = 60°$, $m(\angle Y) = 40°$, then XY .... YZ

(a) < (b) > (c) = (d) ≤

3 If $\overleftrightarrow{DE}$ is the axis of symmetry of $\overline{AB}$ and C is the midpoint of $\overline{AB}$, then all the following is correct except ....

(a) DA = DB (b) CE = CD (c) CB = CA (d) EA = EB

4 If $\overline{AD}$ is a median of ΔABC , M is the point of intersection of the medians , E is the midpoint of $\overline{AD}$ and AD = 9 cm. , then EM = ... cm.

(a) 3 (b) 1 (c) 1.5 (d) 2.5

5 The area of the rhombus whose diagonal lengths are 6 cm. and 8 cm. equals .... cm^2.

(a) 48 (b) 24 (c) 28 (d) 96

6 The triangle which has three axes of symmetry is ....

(a) right. (b) isosceles. (c) scalene. (d) equilateral.

---

2 **Complete each of the following :**

1 If $\angle X$ and $\angle Y$ are two complementary angles and $m(\angle X) = m(\angle Y)$, then $m(\angle X) = ... °$

2 If m (∠ A) = 70° , then the measure of reflex of ∠ A = ...°

3 The point of intersection of the medians of the triangle divides each median in the ratio .... : .... from the base.

4 The number of medians of the obtuse-angled triangle equals ....

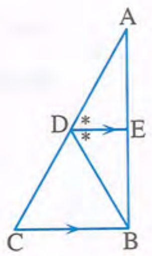
5 The bisector of the vertex angle of an isosceles triangle .... and ....

---

3 [a] In Δ ABC , if AB = AC , m (∠ A) = (x + 40) and m (∠ B) = (2 x – 10)

Find : The measure of each angle of the triangle.

[b] **In the opposite figure :**



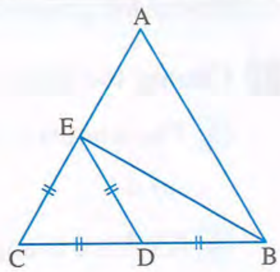
$\overrightarrow{DE}$ bisects ∠ ADB

and $\overrightarrow{DE}$ // $\overline{BC}$

Prove that : Δ DBC is an isosceles triangle.

---

4 [a] **In the opposite figure :**



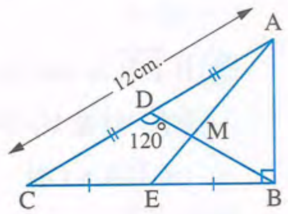
ABC is a triangle where D ∈ $\overline{BC}$, E ∈ $\overline{AC}$

, BD = CD = CE = DE

1 **Prove that :** m (∠ BEC) = 90°

2 If BC = BA , **prove that :** ABC is an equilateral triangle.

[b] **In the opposite figure :**



Δ ABC is right-angled at B , m (∠ BDC) = 120°

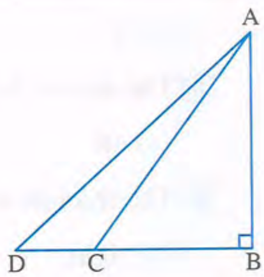
, $\overline{AE}$ and $\overline{BD}$ are two medians , $\overline{AE}$ ∩ $\overline{BD}$ = {M}

and AC = 12 cm.

Find : The length of each of $\overline{BD}$, $\overline{AB}$, $\overline{BM}$ and $\overline{MD}$

---

5 [a] **In the opposite figure :**



Δ ABC is right-angled at B

, C ∈ $\overline{BD}$

Prove that : AD > BC

[b] **Prove that :**

The length of any side in a triangle is less than half of its perimeter.

9 El-Sharkia Governorate

East Zagazig Educational Administration
Omar Al-Farouk Governmental Language School

Answer the following questions :

1 Choose the correct answer :

1. In Δ ABC , if m ($\angle$ B) > m ($\angle$ C) , then ....
   (a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{AC}$
2. The number of axes of symmetry of the equilateral triangle equals ....
   (a) 0 (b) 1 (c) 2 (d) 3
3. The triangle whose side lengths are 2 cm. , $(X + 1)$ cm. and 5 cm. becomes an isosceles triangle when X = ...
   (a) 1 (b) 2 (c) 3 (d) 4
4. In Δ ABC which is right-angled at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals ....
   (a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.
5. ABC is a triangle in which AB = AC and m ($\angle$ A) = 110° , then m ($\angle$ B) = ...°
   (a) 70 (b) 55 (c) 35 (d) 110
6. In Δ ABC , if AB > AC , m ($\angle$ C) = 70° , then m ($\angle$ B) may equal ....°
   (a) 70 (b) 50 (c) 80 (d) 75

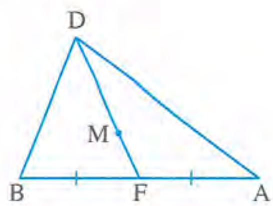
2 Complete the following :

1. In Δ ABC , if AB = 3 cm. and BC = 5 cm. , then AC $\in$] .... , ... [
2. If the measure of an angle in the isosceles triangle is 60° , then the triangle is ....
3. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.
4. The measure of the exterior angle of the equilateral triangle equals ....°
5. **In the opposite figure :**

   If $\overline{DF}$ is a median in Δ ABD

   , M is the intersection point of its medians

   , MF = 2 cm. , then DF = ... cm.



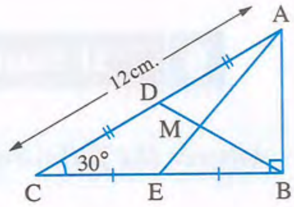
3 [a] **In the opposite figure :**

$m(\angle ABC) = 90°$, $m(\angle C) = 30°$

, D and E are the midpoints of $\overline{AC}$ and $\overline{BC}$

, $\overline{AE} \cap \overline{BD} = \{M\}$, AC = 12 cm.

Find : The length of each of $\overline{AB}$, $\overline{BM}$

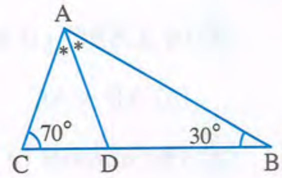


[b] **In the opposite figure :**

$\overrightarrow{AD}$ bisects $\angle BAC$, $m(\angle B) = 30°$

, $m(\angle C) = 70°$

Prove that : Δ ADC is an isosceles triangle.

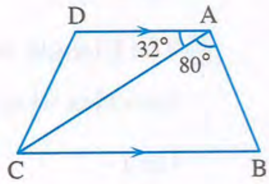


4 [a] **In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 80°$

, $m(\angle CAD) = 32°$

Prove that : AC > AB

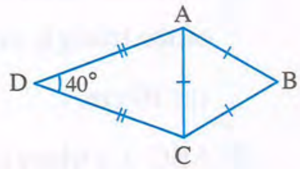


[b] **In the opposite figure :**

AB = BC = AC

, $m(\angle D) = 40°$ and AD = DC

Find : $m(\angle BCD)$



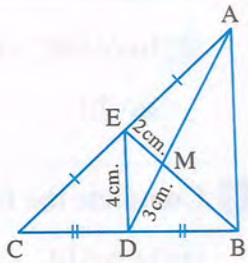
5 [a] **In the opposite figure :**

E and D are the midpoints of $\overline{AC}$ and $\overline{BC}$

, $\overline{AD} \cap \overline{BE} = \{M\}$, ME = 2 cm.

, MD = 3 cm. , DE = 4 cm.

Find : The perimeter of Δ MAB



[b] In Δ XYZ , if XY = 7 cm. , YZ = 8 cm. , ZX = 10 cm.

, then arrange the measures of the angles in a descending order.

10 El-Gharbia Governorate

East Tanta Educational Directorate
Al-Salam Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The number of axes of symmetry of the isosceles triangle equals ....

(a) 0 (b) 1 (c) 2 (d) 3

2 The triangle whose side lengths are 2 cm. , $(X + 3)$ cm. , 5 cm. becomes an isosceles triangle when $X =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

3 In Δ ABC , if m ($\angle$ B) = 130° , then the longest side of it is ....

(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

4 The triangle in which the measures of two angles are 42° , 69° is ....

(a) right. (b) isosceles. (c) scalene. (d) equilateral.

5 Δ XYZ is an isosceles triangle in which m ($\angle$ X) = 100° , then m ($\angle$ Y) = ...°

(a) 100 (b) 60 (c) 80 (d) 40

6 The numbers which can be lengths of sides of a triangle are ....

(a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

2 Complete each of the following :

1 In an isosceles triangle , if any angle has a measure of 60° , then the triangle is ....

2 The intersection point of the medians of a triangle divides each median in the ratio .... from its base.

3 In Δ ABC , if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then BC = ... AC

4 The measure of the exterior angle of an equilateral triangle equals ....°

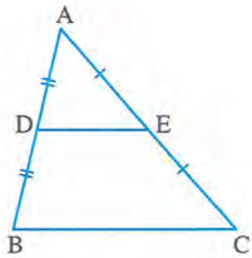
5 **In the opposite figure :**

In Δ ABC , AD = DB

, AE = EC

, then DE = ... BC

, $\overline{DE}$ .... $\overline{BC}$



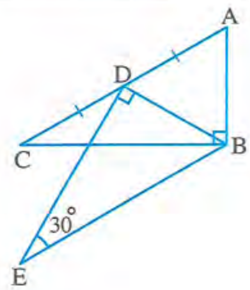
3 [a] In the opposite figure :

m ($\angle$ ABC) = m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30°

, AD = DC

Prove that : AC = BE

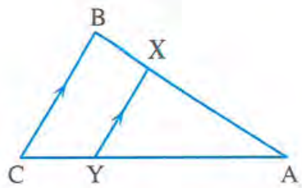


[b] In the opposite figure :

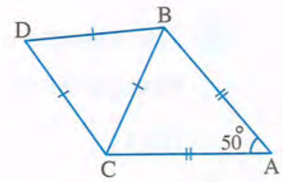
AB > BC

, $\overline{XY}$ // $\overline{BC}$

Prove that : AX > XY



4 [a] **In the opposite figure :**

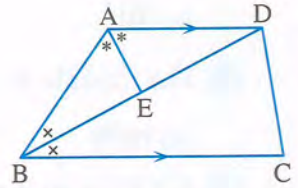


m (∠ A) = 50° , AB = AC

, Δ DBC is an equilateral triangle.

Find : m (∠ ABD)

[b] **In the opposite figure :**



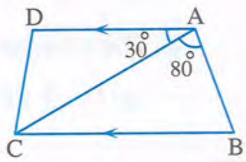
$\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ bisects ∠ ABC

, $\overrightarrow{AE}$ bisects ∠ BAD

Prove that : 1 AB = AD   2 $\overline{AE} \perp \overline{BD}$

---

5 [a] **In the opposite figure :**

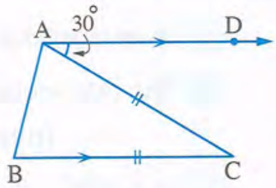


m (∠ BAC) = 80° , m (∠ CAD) = 30°

, $\overline{AD}$ // $\overline{BC}$

Prove that : BC > AB

[b] **In the opposite figure :**



AC = CB , $\overrightarrow{AD}$ // $\overline{BC}$

, m (∠ CAD) = 30°

Find : The measures of the angles of Δ ABC

11 El-Dakahlia Governorate — Maths Supervision

Answer the following questions :

1 Choose the correct answer from those given :

1 The length of the median from the vertex of the right angle in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) twice (c) third (d) quarter

2 In Δ ABC , if AB = AC , m (∠ B) = 50° , then m (∠ A) = ...°

(a) 40 (b) 50 (c) 80 (d) 100

3 The measure of the exterior angle of an equilateral triangle equals ....°

(a) 60 (b) 90 (c) 120 (d) 360

4 If the point A lies on the axis of symmetry of $\overline{CD}$, then $\overline{AC}$ .... $\overline{AD}$

(a) // (b) ⊥ (c) ≡ (d) =

5 The right angle supplements .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

6 In Δ ABC , AB + BC – AC .... zero.

(a) >   (b) =   (c) <   (d) otherwise.

2 **Complete each of the following :**

1 The longest side in the right-angled triangle is ....

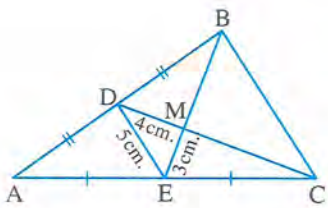
2 ABCD is a parallelogram , m ($\angle$ A) + m ($\angle$ C) = 100° , then m ($\angle$ B) = ...°

3 In Δ ABC , m ($\angle$ A) = 60° , m ($\angle$ B) = 70° , then the triangle has .... axes of symmetry.

4 The straight line passing through the vertex of an isosceles triangle perpendicular to the base ....

5 In a triangle , if the lengths of two sides are 4 cm. and 6 cm. , then the length of the third side $\in$] .... , ... [

3 **[a] In the opposite figure :**



M is the intersection point of the medians of Δ ABC

, D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$

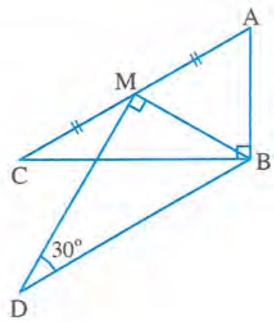
, ME = 3 cm. , MD = 4 cm. , DE = 5 cm.

Find : The perimeter of Δ MBC

[b] Arrange ascendingly the measures of the angles of Δ ABC if

AC = 8 cm. , BC = 10 cm. , AB = 6 cm.

4 **[a] In the opposite figure :**



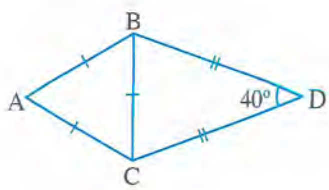
M is the midpoint of $\overline{AC}$

, m ($\angle$ D) = 30°

, m ($\angle$ ABC) = m ($\angle$ BMD) = 90°

Prove that : AC = BD

[b] In the opposite figure :



Δ ABC is equilateral , BD = DC

, m ($\angle$ D) = 40°

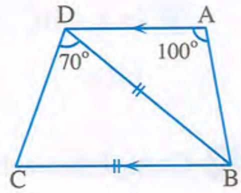
Find : m ($\angle$ ABD)

5 **[a] In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$, m ($\angle$ A) = 100°

, m ($\angle$ BDC) = 70° , BD = BC

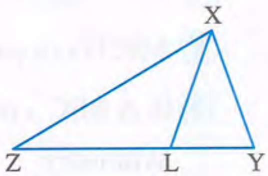
Prove that : Δ ABD is an isosceles triangle.



[b] Using the opposite figure :

Prove that :

The perimeter of Δ XYZ > 2 XL



12 Suez Governorate

Directorate of Education
Inspection of Mathematics

Answer the following questions :

1 **Choose the correct answer :**

1 In Δ ABC , if m ($\angle$ A) = 130° , then the longest side of it is ....

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) the hypotenuse.

2 In any isosceles triangle , the type of the base angle is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

3 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) third (c) quarter (d) twice

4 The triangle whose side lengths are 10 cm. , (X + 3) cm. , 5 cm. becomes an isosceles triangle when X = ...

(a) 5 (b) 2 (c) 7 (d) 10

5 In Δ ABC , if AB = 6 cm. , AC = 7 cm. , then BC $\in$ ....

(a)]6 , 13] (b) [6 , 7] (c)]1 , 13[(d) [1 , 7[

6 If A $\in$ the axis of symmetry of $\overline{BC}$, then AB .... AC

(a) = (b) // (c) ≡ (d) $\perp$

2 **Complete :**

1 The sum of lengths of any two sides in a triangle is .... the length of the third side.

2 The longest side in the right-angled triangle is ....

3 The base angles of an isosceles triangle are ....

4 In Δ ABC , if m ($\angle$ B) + m ($\angle$ C) = 2 m ($\angle$ A) , then m ($\angle$ A) = ...°

5 The bisector of the vertex angle of an isosceles triangle .... , ...

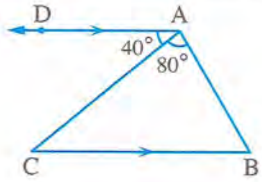
---

3 **[a] In the opposite figure :**

$\overrightarrow{AD} \parallel \overline{BC}$, m ($\angle$ BAC) = 80°

, m ($\angle$ DAC) = 40°

Prove that : BC > AB

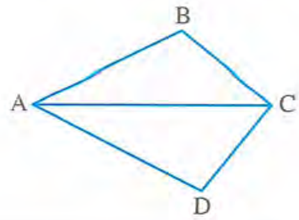


[b] In the opposite figure :

If AB > BC

, AD > DC

Prove that : m ($\angle$ BCD) > m ($\angle$ BAD)



---

4 **[a] In the opposite figure :**

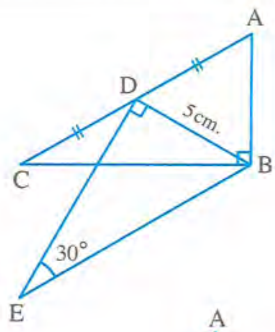
m ($\angle$ ABC) = m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30°

, BD = 5 cm.

, D is the midpoint of $\overline{AC}$

Find : The length of each of $\overline{AC}$ and $\overline{BE}$

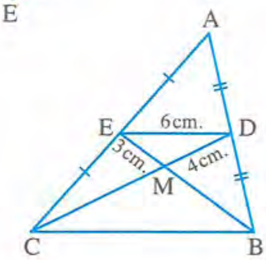


[b] In the opposite figure :

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$, $\overline{BE} \cap \overline{CD} = \{M\}$

, ED = 6 cm. , MD = 4 cm. , ME = 3 cm.

Find : The perimeter of Δ MBC



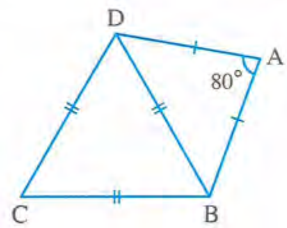
---

5 **[a] In the opposite figure :**

DB = DC = BC , AB = AD

, m ($\angle$ A) = 80°

Find : m ($\angle$ ABC)



[b] In the opposite figure :

m ($\angle$ B) = m ($\angle$ C)

, AB = $(2\mathcal{X} - 1)$ cm.

, AC = $(\mathcal{X} + 3)$ cm.

, BC = $(9 - \mathcal{X})$ cm.

Find : The perimeter of Δ ABC

13 Kafr El-Sheikh Governorate

Math Supervision
Official Language Schools

Answer the following questions :

1 Choose the correct answer :

1. If the lengths of two sides in a triangle are 3 cm. , 7 cm. , then the length of the third side may be .... cm.
   (a) 3 (b) 4 (c) 6 (d) 10
2. The triangle ABC is obtuse-angled at B , then the longest side is ....
   (a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) $\overline{AD}$
3. In the isosceles triangle , if one of its base angles is of measure 40° , then its vertex angle is of measure ....
   (a) 40° (b) 80° (c) 100° (d) 60°
4. The measure of the exterior angle of an equilateral triangle is ....
   (a) 60° (b) 70° (c) 80° (d) 120°
5. In the triangle ABC , if m (∠ B) = 75° , m (∠ C) = 50° , then BC .... AB
   (a) < (b) > (c) = (d) ≡
6. If XA = XB , YA = YB , then $\overleftrightarrow{XY}$ .... $\overline{AB}$
   (a) ⊥ (b) ≡ (c) // (d) =

2 Complete the following :

1. The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.
2. Any point at the axis of symmetry of a line segment is at two equal distances from ....
3. The length of the side opposite to the angle whose measure is 30° in the right-angled triangle equals ....
4. In the right-angled triangle , the length of the median from the vertex of the right angle equals .... the length of the hypotenuse.
5. In Δ ABC , if m (∠ A) = 70° , m (∠ B) = 30° , then the longest side is ....

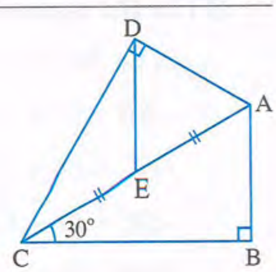
3 [a] In the opposite figure :

ABCD is a quadrilateral

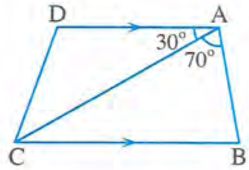
, m (∠ B) = m (∠ ADC) = 90° , m (∠ ACB) = 30°

, E is the midpoint of $\overline{AC}$

Prove that : AB = DE



[b] In the opposite figure :



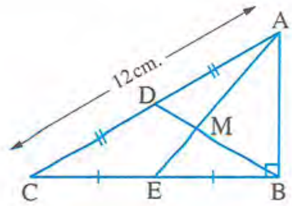
m (∠ BAC) = 70° , m (∠ DAC) = 30°

, $\overline{AD}$ // $\overline{BC}$

Prove that : AC > CB

---

4 **[a] In the opposite figure :**



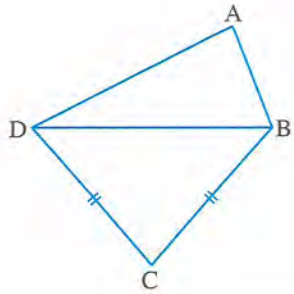
ABC is a right-angled triangle at B

, $\overline{AE}$ and $\overline{BD}$ are two medians of

the triangle intersecting at M , AC = 12 cm.

Calculate : The length of each of $\overline{BD}$ and $\overline{MD}$

[b] In the opposite figure :



ABCD is a quadrilateral in which

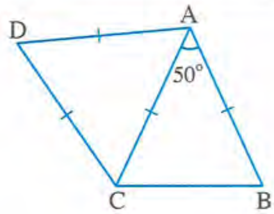
AD > AB and BC = CD

Prove that :

m (∠ ABC) > m (∠ ADC)

---

5 **[a] In the opposite figure :**



AB = AC = CD = DA

, m (∠ BAC) = 50°

Find : m (∠ BCD)

[b] ABC is a triangle in which : m (∠ A) = $(5X + 2)°$, m (∠ B) = $(6X - 10)°$

, m (∠ C) = $(X + 20)°$

Arrange the lengths of the sides of the triangle in an ascending order.

14 Beni Suef Governorate

Directorate of Official Language School
Education Administration

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The point of concurrence of the medians of the triangle divides each median in the ratio .... from the vertex.

(a) 3 : 2 (b) 2 : 1 (c) 1 : 2 (d) 3 : 1

2 The sum of lengths of any two sides in a triangle is .... the length of the third side.

(a) less than (b) greater than (c) equal to (d) half

3 In Δ ABC , if m ($\angle$ C) = 55° and m ($\angle$ A) = 85° , then ....

(a) AB > BC (b) AB < AC (c) BC > AB (d) AB = AC

4 The number of axes of symmetry in the scalene triangle equals ....

(a) zero. (b) 1 (c) 2 (d) 3

5 The triangle whose side lengths are 2 cm. , (X + 3) cm. and 5 cm. becomes an isosceles triangle when X = ...

(a) 1 (b) 2 (c) 3 (d) 4

6 If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then m ($\angle$ ABC) = ...°

(a) 45 (b) 90 (c) 180 (d) 360

---

2 Complete each of the following :

1 Δ ABC is right-angled at B , AB = 6 cm. and AC = 10 cm. , then the length of $\overline{BC}$ equals .... cm.

2 The number of medians of the triangle is ....

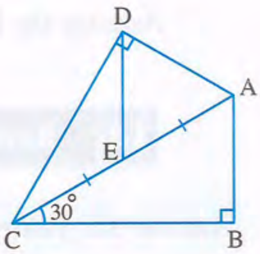
3 In Δ ABC , if m ($\angle$ B) = 55° and m ($\angle$ C) = 70° , then the type of the triangle ABC according to its side lengths is .... triangle.

4 Any point at the axis of symmetry of a line segment is at equal distances from ....

5 In Δ DEF , if DE > EF > DF , then the smallest angle in measure is ....

---

3 [a] In the opposite figure :



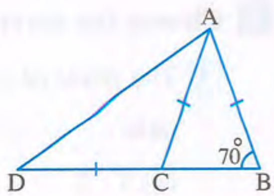
m ($\angle$ ABC) = m ($\angle$ ADC) = 90°

, m ($\angle$ ACB) = 30°

and E is the midpoint of $\overline{AC}$

Prove that : AB = DE

[b] In the opposite figure :

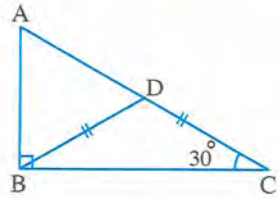


AB = AC = CD

and m ($\angle$ ABC) = 70°

Find : m ($\angle$ BAD)

4 **[a] In the opposite figure :**

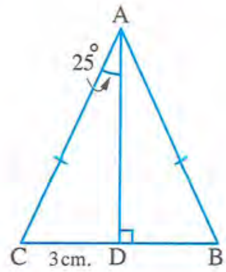


ABC is a right-angled triangle at B

, m (∠ C) = 30° , D ∈ $\overline{AC}$ where DB = DC

Prove that : Δ ABD is an equilateral triangle.

[b] In the opposite figure :



ABC is a triangle in which AB = AC

, CD = 3 cm.

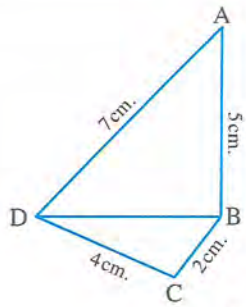
, m (∠ CAD) = 25° and $\overline{AD} \perp \overline{BC}$

Find : 1 The length of $\overline{BC}$

2 m (∠ BAC)

---

5 **[a] In the opposite figure :**



ABCD is a quadrilateral in which AB = 5 cm.

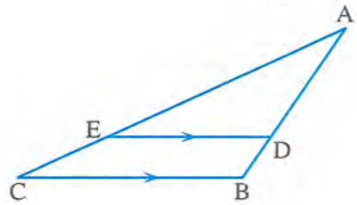
, AD = 7 cm.

, BC = 2 cm. and DC = 4 cm.

Prove that :

m (∠ ABC) > m (∠ ADC)

[b] In the opposite figure :



ABC is an obtuse-angled triangle at B and $\overline{DE}$ // $\overline{BC}$

Prove that : AE > AD

15 Aswan Governorate

Edfu Educational Zone
Edfu Official Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The angle whose measure is more than 90° and less than 180° is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 The point of the intersection of the medians of the triangle divides each of them by the ratio .... from the base.

(a) 2 : 1 (b) 1 : 2 (c) 3 : 4 (d) 1 : 1

3 In ΔXYZ , $XY = XZ$, $m(\angle Y) = 40°$, then $m(\angle X) =$ ....°

(a) 40 (b) 55 (c) 70 (d) 100

4 The measure of the exterior angle of an equilateral triangle is ....°

(a) 120 (b) 60 (c) 90 (d) 30

5 In ΔABC , if $AB > AC$, then $m(\angle B)$ .... $m(\angle C)$

(a) $>$ (b) $<$ (c) $\geq$ (d) $=$

6 In ΔABC , if $2\,m(\angle A) = m(\angle B) + m(\angle C)$, then $m(\angle A) =$ ....°

(a) 45 (b) 90 (c) 60 (d) 120

---

2 Complete the following :

1 In ΔDEF , if $m(\angle E) = 120°$, then the longest side is ....

2 The isosceles triangle has .... axes of symmetry.

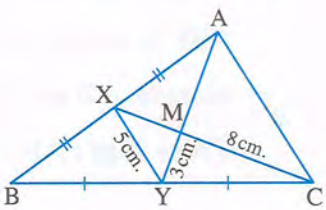
3 If $\overline{AB} \equiv \overline{CD}$ and $AB = 6$ cm. , then $AB + CD =$ .... cm.

4 The length of the side opposite to the angle whose measure is 30° in the right-angled triangle is ....

5 In ΔABC , if $AB = 4$ cm. , $BC = 5$ cm. , then $AC \in]$ .... , ... $[$

---

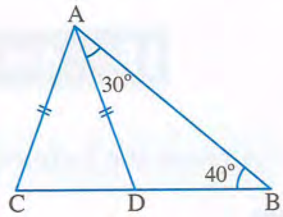
3 [a] In the opposite figure :



ABC is a triangle , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$, $\overline{XC} \cap \overline{AY} = \{M\}$, $XY = 5$ cm. , $CM = 8$ cm. , $YM = 3$ cm.

Find : The perimeter of the triangle MAC

[b] In the opposite figure :

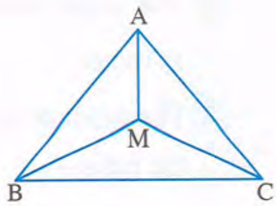


$AD = AC$, $m(\angle DAB) = 30°$, $m(\angle ABD) = 40°$

Prove that : $AB = CB$

---

4 [a] In the opposite figure :



ABC is a triangle in which M is a point inside it.

Prove that :

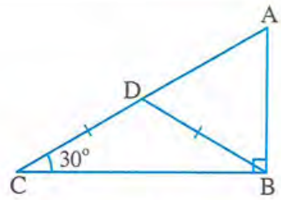
$MA + MB + MC > \frac{1}{2}$ the perimeter of the triangle ABC

[b] XYZ is a triangle in which : m (∠ X) = 40° , m (∠ Y) = 80°

Order the lengths of sides of Δ XYZ in an ascending order.

---

5 **[a] In the opposite figure :**

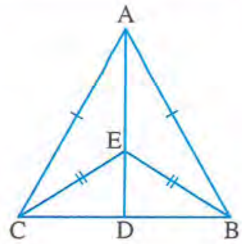


ABC is a right-angled triangle at B

, m (∠ C) = 30° , D $\in \overline{AC}$ where DB = DC

Prove that : Δ ABD is an equilateral triangle.

[b] In the opposite figure :



AB = AC and EB = EC , $\overrightarrow{AE} \cap \overline{BC} = \{D\}$

Prove that :

1. $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$
2. BD = DC

EL-MOASSER

By a group of supervisors

GUIDE ANSWERS

2nd PREP.
2023
FIRST TERM

Maths

Guide Answers

Of

Algebra and Statistics Exercises

Answers of unit one

Answers of Exercise 1

1

Number a	8	125	−27	−1000	$3\frac{3}{8}$	$-\frac{8}{125}$	216	−64
$\sqrt[3]{a}$	2	5	−3	−10	$\frac{3}{2}$	$-\frac{2}{5}$	6	−4

2

1. 6
2. −7
3. 0.1
4. $\frac{-2}{3}$
5. zero
6. −1
7. 6
8. 1
9. zero
10. $-\frac{1}{2}$
11. a
12. $-3a^2$
13. 64
14. 64
15. 25
16. 61

3

1. c
2. b
3. a
4. a
5. d
6. c
7. c
8. d
9. a
10. d
11. c
12. c
13. d
14. b

4

1. 125
2. $-\frac{1}{64}$
3. $\because \sqrt[3]{x} = -2$ $\therefore x = -8$
4. $\because \sqrt[3]{x} = -1 + 3 = 2$ $\therefore x = 8$
5. −2
6. 4
7. $\because x^3 = 32 - 5 = 27$ $\therefore x = 3$
8. $\because x^3 = 54 \div 2 = 27$ $\therefore x = 3$
9. $\because x^3 = -200 \div \frac{1}{5} = -1000$ $\therefore x = -10$

5

1. $\because x^3 = -27$ $\therefore x = \sqrt[3]{-27} = -3$
   $\therefore$ The S.S. $= \{-3\}$
2. $\because 8x^3 = 8 - 7 = 1$ $\therefore x^3 = \frac{1}{8}$
   $\therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ $\therefore$ The S.S. $= \{\frac{1}{2}\}$
3. $\because x^3 = \frac{3}{8} - 16 = -\frac{125}{8}$ $\therefore x = \sqrt[3]{-\frac{125}{8}} = -\frac{5}{2}$
   $\therefore$ The S.S. $= \{-\frac{5}{2}\}$
4. $\because 2x^3 - x^3 = 3 + 5$ $\therefore x^3 = 8$
   $\therefore x = \sqrt[3]{8} = 2$ $\therefore$ The S.S. $= \{2\}$
5. $\because x + 3 = \sqrt[3]{343} = 7$ $\therefore x = 7 - 3 = 4$
   $\therefore$ The S.S. $= \{4\}$
6. $\because 3x + 1 = \sqrt[3]{-8} = -2$ $\therefore 3x = -2 - 1 = -3$
   $\therefore x = -3 \div 3 = -1$ $\therefore$ The S.S. $= \{-1\}$
7. $\because (2x+1)^3 = 20 + 7 = 27$ $\therefore 2x + 1 = \sqrt[3]{27} = 3$
   $\therefore 2x = 3 - 1 = 2$ $\therefore x = 2 \div 2 = 1$
   $\therefore$ The S.S. $= \{1\}$
8. $\because (5x-2)^3 = 18 - 10 = 8$ $\therefore 5x - 2 = \sqrt[3]{8} = 2$
   $\therefore 5x = 2 + 2 = 4$ $\therefore x = \frac{4}{5}$
   $\therefore$ The S.S. $= \{\frac{4}{5}\}$

6

1. $\sqrt[3]{2\frac{1}{4} \div \frac{2}{3}} = \sqrt[3]{\frac{9}{4} \times \frac{3}{2}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$
2. $-\sqrt[3]{2^9 \times 3^6} = -\sqrt[3]{(2^3 \times 3^2)^3} = -2^3 \times 3^2 = -8 \times 9 = -72$
3. $\sqrt{\sqrt[3]{729}} = \sqrt{9} = 3$
4. $\sqrt[3]{\sqrt[3]{512}} = \sqrt[3]{8} = 2$
5. $\sqrt{27\sqrt[3]{27}} = \sqrt{27 \times 3} = \sqrt{81} = 9$

7

The edge length of the cube $= \sqrt[3]{27} = 3$ cm.
The area of one face $= 3^2 = 9$ cm^2.

8

The edge length of the cube $= \sqrt[3]{216} = 6$ cm.
$\therefore$ Its total area $= 6 \times 6^2 = 216$ cm^2.

9

Let the number be x $\therefore \frac{1}{2}x^3 = 32$
$\therefore x^3 = 64$ $\therefore x = 4$
$\therefore$ The number $= 4$

10

The length of the inner edge $= \sqrt[3]{1000} = 10$ cm.

11

The volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{1372}{81}\pi$
$\therefore r^3 = \frac{1372}{81} \times \frac{3}{4} = \frac{343}{27}$
$\therefore r = \sqrt[3]{\frac{343}{27}} = \frac{7}{3}$
$\therefore$ The diameter length of the sphere
$= 2 \times \frac{7}{3} = \frac{14}{3}$ length unit.

12

$\therefore$ The volume of the sphere $= \frac{4}{3}\pi r^3 = 113.04$

$\therefore \frac{4}{3} \times 3.14 \times r^3 = 113.04$

$\therefore r^3 = 27 \qquad \therefore r = \sqrt[3]{27} = 3$ cm.

$\therefore$ The diameter length of the sphere $= 2 \times 3 = 6$ cm.

13

1 $\because (x^2+6)^3 = 1000 \qquad \therefore x^2 + 6 = 10$

$\therefore x^2 = 4 \qquad \therefore x = \pm 2$

$\therefore$ The S.S. $= \{2, -2\}$

2 $\because (x^3 - 14)^2 = 169 \qquad \therefore x^3 - 14 = \pm 13$

$\therefore x^3 = 14 \pm 13 \qquad \therefore x^3 = 27 \quad \therefore x = 3$

or $x^3 = 1 \qquad \therefore x = 1$

$\therefore$ The S.S. $= \{3, 1\}$

3 Cubing the two sides $\qquad \therefore (x-1)^2 = 25$

$\therefore x - 1 = \pm 5 \qquad \therefore x = 6$ or $x = -4$

$\therefore$ The S.S. $= \{6, -4\}$

4 $\because \sqrt[3]{(x-2)(x-2)^2} = 3 \qquad \therefore \sqrt[3]{(x-2)^3} = 3$

$\therefore x - 2 = 3 \qquad \therefore x = 5$

$\therefore$ The S.S. $= \{5\}$

14

Cubing the two sides

$\therefore \sqrt{x} + 19 = 27 \qquad \therefore \sqrt{x} = 8$

Squaring the two sides

$\therefore x = 64 \qquad \therefore \sqrt[3]{x} = \sqrt[3]{64} = 4$

15

Let the age of the grandfather be x year.

$\therefore$ The age of the man $= \frac{1}{2}x$ year.

The age of the grandson (the elder) $= \sqrt{x}$ year.

The age of the grandson (the middle) $= \sqrt[3]{x}$ year.

The age of the granddaughter $= \frac{\sqrt{x}}{\sqrt[3]{x}}$ year.

$\because \sqrt{x} = 2\sqrt[3]{x}$, then cubing the two sides.

$\therefore x\sqrt{x} = 8x$, then squaring the two sides.

$\therefore x^3 = 64x^2 \qquad \therefore x = 64$

$\therefore$ The age of the grandfather = 64 years.

The age of the grandson (the elder) $= \sqrt{64} = 8$ years.

The age of the grandson (the middle) $= \sqrt[3]{64} = 4$ years.

The age of the granddaughter $= 8 \div 4 = 2$ years.

Answers of Exercise 2

1

The rational numbers are No.

1, 2, 3, 4, 5, 8, 9,

11, 13, 14, 16, 17, 18, 19

The remained numbers are irrational.

2 1 3.32 2 1.9 3 -2.1

3

1 $\because \sqrt{4} < \sqrt{5} < \sqrt{9} \qquad \therefore 2 < \sqrt{5} < 3$

$\therefore$ The two numbers are 2 and 3

2 $\because \sqrt{9} < \sqrt{12} < \sqrt{16} \qquad \therefore 3 < \sqrt{12} < 4$

$\therefore$ The two numbers are 3 and 4

3 $\because \sqrt[3]{8} < \sqrt[3]{10} < \sqrt[3]{27} \qquad \therefore 2 < \sqrt[3]{10} < 3$

$\therefore$ The two numbers are 2 and 3

4 $\because \sqrt[3]{-27} < \sqrt[3]{-20} < \sqrt[3]{-8} \quad \therefore -3 < \sqrt[3]{-20} < -2$

$\therefore$ The two numbers are -2 and -3

4

1 $\because \sqrt{1} < \sqrt{2} < \sqrt{4} \qquad \therefore 1 < \sqrt{2} < 2 \quad \therefore x = 1$

2 $\because \sqrt{64} < \sqrt{80} < \sqrt{81} \qquad \therefore 8 < \sqrt{80} < 9 \quad \therefore x = 8$

3 $\because \sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8} \qquad \therefore 1 < \sqrt[3]{5} < 2 \quad \therefore x = 1$

4 $\because \sqrt[3]{27} < \sqrt[3]{50} < \sqrt[3]{64} \qquad \therefore 3 < \sqrt[3]{50} < 4 \quad \therefore x = 3$

5 $\because \sqrt[3]{-125} < \sqrt[3]{-100} < \sqrt[3]{-64}$

$\therefore -5 < \sqrt[3]{-100} < -4 \qquad \therefore x = -5$

6 $\because \sqrt{25} < \sqrt{35} < \sqrt{36} \qquad \therefore 5 < \sqrt{35} < 6 \quad \therefore x = 5$

5

1 $\because \sqrt{16} < \sqrt{20} < \sqrt{25} \qquad \therefore 4 < \sqrt{20} < 5$

$\because (4.1)^2 = 16.81, (4.2)^2 = 17.64, (4.3)^2 = 18.49,$

$(4.4)^2 = 19.36, (4.5)^2 = 20.25$

$\therefore 4.4 < \sqrt{20} < 4.5 \qquad \therefore \sqrt{20} \approx 4.4$ or 4.5

Using the calculator $\sqrt{20} \approx 4.47$

2 $\because \sqrt[3]{8} < \sqrt[3]{17} < \sqrt[3]{27}$ $\therefore 2 < \sqrt[3]{17} < 3$

$\because (2.1)^3 = 9.261$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

$(2.4)^3 = 13.824$, $(2.5)^3 = 15.625$, $(2.6)^3 = 17.576$

$\therefore 2.5 < \sqrt[3]{17} < 2.6$

$\sqrt[3]{17} \approx 2.5$ or 2.6

Using the calculator $\sqrt[3]{17} \approx 2.57$

3 $\because \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\therefore 2 < \sqrt{5} < 3$

$\because (2.1)^2 = 4.41$, $(2.2)^2 = 4.84$, $(2.3)^2 = 5.29$

$\therefore 2.2 < \sqrt{5} < 2.3$ $\therefore 3.2 < \sqrt{5} + 1 < 3.3$

$\therefore \sqrt{5} + 1 \approx 3.2$ or 3.3

Using the calculator $\sqrt{5} + 1 \approx 3.24$

4 $\because \sqrt[3]{8} < \sqrt[3]{9} < \sqrt[3]{27}$ $\therefore 2 < \sqrt[3]{9} < 3$

$\because (2.1)^3 = 9.261$ $\therefore 2 < \sqrt[3]{9} < 2.1$

$\therefore 1 < \sqrt[3]{9} - 1 < 1.1$ $\therefore \sqrt[3]{9} - 1 \approx 1$ or 1.1

Using the calculator $\sqrt[3]{9} - 1 \approx 1.08$

6

1 d	2 b	3 b	4 c
5 c	6 b	7 b	8 d
9 c	10 c	11 d	

7

1 $x^2 = \frac{10}{5} = 2$ $\therefore x = \pm\sqrt{2}$ $\therefore x \in \grave{\mathbb{Q}}$

2 $x^2 = \frac{9}{4}$ $\therefore x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$ $\therefore x \in \mathbb{Q}$

3 $x = \sqrt[3]{125}$ $\therefore x = 5$ $\therefore x \in \mathbb{Q}$

4 $x^3 = \frac{27}{3} = 9$ $\therefore x = \sqrt[3]{9}$ $\therefore x \in \grave{\mathbb{Q}}$

5 $x^2 = \frac{10}{0.1} = 100$ $\therefore x = \pm\sqrt{100} = \pm 10$ $\therefore x \in \mathbb{Q}$

6 $x^3 = \frac{-8}{0.001} = -8000$

$\therefore x = \sqrt[3]{-8000} = -20$ $\therefore x \in \mathbb{Q}$

7 $x - 1 = \pm\sqrt{4} = \pm 2$ $\therefore x = 2 + 1 = 3$

or $x = -2 + 1 = -1$ $\therefore x \in \mathbb{Q}$

8 $x - 5 = \sqrt[3]{1} = 1$ $\therefore x = 1 + 5 = 6$ $\therefore x \in \mathbb{Q}$

8

1 $x^2 = 13$ $\therefore x = \pm\sqrt{13}$

$\therefore$ The S.S. $= \{\sqrt{13} , -\sqrt{13}\}$

2 $x^3 = 16$ $\therefore x = \sqrt[3]{16}$ $\therefore$ The S.S. $= \{\sqrt[3]{16}\}$

3 $x^2 = \frac{25}{2} \times \frac{5}{2} = \frac{125}{4}$ $\therefore x = \pm\sqrt{\frac{125}{4}}$

$\therefore$ The S.S. $= \{\sqrt{\frac{125}{4}} , -\sqrt{\frac{125}{4}}\}$

4 $x^3 = -2 \times \frac{4}{5} = -\frac{8}{5}$ $\therefore x = \sqrt[3]{-\frac{8}{5}}$

$\therefore$ The S.S. $= \{\sqrt[3]{-\frac{8}{5}}\}$

5 $125\, x^3 = 27$ $\therefore x^3 = \frac{27}{125}$

$\therefore x = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

$\therefore$ The S.S. $= \varnothing$ because $\frac{3}{5} \notin \grave{\mathbb{Q}}$

6 $\frac{1}{4}\, x^2 = 64$ $\therefore x^2 = 64 \times 4 = 256$

$\therefore x = \pm\sqrt{256} = \pm 16$

$\therefore$ The S.S. $= \varnothing$ because $16 \notin \grave{\mathbb{Q}}$, $-16 \notin \grave{\mathbb{Q}}$

7 $\because (x^3 + 5)(x^2 - 3) = 0$

$\therefore x^3 + 5 = 0$ $\therefore x^3 = -5$ $\therefore x = -\sqrt[3]{5}$

or $x^2 - 3 = 0$ $\therefore x^2 = 3$ $\therefore x = \pm\sqrt{3}$

$\therefore$ The S.S. $= \{-\sqrt[3]{5} , \sqrt{3} , -\sqrt{3}\}$

8 $\because (x + \sqrt{7})(x^3 - 6) = 0$

$\therefore x + \sqrt{7} = 0$ $\therefore x = -\sqrt{7}$

or $x^3 - 6 = 0$ $\therefore x^3 = 6$

$\therefore x = \sqrt[3]{6}$ $\therefore$ The S.S. $= \{-\sqrt{7} , \sqrt[3]{6}\}$

9

1 $\because (1.4)^2 = 1.96$, $(1.5)^2 = 2.25$, $(\sqrt{2})^2 = 2$

$\therefore \sqrt{2}$ is included between 1.4 , 1.5

2 $\because (3.31)^2 \approx 10.96$, $(3.32)^2 \approx 11.02$, $(\sqrt{11})^2 = 11$

$\therefore \sqrt{11}$ is included between 3.31 , 3.32

3 $\because (1.2)^3 = 1.728$, $(1.3)^3 = 2.197$, $(\sqrt[3]{2})^3 = 2$

$\therefore \sqrt[3]{2}$ is included between 1.2 , 1.3

4 $\because (2.4)^3 = 13.824$, $(2.5)^3 = 15.625$

$(\sqrt[3]{15})^3 = 15$

$\therefore \sqrt[3]{15}$ is included between 2.4 , 2.5

5 $\because (-2.6)^3 = -17.576$, $(-2.5)^3 = -15.625$

$(\sqrt[3]{-17})^3 = -17$

$\therefore \sqrt[3]{-17}$ is included between -2.6 , -2.5

6 $\because 2.7 - 1 = 1.7 , (1.7)^2 = 2.89$

$2.8 - 1 = 1.8 , (1.8)^2 = 3.24$

$\sqrt{3} + 1 - 1 = \sqrt{3} , (\sqrt{3})^2 = 3$

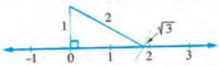
$\therefore \sqrt{3}$ is included between 1.7 , 1.8

$\therefore \sqrt{3} + 1$ is included between 2.7 , 2.8

10

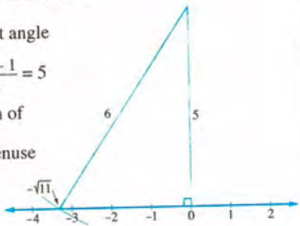
1 The length of one side of the right angle $= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



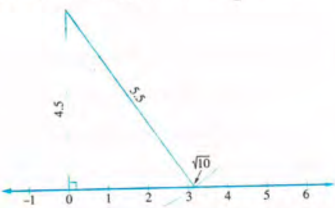
2 The length of one side of the right angle equals $\frac{11-1}{2} = 5$

The length of the hypotenuse $= \frac{11+1}{2} = 6$



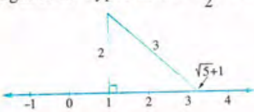
3 The length of one side of the right angle $= \frac{10-1}{2} = 4.5$

The length of the hypotenuse $= \frac{10+1}{2} = 5.5$



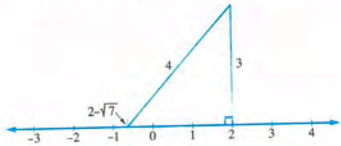
4 The length of one side of the right angle $= \frac{5-1}{2} = 2$

The length of the hypotenuse $= \frac{5+1}{2} = 3$



5 The length of one side of the right angle $= \frac{7-1}{2} = 3$

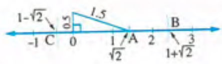
The length of the hypotenuse $= \frac{7+1}{2} = 4$



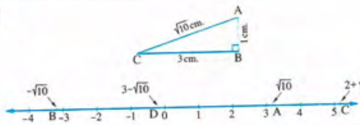
11

The length of one side of the right angle $= \frac{2-1}{2} = 0.5$

The length of the hypotenuse $= \frac{2+1}{2} = 1.5$



12



13

The length of the side of the square $= \sqrt{10}$ cm.

The square of the length of the diagonal

$= (\sqrt{10})^2 + (\sqrt{10})^2 = 10 + 10 = 20$

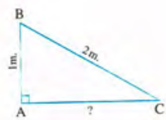
$\therefore$ The length of the diagonal $= \sqrt{20}$ cm.

14

$\because$ The length of the tree = 3 m.

$\therefore$ AB + BC = 3 m.

, $\because$ the length of the left part of the tree = 1 m.



$\therefore$ BC = 3 – 1 = 2 m.

$\because$ In Δ ABC : m ($\angle$ A) = 90°

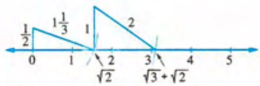
$\therefore (AC)^2 = (BC)^2 - (AB)^2 = 4 - 1 = 3$

$\therefore AC = \sqrt{3}$ m.

$\therefore$ The distance between the base of the tree and the point of touching of its top with the ground $= \sqrt{3}$ m.

15

We represent on the number line the point representing the number $\sqrt{3}+\sqrt{2}$ as shown in the figure :



We find that the point representing the number $\sqrt{3}+\sqrt{2}$ lies between the point representing the number 3 and the point representing the number 4

i.e. $\sqrt{3}+\sqrt{2}$ lies between 3 and 4

Answers of Exercise 3

1

The number	Natural	Integer	Rational	Irrational	Real
-5	✗	✓	✓	✗	✓
$\sqrt{2}$	✗	✗	✗	✓	✓
$1\frac{1}{2}$	✗	✗	✓	✗	✓
$\sqrt[3]{9}$	✗	✗	✗	✓	✓
$\lvert-2\rvert$	✓	✓	✓	✗	✓
$-\sqrt{4}$	✗	✓	✓	✗	✓
$\frac{5}{2}$	✗	✗	✓	✗	✓
0.3	✗	✗	✓	✗	✓
$\sqrt{-1}$	✗	✗	✗	✗	✗

2

1 positive 2 negative 3 positive

4 positive 5 negative 6 positive

3

1 $>$ 2 $>$ 3 $<$

4 $<$ 5 $>$ 6 $=$

7 $>$ 8 $>$ 9 $>$

4

1 a 2 d 3 b 4 a

5 d 6 c 7 a 8 b

9 b 10 a 11 d 12 a

13 c 14 d

5

1 The ascending order is :

$-\sqrt{11}$, $-\sqrt{7}$, $-\sqrt{3}$, $\sqrt{5}$, $\sqrt{8}$ and $\sqrt{15}$

2 $\because 0.6=\sqrt{0.36}$, $\sqrt[3]{-1}=-1=-\sqrt{1}$

$\therefore$ The ascending order is :

$-\sqrt{45}$, $-\sqrt{1}$, $\sqrt{0.36}$, $\sqrt{20}$ and $\sqrt{27}$

i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0.6 , $\sqrt{20}$ and $\sqrt{27}$

6

1 $\because 8=\sqrt{64}$

$\therefore$ The descending order is :

$\sqrt{70}$, $\sqrt{64}$, $\sqrt{62}$ and $-\sqrt{50}$

i.e. $\sqrt{70}$, 8 , $\sqrt{62}$ and $-\sqrt{50}$

2 $\because 9=\sqrt{81}$

$\therefore$ The descending order is :

$\sqrt{101}$, $\sqrt{81}$, $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$

i.e. $\sqrt{101}$, 9 , $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$

7

$\because 2^2=4$ $\therefore 4>3>2>\frac{3}{2}>0$

$\therefore 2>\sqrt{3}>\sqrt{2}>\sqrt{\frac{3}{2}}>0$

$\therefore$ The positive irrational numbers are

$\sqrt{3}$, $\sqrt{2}$ and $\sqrt{\frac{3}{2}}$

(There are other solutions)

8

The irrational numbers are

$-\sqrt{5}$, $-\sqrt{3}$ and $-\sqrt{2}$ (There are other solutions)

9

$\because (15)^2=225$, $(17)^2=289$

Then choosing 4 integers included between 225 , 289

(except 256 because $\sqrt{256}=16\in\mathbb{Q}$)

$\therefore 225<235<245<255<265<289$

$\therefore 15<\sqrt{235}<\sqrt{245}<\sqrt{255}<\sqrt{265}<17$

$\therefore$ The four irrational numbers are

$\sqrt{235}$, $\sqrt{245}$, $\sqrt{255}$ and $\sqrt{265}$

(There are other solutions)

10

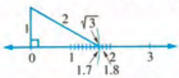
Using the calculator $\sqrt{3} \approx 1.73$

(to the nearest hundredth)

$\therefore 1.7 < \sqrt{3} < 1.8$ for representing $\sqrt{3}$

$\therefore$ The length of the hypotenuse $= \frac{3+1}{2} = 2$

, the length of one side of the right angle $= \frac{3-1}{2} = 1$



11

1. $x^2 = 6$ $\qquad \therefore x = \pm\sqrt{6} \approx \pm 2.45$
2. $x^2 = 24 \times \frac{4}{3} = 32$ $\qquad \therefore x = \pm\sqrt{32} \approx \pm 5.66$
3. $\frac{1}{2} x^2 = 5$ $\qquad \therefore x^2 = 5 \times 2 = 10$
   $\therefore x = \pm\sqrt{10}$ $\qquad \therefore x \approx \pm 3.16$
4. $5x^3 = -1$ $\qquad \therefore x^3 = -\frac{1}{5}$
   $\therefore x = \sqrt[3]{-\frac{1}{5}} \approx -0.58$
5. $\frac{3}{4} x^2 = -13$ $\qquad \therefore x^2 = -13 \times \frac{4}{3} = -\frac{52}{3}$
   (has no solution in $\mathbb{R}$)
6. $\frac{2}{x^3} = 16$ $\qquad \therefore x^3 = \frac{1}{8}$ $\qquad \therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$
7. $\because (x^2 - 9)(x^3 - 5) = 0$
   $\therefore x^2 - 9 = 0$ $\qquad \therefore x^2 = 9$
   $\therefore x = \pm\sqrt{9} = \pm 3$
   or $x^3 - 5 = 0$ $\qquad \therefore x^3 = 5$
   $\therefore x = \sqrt[3]{5} \approx 1.71$
8. $\because (2x^3 - 5)(x^2 + 1) = 0$
   $\therefore 2x^3 - 5 = 0$ $\qquad \therefore 2x^3 = 5$
   $\therefore x^3 = \frac{5}{2}$ $\qquad \therefore x = \sqrt[3]{\frac{5}{2}} \approx 1.36$
   or $x^2 + 1 = 0$
   $\therefore x^2 = -1$ (has no solution in $\mathbb{R}$)

12

The side length $= \sqrt{5}$ cm. , $\sqrt{5} \notin \mathbb{Q}$

13

The edge length $= \sqrt[3]{1.728} = \frac{6}{5}$ cm. , $\frac{6}{5} \in \mathbb{Q}$

14

$\because$ The total area of the cube $= 6\,\ell^2$

$\therefore 13.5 = 6\,\ell^2$ $\qquad \therefore \frac{13.5}{6} = \ell^2$

$\therefore \ell = \sqrt{\frac{13.5}{6}} = 1.5$ cm. , $1.5 \in \mathbb{Q}$

15

The diagonal length $= \sqrt{6^2 + 6^2} = \sqrt{72}$ cm.

16

The side length $= \sqrt{32}$ cm.

$\therefore$ The diagonal length $= \sqrt{(\sqrt{32})^2 + (\sqrt{32})^2}$

$= \sqrt{32 + 32} = \sqrt{64} = 8$ cm.

17

The length of the hypotenuse $= \sqrt{5^2 + 5^2} = \sqrt{50}$ cm.

18

The diagonal length of the rectangle

$= \sqrt{(5)^2 + (7)^2} = \sqrt{74}$ cm.

$\because$ The area of the square = the area of the rectangle $= 5 \times 7 = 35$ cm.^{2}

$\therefore$ The side length of the square $= \sqrt{35}$ cm.

$\therefore$ The diagonal length of the square $= \sqrt{35 + 35}$

$= \sqrt{70}$ cm.

19

Cubing the two sides then squaring them we find that

$(\sqrt[3]{3})^3 = 3$, $3^2 = 9$, $(\sqrt{2})^3 = 2\sqrt{2}$, $(2\sqrt{2})^2 = 8$

$\because 9 > 8$ $\qquad \therefore \sqrt[3]{3} > \sqrt{2}$

20

Let the other number $= x$ $\qquad \therefore x^2 + 2^2 = 7$

$\therefore x^2 = 7 - 4 = 3$ $\qquad \therefore x = \pm\sqrt{3}$

$\therefore$ The other number is $\sqrt{3}$ or $-\sqrt{3}$

Answers of Exercise 4

1

2. $\{x : 1 \le x < 3 , x \in \mathbb{R}\}$

3. $]0 , 3]$

4. $]-2 , 3[$, $\{x : -2 < x < 3 , x \in \mathbb{R}\}$
5. $\{x : x \le 1 , x \in \mathbb{R}\}$

6. $]0 , \infty[$, $\{x : x > 0 , x \in \mathbb{R}\}$
7. $]-\infty , 4[$

8. $\{x : x \ge -2 , x \in \mathbb{R}\}$

2

1 c 2 a 3 b 4 c 5 d

3

1 $\in$ 2 $\notin$ 3 $\in$ 4 $\in$ 5 $\in$

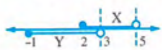
6 $\in$ 7 $\in$ 8 $\notin$ 9 $\notin$ 10 $\notin$

4

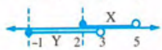
1 $[-1, 5[$

2 $[2, 3[$

3 $[3, 5[$



4 $[-1, 2[$

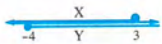


5 $]-\infty, 2[\cup [5, \infty[$

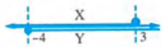
6 $]-\infty, -1[\cup [3, \infty[$

5

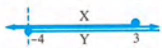
1 $\mathbb{R}$



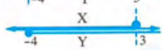
2 $[-4, 3]$



3 $]-\infty, -4[$



4 $]3, \infty[$



5 $]3, \infty[$

6 $]-\infty, -4[$

6 Use the number line to get the following results :

1 $[-1, \infty[$ 2 $[3, 4]$ 3 $[-1, 3[$

4 $[-1, 4[- \{3\}$ 5 $\{3, 4\}$

6 $]4, \infty[$ 7 $]-\infty, -1[\cup]4, \infty[$

8 $]-\infty, 3[$

7 Use the number line to get the following results :

1 $[2, 4]$ 2 $[-1, 5]$ 3 $]0, 1[$

4 $]-2, 3]$ 5 $[3, 6]$ 6 $[-1, 2[$

7 $[-3, 2] - \{0\}$ 8 $\varnothing$

9 $\varnothing$ 10 $[-2, 1[\cup]2, 4]$

11 $\varnothing$ 12 $\{-1, 5\}$

8 Use the number line to get the following results :

1 $[-3, \infty[$ 2 $[2, 3[$ 3 $[-4, 3]$

4 $\mathbb{R}$ 5 $]-\infty, -1[$ 6 $]-\infty, -3[$

7 $]0, 2]$ 8 $\mathbb{R} - [3, 4]$

9

1 $[3, 5]$ 2 $[3, 5]$ 3 $\{3, 5\}$ 4 $\varnothing$

5 $]3, 5[$ 6 $]3, 5[$ 7 $\varnothing$ 8 $\{3, 5\}$

9 $[3, 5[$ 10 $[3, 5[$ 11 $\{3, 4\}$ 12 $]-3, 5]$

10

1 $]1, 7[$ 2 $]-3, 0[$ 3 $[3, 4]$ 4 $]2, 5[$

5 $\{5\}$ 6 $[3, 4[$ 7 $\{2, 7\}$ 8 $\{4\}$

9 $[3, 4[$ 10 $]0, 1[$

11

1 b 2 d 3 c

4 b 5 b 6 d

12

1 $[-3, 3]$ 2 $\mathbb{R}$ 3 $]-\infty, -1[$

4 $]-\infty, -3[$ 5 $]-2, 0]$ 6 $[0, 2]$

7 $\{1, 2\}$ 8 $\{0, 1\}$

9 $\{-1, 0, 1, 2\}$ 10 $]0, 5]$

11 $[-3, 0[$

13

1 $[-3, 1[$ 2 $\mathbb{R} -]-3, 1]$ 3 $]-3, 1[$

4 $\mathbb{R} -]-3, 1[$ 5 $[-3, 3[- \{1\}$

14

Let X be the temperature degrees needed to keep the first kind.

, Y be the temperature degrees needed to keep the second kind.

$\therefore X = [-3, 4]$, $Y = [2, 10]$

$\therefore$ The temperature needed to keep the two kinds altogether at the same place $= X \cap Y = [2, 4]$

15

1 d 2 c 3 c 4 c 5 b

6 d 7 c 8 c 9 c

16

$\because X \subset Y \quad \therefore X = X \cap Y = [4, 7]$

$, Y = X \cup Y = [3, 7], Y - X = [3, 4[$

Answers of Exercise 5

1

1. $3\sqrt{3}$
2. zero
3. $-\sqrt[3]{7}$
4. $6\sqrt{5}$

2

1. $3\sqrt{5}$
2. $3\sqrt{3} - 1$
3. $8\sqrt{7} - 3\sqrt{2}$
4. $7\sqrt{2} - 2\sqrt[3]{2}$
5. $\sqrt{2}$
6. $8 \times \frac{1}{2} + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3} = 4 + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3}$
   $= -3\sqrt[3]{3}$

3

1. 3
2. -30
3. $6\sqrt{2}$
4. 1
5. $15\sqrt{3}$
6. $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \times \frac{5\sqrt{7}}{20\sqrt{3}} = 1$

4

1. $2\sqrt{2} + 2\sqrt{5}$
2. $5\sqrt{2} + 2$
3. $7 + 2\sqrt{7}$
4. $5\sqrt{3} + 3$
5. $-6\sqrt{5} + 10$
6. $2 - 7 + 3\sqrt{7} = -5 + 3\sqrt{7}$
7. $-24 - 6\sqrt{3} + 6\sqrt{3} = -24$
8. $3\sqrt{5} - 5 - 2 - 2\sqrt{5} = \sqrt{5} - 7$

5

1. $(\sqrt{2})^2 - (1)^2 = 2 - 1 = 1$
2. $(4)^2 - (3\sqrt{2})^2 = 16 - 18 = -2$
3. $(\sqrt{5})^2 - 2 \times 1 \times \sqrt{5} + (-1)^2 = 5 - 2\sqrt{5} + 1$
   $= 6 - 2\sqrt{5}$
4. $(2\sqrt{3})^2 + 2 \times 4 \times 2\sqrt{3} + (4)^2 = 12 + 16\sqrt{3} + 16$
   $= 28 + 16\sqrt{3}$
5. $3 + \sqrt{3} - 2 = 1 + \sqrt{3}$
6. $(5)^2 - 2 \times 5 \times \sqrt{3} + (-\sqrt{3})^2 - 28$
   $= 25 - 10\sqrt{3} + 3 - 28 = -10\sqrt{3}$

6

1. $\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$
2. $\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$
3. $-\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$
4. $\frac{8}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{8\sqrt{6}}{6} = \frac{4\sqrt{6}}{3}$
5. $\frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$
6. $\frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$
7. $\frac{25}{2\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{25\sqrt{10}}{20} = \frac{5\sqrt{10}}{4}$
8. $\frac{\sqrt{2} + 3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2 + 3\sqrt{2}}{2}$
9. $\frac{\sqrt{5} - 15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5 - 15\sqrt{5}}{10} = \frac{1 - 3\sqrt{5}}{2}$

7

1. c
2. d
3. b
4. c
5. a
6. d
7. c
8. b
9. d
10. b
11. d
12. c

8

1. 1 , zero
2. $\sqrt{2} - 1$
3. $5\sqrt{3}$
4. 1
5. $2, \sqrt{3}$
6. $\frac{2}{3}$
7. $4\sqrt{3}$
8. $3 + 2\sqrt{2}$
9. $\pm\sqrt{5}$
10. $8\sqrt{2}$
11. 60 cm^2
12. The additive inverse
13. $\mathbb{R}$

9

1. $\sqrt{5} - 2 + \sqrt{5} + 2 = 2\sqrt{5}$
2. $\sqrt{5} - 2 - \sqrt{5} - 2 = -4$
3. $(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$
4. $x^2 - y^2 = (x - y)(x + y) = (-4)(2\sqrt{5}) = -8\sqrt{5}$
5. $x^2 + 2xy + y^2 = (x + y)^2 = (2\sqrt{5})^2 = 20$
6. $x^2 - 2xy + y^2 = (x - y)^2 = (-4)^2 = 16$

10

$\because x=\sqrt[3]{\sqrt{2}\times\sqrt{2}\times\sqrt{2}}=\sqrt{2}$

$\therefore (x+\sqrt{2})^2=(\sqrt{2}+\sqrt{2})^2$

$=(2\sqrt{2})^2=8$

11

$\because a=2\sqrt{2}+2 \quad , \quad b=2\sqrt{2}-2$

$\therefore ab=(2\sqrt{2}+2)(2\sqrt{2}-2)$

$=(2\sqrt{2})^2-(2)^2=8-4=4$

$, a-b=(2\sqrt{2}+2)-(2\sqrt{2}-2)$

$=2\sqrt{2}+2-2\sqrt{2}+2=4$

$\therefore ab=a-b$

12

1 $x\approx 4+2=6$

and using the calculator $x\approx 5.9$ (accepted estimation)

$, y\approx 4-3=1$

and using the calculator $y\approx 1.08$ (accepted estimation)

2 $x\times y\approx 6\times 1=6$

and using the calculator , the expression ≈ 6.3 (accepted estimation)

3 $x+y\approx 6+1=7$

and using the calculator , the expression ≈ 6.9 (accepted estimation)

13

The perimeter $=2(6+\sqrt{5}+6-\sqrt{5})=2\times 12$

$=24$ cm.

The area $=(6+\sqrt{5})(6-\sqrt{5})=36-5=31$ cm^2.

14

The expression $=a(a-b)^3+b(b-a)^3$

$=a(a-b)^3-b(a-b)^3$

$=(a-b)^3(a-b)=(a-b)^4$

$=(2\sqrt{3})^4=144$

15

$(\sqrt{a}-1)\times\dfrac{\sqrt{a}+1}{4}=1$

$\therefore \dfrac{a-1}{4}=1 \qquad \therefore a-1=4 \qquad \therefore a=5$

16

$x=\sqrt{2} \; , \; y=\dfrac{\sqrt{2}}{2} \; , \; z=\dfrac{\sqrt{2}}{4}$

$\therefore x^2+2y^2+4z^2=(\sqrt{2})^2+2\times\left(\dfrac{\sqrt{2}}{2}\right)^2+4\left(\dfrac{\sqrt{2}}{4}\right)^2$

$=2+2\times\dfrac{2}{4}+4\times\dfrac{2}{16}=3\dfrac{1}{2}$

17

$\dfrac{1}{2}(2y)=1-\sqrt{2} \quad \therefore y=1-\sqrt{2}$

$\therefore x=-1+\sqrt{2}$

$\therefore xy-2\sqrt{2}=(-1+\sqrt{2})(1-\sqrt{2})-2\sqrt{2}$

$=-1+\sqrt{2}+\sqrt{2}-2-2\sqrt{2}=-3$

Answers of Exercise 6

1

1 $\sqrt{4\times 3}=2\sqrt{3}$ 2 $\sqrt{4\times 7}=2\sqrt{7}$

3 $2\sqrt{36\times 2}=2\times 6\sqrt{2}=12\sqrt{2}$

4 $\frac{2}{5}\sqrt{100\times 10}=\frac{2}{5}\times 10\sqrt{10}=4\sqrt{10}$

5 $\sqrt{4\times\frac{1}{2}}=\sqrt{2}$ 6 $2\sqrt{\frac{2}{3}\times 9}=2\sqrt{6}$

2

1 $5\sqrt{2}+2\sqrt{2}=7\sqrt{2}$

2 $2\sqrt{5}-3\sqrt{5}=-\sqrt{5}$

3 $3\sqrt{2}+2\sqrt{2}-3\sqrt{2}=2\sqrt{2}$

4 $7\sqrt{2}-8\sqrt{2}-3\sqrt{2}+4\sqrt{2}=$ zero

5 $2\times 3\sqrt{2}+5\sqrt{2}+\frac{1}{3}\times 9\sqrt{2}$

$=6\sqrt{2}+5\sqrt{2}+3\sqrt{2}=14\sqrt{2}$

6 $7\sqrt{2}+5\sqrt{2}-\frac{1}{2}\times 10\sqrt{2}-\sqrt{2}$

$=7\sqrt{2}+5\sqrt{2}-5\sqrt{2}-\sqrt{2}=6\sqrt{2}$

7 $3\sqrt{3}+5\times 3\sqrt{2}-10\sqrt{3}=15\sqrt{2}-7\sqrt{3}$

3

1 $2\sqrt{5}+4\times 2\sqrt{5}-\sqrt{25\times\frac{1}{5}}=2\sqrt{5}+8\sqrt{5}-\sqrt{5}$

$=9\sqrt{5}$

2 $4\sqrt{2}-6\sqrt{2}+3\sqrt{4\times\frac{1}{2}}=4\sqrt{2}-6\sqrt{2}+3\sqrt{2}$
$=\sqrt{2}$

3 $2\sqrt{5}+2\sqrt{9\times\frac{1}{3}}-2\sqrt{3}-\sqrt{25\times\frac{1}{5}}$
$=2\sqrt{5}+2\sqrt{3}-2\sqrt{3}-\sqrt{5}=\sqrt{5}$

4 $\sqrt{3}+\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\sqrt{12}=\sqrt{3}+\sqrt{3}-2\sqrt{3}=$ zero

5 $3\sqrt{2}-\sqrt{\frac{12}{6}}=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

6 $5+3\sqrt{2}-\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=5+3\sqrt{2}-3\sqrt{2}=5$

4

1 $10\sqrt{6}$

2 $6\sqrt{36}=6\times6=36$

3 $2\sqrt{50}=2\times5\sqrt{2}=10\sqrt{2}$

4 $\sqrt{\frac{2}{7}\times\frac{7}{2}}=\sqrt{1}=1$

5 $3\sqrt{\frac{15}{5}}=3\sqrt{3}$

6 $12\times\sqrt{\frac{2}{3}\times54}=12\sqrt{36}=12\times6=72$

5

1 $\sqrt{18}-\sqrt{12}=3\sqrt{2}-2\sqrt{3}$

2 $20+5\sqrt{24}=20+5\times2\sqrt{6}=20+10\sqrt{6}$

3 $(3\sqrt{5})^2-(\sqrt{7})^2=45-7=38$

4 $(\sqrt{3})^2-2\times\sqrt{3}\times\sqrt{2}+(-\sqrt{2})^2=3-2\sqrt{6}+2$
$=5-2\sqrt{6}$

5 $(\sqrt{3})^2+2\times\sqrt{3}\times\sqrt{5}+(\sqrt{5})^2-2\sqrt{15}$
$=3+2\sqrt{15}+5-2\sqrt{15}=8$

6 $3\sqrt{2}-\frac{12}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}+2\sqrt{6}-3\sqrt{2}$
$=3\sqrt{2}-2\sqrt{6}+2\sqrt{6}-3\sqrt{2}=$ zero

6

1 $\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

2 $\sqrt{\frac{5}{3}}=\frac{\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{15}}{3}$

3 $\frac{5\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{15}}{5}=\sqrt{15}$

4 $\frac{4\sqrt{3}-\sqrt{2}}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{12-\sqrt{6}}{6}$

7

1 a	2 b	3 c	4 a	5 b
6 c	7 a	8 a	9 c	10 b

8

1 $\frac{1}{2}$ 2 $\sqrt{2}$ 3 $\sqrt{3}$ 4 $\frac{3}{2}$

5 -2 6 $\sqrt{125}$ 7 $\pm\frac{2\sqrt{2}}{3}$ 8 20 , zero

9

1 $x+y=3+\sqrt{5}+1-\sqrt{5}=4$
$x\times y=(3+\sqrt{5})(1-\sqrt{5})=3-2\sqrt{5}-5$
$=-2-2\sqrt{5}$

2 $x+y=\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$
$x\times y=(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})=3-2=1$

3 $x+y=5-3\sqrt{2}+5-3\sqrt{2}=10-6\sqrt{2}$
$x\times y=(5-3\sqrt{2})(5-3\sqrt{2})$
$=25-30\sqrt{2}+18=43-30\sqrt{2}$

10

$\because x=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$, $y=\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

$\therefore 6(x+y)=6\left(\frac{\sqrt{6}}{3}+\frac{\sqrt{6}}{2}\right)=6\times\frac{\sqrt{6}}{3}+6\times\frac{\sqrt{6}}{2}$
$=2\sqrt{6}+3\sqrt{6}=5\sqrt{6}$

11

$\because x=\frac{10}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{10\sqrt{5}}{5}=2\sqrt{5}$

, $y=3\sqrt{5}+\sqrt{2}$, $z=2\sqrt{2}+\sqrt{5}$

$\therefore (x-y+z)^2$
$=(2\sqrt{5}-3\sqrt{5}-\sqrt{2}+2\sqrt{2}+\sqrt{5})^2=(\sqrt{2})^2=2$

12 We know that $(x+y)^2=x^2+2xy+y^2$
$\therefore x^2+2xy+y^2=(2\sqrt{5}+\sqrt{2}+2\sqrt{5}-\sqrt{2})^2$
$=(4\sqrt{5})^2=16\times5=80$

13

$\because x=\sqrt{7}+\frac{1}{2}\times 2\sqrt{3}=\sqrt{7}+\sqrt{3}$

$, y=\frac{1}{3}\times 3\sqrt{7}-\sqrt{3}=\sqrt{7}-\sqrt{3}$

$\therefore x^2y^2=(xy)^2=\left((\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})\right)^2$

$=(7-3)^2=4^2=16$

14

The perimeter of Δ ABC

$=\sqrt{28}+28\sqrt{\frac{1}{7}}+5\sqrt{7}$

$=\sqrt{4\times 7}+4\sqrt{49\times\frac{1}{7}}+5\sqrt{7}$

$=2\sqrt{7}+4\sqrt{7}+5\sqrt{7}=11\sqrt{7}$ cm.

15

1. The area of one square $=\frac{300}{6}=50$ cm^2.

   $\therefore$ The side length of one square $=\sqrt{50}=5\sqrt{2}$ cm.

   $\therefore$ The perimeter of the figure $=14\times 5\sqrt{2}$

   $=70\sqrt{2}$ cm.

2. The area of one square $=\frac{72}{6}=12$ cm^2.

   $\therefore$ The side length of one square $=\sqrt{12}=2\sqrt{3}$ cm.

   $\therefore$ The perimeter of the figure $=14\times 2\sqrt{3}$

   $=28\sqrt{3}$ cm.

3. The area of one square $=\frac{40}{5}=8$ cm^2.

   $\therefore$ The side length of one square $=\sqrt{8}=2\sqrt{2}$ cm.

   $\therefore$ The perimeter of the figure $=12\times 2\sqrt{2}$

   $=24\sqrt{2}$ cm.

16

$a^{x+y}=a^x\times a^y=a^x\div a^{-y}=6\div\sqrt{3}=\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{6\sqrt{3}}{3}=2\sqrt{3}$

17

1. $\frac{(\sqrt{5})^3\times(\sqrt{5})^5}{(\sqrt{2})^6\times(\sqrt{5})^6}=\frac{(\sqrt{5})^{3+5-6}}{(\sqrt{2})^6}=\frac{5}{8}$

2. $\frac{2\sqrt{2}\times(\sqrt{2})^{-3}\times(\sqrt{3})^{-3}}{(\sqrt{3})^{-3}}=2\times(\sqrt{2})^{-2}$

   $=\frac{2}{(\sqrt{2})^2}=\frac{2}{2}=1$

18

$\because 3\sqrt{3}+\sqrt{4\times\frac{1}{2}}+3\sqrt{2}+2\sqrt{3}-5\sqrt{2}$

$=x\sqrt{2}+y\sqrt{3}$

$\therefore 3\sqrt{3}+\sqrt{2}+3\sqrt{2}+2\sqrt{3}-5\sqrt{2}=x\sqrt{2}+y\sqrt{3}$

$\therefore -\sqrt{2}+5\sqrt{3}=x\sqrt{2}+y\sqrt{3}$ $\therefore x=-1$, $y=5$

Answers of Exercise 7

1

1. $\sqrt{5}-\sqrt{3}$
2. $5+2\sqrt{7}$
3. $\because \sqrt{5}+\frac{2}{\sqrt{2}}=\sqrt{5}+\sqrt{2}$

   $\therefore$ The conjugate number $=\sqrt{5}-\sqrt{2}$

2

1. $\frac{5}{\sqrt{7}-\sqrt{2}}\times\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}=\frac{5(\sqrt{7}+\sqrt{2})}{7-2}=\sqrt{7}+\sqrt{2}$

2. $\frac{\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{\sqrt{3}\,(2+\sqrt{3})}{4-3}=2\sqrt{3}+3$

3. $\frac{\sqrt{7}+3}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}=\frac{16+6\sqrt{7}}{7-9}=-8-3\sqrt{7}$

3

$\because x=\frac{2}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$=\sqrt{7}+\sqrt{5}$

$\therefore (x+y)^2=\left(\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}\right)^2$

$=\left(2\sqrt{7}\right)^2=28$

4

$x^2y^2=(xy)^2=\left(\frac{4}{\sqrt{7}-\sqrt{3}}\times\frac{4}{\sqrt{7}+\sqrt{3}}\right)^2$

$=\left(\frac{16}{7-3}\right)^2=4^2=16$

5

L.H.S. $=\frac{4}{x}+2x$

$=\frac{4}{\sqrt{5}+\sqrt{3}}+2(\sqrt{5}+\sqrt{3})$

$=\frac{4(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}+2(\sqrt{5}+\sqrt{3})$

$= \frac{4(\sqrt{5}-\sqrt{3})}{2} + 2(\sqrt{5}+\sqrt{3})$

$= 2(\sqrt{5}-\sqrt{3}) + 2(\sqrt{5}+\sqrt{3})$

$= 2\sqrt{5} - 2\sqrt{3} + 2\sqrt{5} + 2\sqrt{3}$

$= 4\sqrt{5} =$ R.H.S.

6

$\because b = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$

$= \sqrt{3}-\sqrt{2}$

We know that : $(a-b)(a+b) = a^2 - b^2$

$\therefore (\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})$

$= 2\sqrt{2} \times 2\sqrt{3} = 4\sqrt{6}$

7

$\because y = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5}+\sqrt{3}$

$\therefore x^2 + 2xy + y^2 = (x+y)^2$

$= (\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3})^2 = (2\sqrt{5})^2 = 20$

8

$\because y = \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$= \sqrt{5}+\sqrt{2}$

$, x = \sqrt{5}-\sqrt{2}$

$\therefore x$ and y are two conjugate numbers.

$\therefore x^2 - 2xy + y^2 = (x-y)^2 = (\sqrt{5}-\sqrt{2}-\sqrt{5}-\sqrt{2})^2$

$= (-2\sqrt{2})^2 = 8$

9

$\because x = 3+\sqrt{5}, y = \frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{4(3-\sqrt{5})}{9-5}$

$= 3-\sqrt{5}$

$\therefore x$ and y are two conjugate numbers

1. $xy = (3+\sqrt{5})(3-\sqrt{5}) = 9-5 = 4$
2. $x^2 + y^2 = (x+y)^2 - 2xy$

$= (3+\sqrt{5}+3-\sqrt{5})^2 - 2 \times 4 = 36 - 8 = 28$

10

$\because x = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5}+\sqrt{3}$

$, y = \frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{5-3}$

$= \sqrt{5}-\sqrt{3}$

$\therefore x^2 - xy + y^2 = (x-y)^2 + xy$

$= (\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

$= (2\sqrt{3})^2 + 2 = 14$

11

$\frac{x+y}{xy-1} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})-1}$

$= \frac{2\sqrt{5}}{5-2-1} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

12

$\because a = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$= \sqrt{7}+\sqrt{3}$

$, b = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{7-3}$

$= \sqrt{7}-\sqrt{3}$

$\therefore \frac{a-b}{ab} = \frac{\sqrt{7}+\sqrt{3}-\sqrt{7}+\sqrt{3}}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{2\sqrt{3}}{7-3} = \frac{\sqrt{3}}{2}$

13

$\because x = 2\sqrt{2}-\sqrt{3}, y = \frac{5}{2\sqrt{2}-\sqrt{3}}$

$\therefore y = \frac{5}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{5(2\sqrt{2}+\sqrt{3})}{8-3}$

$= 2\sqrt{2}+\sqrt{3}$

$\therefore x$ and y are conjugate numbers.

$\therefore \frac{x+y}{xy} = \frac{2\sqrt{2}-\sqrt{3}+2\sqrt{2}+\sqrt{3}}{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})}$

$= \frac{4\sqrt{2}}{8-3} = \frac{4\sqrt{2}}{5}$

14

$\because x=\dfrac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{5\sqrt{10}+15}{5}$

$=\sqrt{10}+3$

$, y=\dfrac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{2\sqrt{10}-6}{2}=\sqrt{10}-3$

1 $x^2+y^2=(x+y)^2-2xy$

$=\left(\sqrt{10}+3+\sqrt{10}-3\right)^2-2\left(\sqrt{10}+3\right)\left(\sqrt{10}-3\right)$

$=\left(2\sqrt{10}\right)^2-2\times(10-9)=40-2=38$

2 $xy=\left(\sqrt{10}+3\right)\left(\sqrt{10}-3\right)=10-9=1$

$\therefore x^2+y^2=38\,xy$

15

$\because x=\dfrac{1}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}=\dfrac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$

$, y=\dfrac{12}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{12\sqrt{3}}{3}=4\sqrt{3}$

$\therefore x^2+y=\left(2-\sqrt{3}\right)^2+4\sqrt{3}$

$=4-4\sqrt{3}+3+4\sqrt{3}=7$

16

$\because y=\sqrt{3}-\sqrt{2}$

$, x=\dfrac{1}{\sqrt{3}-\sqrt{2}}\times\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\sqrt{3}+\sqrt{2}$

$\therefore (x+y)^2=\left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right)^2$

$=\left(2\sqrt{3}\right)^2=12$

17

$\because xy=1$

$\therefore y=\dfrac{1}{x}=\dfrac{1}{\sqrt{13}+\sqrt{6}}=\dfrac{1}{\sqrt{13}+\sqrt{6}}\times\dfrac{\sqrt{13}-\sqrt{6}}{\sqrt{13}-\sqrt{6}}$

$=\dfrac{\sqrt{13}-\sqrt{6}}{7}$

$\therefore x^2-49y^2=(x-7y)(x+7y)$

$=\left(\sqrt{13}+\sqrt{6}-7\left(\dfrac{\sqrt{13}-\sqrt{6}}{7}\right)\right)$

$\left(\sqrt{13}+\sqrt{6}+7\left(\dfrac{\sqrt{13}-\sqrt{6}}{7}\right)\right)$

$=\left(\sqrt{13}+\sqrt{6}-\sqrt{13}+\sqrt{6}\right)\left(\sqrt{13}+\sqrt{6}+\sqrt{13}-\sqrt{6}\right)$

$=2\sqrt{6}\times 2\sqrt{13}=4\sqrt{78}$

18

$\because x=\dfrac{4\left(\sqrt{7}+\sqrt{3}\right)}{\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{7}+\sqrt{3}\right)}=\dfrac{4\left(\sqrt{7}+\sqrt{3}\right)}{7-3}$

$=\sqrt{7}+\sqrt{3}$

$, y=\sqrt{7}-\sqrt{3}$

$\therefore$ x and y are two conjugate numbers.

$\therefore x^2y^2=(xy)^2=\left[\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right)\right]^2$

$=(7-3)^2=4^2=16$

19

$\because y=\dfrac{2}{\sqrt{7}+\sqrt{5}}\times\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}=\dfrac{2\left(\sqrt{7}-\sqrt{5}\right)}{7-5}$

$=\sqrt{7}-\sqrt{5}$

$\therefore \dfrac{x+y}{xy}=\dfrac{\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}}{\left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)}=\dfrac{2\sqrt{7}}{7-5}=\sqrt{7}$

20

$\because x=\dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}\times\dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\dfrac{11+2\sqrt{30}}{6-5}$

$=11+2\sqrt{30}$

$, \dfrac{1}{x}=\dfrac{1}{11+2\sqrt{30}}\times\dfrac{11-2\sqrt{30}}{11-2\sqrt{30}}=\dfrac{11-2\sqrt{30}}{121-120}$

$=11-2\sqrt{30}$

$\therefore x+\dfrac{1}{x}=11+2\sqrt{30}+11-2\sqrt{30}=22$

21

1 4	2 $3-\sqrt{2}, 7$	3 $\sqrt{5}-\sqrt{3}$
4 $1-\sqrt{7}$	5 $\sqrt{3}-\sqrt{2}$	6 20
7 20	8 $\sqrt{5}+2$	9 $\left(-1, 2\sqrt{3}\right)$
10 -1		

22

1 $\dfrac{11}{2\sqrt{5}+3}=\dfrac{11\left(2\sqrt{5}-3\right)}{\left(2\sqrt{5}+3\right)\left(2\sqrt{5}-3\right)}$

$=\dfrac{11\left(2\sqrt{5}-3\right)}{20-9}=2\sqrt{5}-3$

$\therefore a=2, b=-3$

2 $\frac{3}{2\sqrt{2}-\sqrt{5}}=\frac{3(2\sqrt{2}+\sqrt{5})}{(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})}$

$=\frac{3(2\sqrt{2}+\sqrt{5})}{8-5}=2\sqrt{2}+\sqrt{5}$

$\therefore a=2, b=1$

3 $\frac{7}{\sqrt{8}+1}=\frac{7(\sqrt{8}-1)}{(\sqrt{8}+1)(\sqrt{8}-1)}$

$=\frac{7(\sqrt{8}-1)}{8-1}=\sqrt{8}-1$

$=2\sqrt{2}-1=a+b\sqrt{2}$

$\therefore a=-1, b=2$

23

1 The expression $=\frac{4(\sqrt{5}-\sqrt{3})+4(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$=\frac{4\sqrt{5}-4\sqrt{3}+4\sqrt{5}+4\sqrt{3}}{5-3}$

$=4\sqrt{5}$

2 The expression $=\frac{(\sqrt{6}-\sqrt{5})^2-(\sqrt{6}+\sqrt{5})^2}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})}$

$=\frac{6+5-2\sqrt{30}-6-5-2\sqrt{30}}{6-5}$

$=-4\sqrt{30}$

3 The expression $=5\sqrt{3}-5+\frac{10(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$=5\sqrt{3}-5+5(\sqrt{3}+1)$

$=5\sqrt{3}-5+5\sqrt{3}+5=10\sqrt{3}$

24

$(x+y)^2=x^2+2xy+y^2$

$=\left(\sqrt{4+\sqrt{7}}\right)^2+2\left(\sqrt{4+\sqrt{7}}\right)\left(\sqrt{4-\sqrt{7}}\right)+\left(\sqrt{4-\sqrt{7}}\right)^2$

$=4+\sqrt{7}+2\sqrt{\left(4+\sqrt{7}\right)\left(4-\sqrt{7}\right)}+4-\sqrt{7}$

$=8+2\sqrt{16-7}=8+2\sqrt{9}=8+6=14$

25

$xy^{-1}+yx^{-1}=\frac{x}{y}+\frac{y}{x}=\frac{x^2+y^2}{xy}$

$=\frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}+1)(\sqrt{5}-1)}$

$=\frac{6+2\sqrt{5}+6-2\sqrt{5}}{5-1}=\frac{12}{4}=3$

26

$\because \frac{3x^2+3y^2}{xy}=\frac{3(x^2+y^2)}{xy}$

$=3\left(\frac{x^2}{xy}+\frac{y^2}{xy}\right)=3\left(\frac{x}{y}+\frac{y}{x}\right)$

$, \because \frac{x}{y}=\sqrt{3}-\sqrt{2}$

$\therefore \frac{y}{x}=\frac{1}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\sqrt{3}+\sqrt{2}$

$\therefore$ The expression $=3(\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2})$

$=3(2\sqrt{3})=6\sqrt{3}$

27

$\because \frac{x^8y^9-y}{(x+y)^5}=\frac{y(x^8y^8-1)}{(x+y)^5}$ (1)

$, x^8y^8=(xy)^8=[(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})]^8$

$=1^8=1$

, from (1) : $\therefore \frac{x^8y^9-y}{(x+y)^5}=\frac{(\sqrt{7}-\sqrt{6})(1-1)}{(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6})^5}$

$=\frac{\text{zero}}{(2\sqrt{7})^5}=$ zero

Answers of Exercise 8

1

1 $\sqrt[3]{8\times 2}=2\sqrt[3]{2}$

2 $\sqrt[3]{-27\times 2}=-3\sqrt[3]{2}$

3 $2\sqrt[3]{125\times 2}=2\times 5\sqrt[3]{2}=10\sqrt[3]{2}$

4 $\frac{2}{3}\sqrt[3]{-27\times 5}=\frac{2}{3}\times -3\sqrt[3]{5}=-2\sqrt[3]{5}$

5 $\sqrt[3]{\frac{1}{3}\times 27}=\sqrt[3]{9}$

6 $-2\sqrt[3]{\frac{2}{5}\times 125}=-2\sqrt[3]{50}$

2

1 $\sqrt[3]{2\times 32}=\sqrt[3]{64}=4$ 2 $\sqrt[3]{\frac{72}{9}}=\sqrt[3]{8}=2$

3 $\frac{4}{2}\sqrt[3]{\frac{-54}{-2}}=2\sqrt[3]{27}=2\times 3=6$

4 $\frac{1}{2}\times 6\sqrt[3]{10\times 100}=3\sqrt[3]{1000}=3\times 10=30$

5 $\sqrt[3]{\frac{2}{5}\times\frac{4}{25}}=\sqrt[3]{\frac{8}{125}}=\frac{2}{5}$

6 $\sqrt[3]{\frac{3}{4}\div\frac{2}{9}}=\sqrt[3]{\frac{3}{4}\times\frac{9}{2}}=\sqrt[3]{\frac{27}{8}}=\frac{3}{2}$

3

1 $2\sqrt[3]{2}-\sqrt[3]{2}=\sqrt[3]{2}$ 2 $5-2\sqrt[3]{3}$

3 $3\sqrt[3]{3}-2\sqrt[3]{3}=\sqrt[3]{3}$

4 $3\sqrt[3]{2}+2\sqrt[3]{2}-5\sqrt[3]{2}=$ zero

5 $2\times 3\sqrt[3]{2}-5\sqrt[3]{2}+2\sqrt[3]{2}=3\sqrt[3]{2}$

6 $2\sqrt[3]{2}-\frac{1}{3}\times 3\sqrt[3]{2}-\sqrt[3]{2}=$ zero

7 $2\sqrt[3]{2}+\sqrt[3]{250}=2\sqrt[3]{2}+5\sqrt[3]{2}=7\sqrt[3]{2}$

8 $2\sqrt[3]{3}-2\times 3\sqrt[3]{\frac{125}{9}}=2\sqrt[3]{3}-2\sqrt[3]{\frac{125}{9}\times 27}$

$=2\sqrt[3]{3}-2\times 5\sqrt[3]{3}=-8\sqrt[3]{3}$

4

1 The left hand side $=4\sqrt[3]{2}+2\sqrt[3]{2}-2\times 3\sqrt[3]{2}=$ zero

= the right hand side.

2 The left hand side $=3\sqrt[3]{2}\times 2\sqrt[3]{2}\div(6\sqrt[3]{4})$

$=6\sqrt[3]{4}\div 6\sqrt[3]{4}=1=$ the right hand side.

5

1 $3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{27\times\frac{1}{9}}=3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{3}=$ zero

2 $3\sqrt[3]{2}-4\sqrt[3]{\frac{1}{4}\times 8}+5\times 2\sqrt[3]{2}$

$=3\sqrt[3]{2}-4\sqrt[3]{2}+10\sqrt[3]{2}=9\sqrt[3]{2}$

3 $3\sqrt[3]{4}-2\sqrt[3]{4}-\sqrt[3]{\frac{4}{8}}=3\sqrt[3]{4}-2\sqrt[3]{4}-\frac{1}{2}\sqrt[3]{4}=\frac{1}{2}\sqrt[3]{4}$

4 $\sqrt[3]{3}-\sqrt[3]{24}+\sqrt[3]{27\times\frac{1}{9}}=\sqrt[3]{3}-2\sqrt[3]{3}+\sqrt[3]{3}=$ zero

6

1 $\frac{7}{3}\times 3\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}$

$=7\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}=5\sqrt[3]{2}$

2 $3\sqrt{3}+\frac{1}{3}\times 3-3\sqrt{9\times\frac{1}{3}}-1$

$=3\sqrt{3}+1-3\sqrt{3}-1=$ zero

3 $-2\sqrt[3]{2}+\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}-2\sqrt{7}+3\sqrt[3]{2}$

$=-2\sqrt[3]{2}+2\sqrt{7}-2\sqrt{7}+3\sqrt[3]{2}=\sqrt[3]{2}$

4 $3\sqrt{2}+3\sqrt[3]{2}-\sqrt{\frac{216}{12}}-2\sqrt[3]{2}$

$=3\sqrt{2}+\sqrt[3]{2}-\sqrt{18}=3\sqrt{2}+\sqrt[3]{2}-3\sqrt{2}=\sqrt[3]{2}$

5 $5\sqrt{2}-\frac{1}{2}\times 10\sqrt{2}+\sqrt[3]{125}=5\sqrt{2}-5\sqrt{2}+5=5$

7

$4\sqrt[3]{2}\,(3\sqrt[3]{4}+10\sqrt[3]{4}-\sqrt[3]{\frac{8}{2}})$

$=4\sqrt[3]{2}\,(3\sqrt[3]{4}+10\sqrt[3]{4}-\sqrt[3]{4})$

$=4\sqrt[3]{2}\times 12\sqrt[3]{4}=48\sqrt[3]{8}=96$

8

1 c 2 a 3 c 4 d 5 a

9

1 -2 2 9 3 250

4 $\frac{2}{3}\sqrt[3]{7}$ 5 $-\frac{1}{2}$ 6 1

10

1 $(\sqrt[3]{5}+1-\sqrt[3]{5}+1)^5=2^5=32$

2 $(\sqrt[3]{5}+1+\sqrt[3]{5}-1)^3=(2\sqrt[3]{5})^3=8\times 5=40$

11

$\because x-y=3+\sqrt[3]{6}-3+\sqrt[3]{6}=2\sqrt[3]{6}$

$, x+y=3+\sqrt[3]{6}+3-\sqrt[3]{6}=6$

$\therefore\left(\frac{x-y}{x+y}\right)^3=\left(\frac{2\sqrt[3]{6}}{6}\right)^3=\left(\frac{\sqrt[3]{6}}{3}\right)^3=\frac{(\sqrt[3]{6})^3}{3^3}$

$=\frac{6}{27}=\frac{2}{9}$

12

$2\sqrt[3]{4}+2\sqrt[3]{8\times\frac{1}{2}}-4\sqrt[3]{-2}\times\sqrt[3]{-2}+1-\frac{4}{2}$

$=2\sqrt[3]{4}+2\sqrt[3]{4}-4\sqrt[3]{4}+1-2=-1$

13

$\because$ The edge length of one cube $= \sqrt[3]{24} = 2\sqrt[3]{3}$ dm.

$\therefore$ The area of one face of one cube

$= 2\sqrt[3]{3} \times 2\sqrt[3]{3} = 4\sqrt[3]{9}$ dm^2

$\therefore$ The area of the using ground

$= 5 \times 4\sqrt[3]{9} = 20\sqrt[3]{9}$ dm^2

14

L.H.S. $= x^2 + y^2 = (x + y)^2 - 2xy$

$= (\sqrt[3]{2} + 1 + \sqrt[3]{2} - 1)^2 - 2(\sqrt[3]{2} + 1)(\sqrt[3]{2} - 1)$

$= (2\sqrt[3]{2})^2 - 2((\sqrt[3]{2})^2 - 1)$

$= 4\sqrt[3]{4} - 2\sqrt[3]{4} + 2 = 2\sqrt[3]{4} + 2 =$ R.H.S.

15

$$\frac{2}{\sqrt[3]{2}} = \frac{2 \times \sqrt[3]{4}}{\sqrt[3]{2} \times \sqrt[3]{4}} = \frac{2\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{2\sqrt[3]{4}}{2} = \sqrt[3]{4}$$

Another solution : $\frac{2}{\sqrt[3]{2}} = \frac{\sqrt[3]{8}}{\sqrt[3]{2}} = \sqrt[3]{\frac{8}{2}} = \sqrt[3]{4}$

Answers of Exercise 9

1

1 125   2 96   3 $4\ell^2$ 4 $6\ell^2$ 5 $8\ell^3$

2

$\because$ Area of one face $= \frac{36}{4} = 9$ cm^2

$\therefore$ The edge length of the cube $= \sqrt{9} = 3$ cm.

1 Its total area $= 6\ell^2 = 6 \times 3^2 = 54$ cm^2

2 Its volume $= \ell^3 = 3^3 = 27$ cm^3

3

The edge length of the cube $= \frac{12}{4} = 3$ cm.

1 Its volume $= \ell^3 = 3^3 = 27$ cm^3

2 Its lateral area $= 4\ell^2 = 4 \times 3^2 = 36$ cm^2

4

The edge length of the cube $= \frac{60}{12} = 5$ cm.

1 Its volume $= \ell^3 = 5^3 = 125$ cm^3

2 Its total area $= 6\ell^2 = 6 \times 5^2 = 150$ cm^2

5

1 d   2 c   3 d   4 a   5 b

6 d   7 b   8 d   9 a

6

1 The volume of the cuboid $= x \times y \times z$

$= 9 \times 10 \times 5 = 450$ cm^3

2 Its lateral area $= 2(x + y) \times z = 2(9 + 10) \times 5$

$= 190$ cm^2

3 Its total area $= 2(xy + yz + zx)$

$= 2(9 \times 10 + 10 \times 5 + 5 \times 9)$

$= 2(90 + 50 + 45) = 370$ cm^2

7

The volume $= x \times y \times z = \sqrt{2} \times \sqrt{3} \times \sqrt{6} = 6$ cm^3

8

$\because$ The area of the base $= \sqrt{3}(\sqrt{3} - 1) = (3 - \sqrt{3})$ cm^2

The volume = the area of the base × height

$= (3 - \sqrt{3})(3 + \sqrt{3}) = 9 - 3 = 6$ cm^3

9

$\because$ The lateral area = the perimeter of the base × height

$\therefore$ The height $= \frac{480}{4 \times 10} = 12$ cm.

10

The area of the base $= \frac{\text{volume}}{\text{height}} = \frac{720}{5} = 144$ cm^2

$\therefore$ The side length of the base $= \sqrt{144} = 12$ cm.

$\therefore$ The total area $= 2(xy + yz + zx)$

$= 2 \times (12 \times 12 + 12 \times 5 + 12 \times 5)$

$= 528$ cm^2.

11

$\because$ The area of the face of the cube $= \frac{294}{6} = 49$ cm^2

$\therefore$ The edge length $= \sqrt{49} = 7$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 7 \times 7 \times 7 = 343$ cm^3

, $\because$ the volume of the cuboid $= x \times y \times z$

$= 7\sqrt{2} \times 5\sqrt{2} \times 5$

$= 350$ cm^3

$\therefore$ The volume of the cuboid is greater than the volume of the cube

12

The volume of the cuboid $= x \times y \times z = 17 \times 7 \times 4$

$= 476 \text{ cm}^3$.

, the total area $= 2(x + y) \times z + x y$

$= 2(17 + 7) \times 4 + 17 \times 7$

$= 192 + 119 = 311 \text{ cm}^2$.

13

The circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 10.5$

$= 66$ cm.

The area of the circle $= \pi r^2 = \frac{22}{7} \times (10.5)^2$

$= 346.5 \text{ cm}^2$.

14

$\because$ The area of the circle $= \pi r^2$

$\therefore 154 = \frac{22}{7} r^2 \qquad \therefore r^2 = \frac{154 \times 7}{22} = 49$

$\therefore r = \sqrt{49} = 7$ cm.

The circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

The diameter length $= 2 \times 7 = 14$ cm.

15

$\because$ The area of the circle $= \pi r^2$

$\therefore 64\pi = \pi r^2 \qquad \therefore r^2 = 64$

$\therefore r = \sqrt{64} = 8$ cm.

The circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 8 \approx 50$ cm.

16

$\because$ The area of the circle $= 2 \times 12.32 = 24.64 \text{ cm}^2$.

$\therefore \pi r^2 = 24.64 \qquad \therefore r^2 = 24.64 \times \frac{7}{22} = 7.84$

$\therefore r = \sqrt{7.84} = 2.8$ cm.

$\therefore$ The perimeter of the figure $= \pi r + 2r$

$= \frac{22}{7} \times 2.8 + 2 \times 2.8 = 14.4$ cm.

17

The area of the shaded part = the area of the great circle

– the area of the small circle $= \pi r_1^2 - \pi r_2^2$

$= \pi \times 25 - \pi \times 9 = 16\pi \text{ cm}^2$.

18

Let the radius length of the circle $= x$ cm.

$\therefore$ The side length of the square $= 2x$ cm.

$\therefore$ The area of the shaded part

$= \frac{\text{the area of the square} - \text{the area of the circle}}{2}$

$= \frac{4x^2 - \pi x^2}{2} = 10\frac{5}{7} = \frac{75}{7}$

$\therefore 4x^2 - \frac{22}{7} x^2 = \frac{75}{7} \times 2$

$\therefore \frac{6}{7} x^2 = \frac{150}{7} \qquad \therefore x^2 = \frac{150}{7} \times \frac{7}{6} = 25$

$\therefore x = \sqrt{25} = 5$ cm.

$\therefore$ The perimeter of the shaded part

$= \frac{1}{2}$ the circumference of the circle

$+ \frac{1}{2}$ the perimeter of the square.

$= \frac{22}{7} \times 5 + 20 = 35\frac{5}{7}$ cm.

19

In the right-angled triangle ABD at A :

$\because (AB)^2 + (AD)^2 = (BD)^2$ (but AB = AD)

$\therefore 2(AB)^2 = (14)^2 \qquad \therefore (AB)^2 = \frac{196}{2} = 98$

$\therefore AB = \sqrt{98} = 7\sqrt{2}$ cm.

$\therefore$ The area of the shaded part

$= \frac{\text{the area of the circle} - \text{the area of the square}}{4}$

$= \frac{\frac{22}{7} \times (7)^2 - 7\sqrt{2} \times 7\sqrt{2}}{4} = \frac{154 - 98}{4} = 14 \text{ cm}^2$.

The perimeter of the shaded part

$= \frac{1}{4}$ the circumference of the circle

+ side length of the square

$= \frac{1}{4} \times 2 \times \frac{22}{7} \times 7 + 7\sqrt{2} = (11 + 7\sqrt{2})$ cm.

20

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (14)^2 \times 20 = 12320 \text{ cm}^3$.

The total area of the cylinder $= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 20 + 2 \times \frac{22}{7} \times (14)^2 = 2992 \text{ cm}^2$.

21

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 924 = \frac{22}{7} \times r^2 \times 6$

$\therefore r^2 = \frac{924 \times 7}{6 \times 22} = 49 \qquad \therefore r = 7$ cm.

$\therefore$ The lateral area $= 2\pi r h = 2 \times \frac{22}{7} \times 7 \times 6$

$= 264 \text{ cm}^2$.

22

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100 \qquad \therefore r = 10$ cm.

$\therefore$ The total area $= 2\pi r h + 2\pi r^2$

$= 2 \times 3.14 \times 10 \times 24 + 2 \times 3.14 \times (10)^2 = 2135.2$ cm^2.

23

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10 = 1540$ cm^3.

The volume of the cube $= \ell^3 = (11)^3 = 1331$ cm^3.

$\therefore$ The volume of the cylinder is greater than the volume of the cube.

24

1. $2\pi r h$, $\pi r^2 h$
2. 2 cm.
3. 20 cm.
4. r cm.
5. r cm.

25

The circumference of the base $= 2\pi r$

$\therefore 44 = 2 \times \frac{22}{7} \times r$

$\therefore r = \frac{44}{2} \times \frac{7}{22} = 7$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 25 = 3850$ cm^3.

26

The lateral area $= 2\pi r h$

$\therefore 52 = 2 \times \frac{22}{7} \times 4 \times h$

$\therefore h = \frac{52 \times 7}{2 \times 22 \times 4} = \frac{91}{44}$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 4^2 \times \frac{91}{44} = 104$ cm^3.

27

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 36\pi = \pi \times r^2 \times 4$

$\therefore r^2 = \frac{36}{4} = 9 \qquad \therefore r = 3$ cm.

$\therefore$ The edge length of the cube $= 3$ cm.

$\therefore$ The total area of the cube $= 6\ell^2 = 6 \times 3^2 = 54$ cm^2.

28

$\because$ The volume of the cylinder $= \pi r^2 h$

$\because h = r$

$\therefore$ The volume of the cylinder $= \pi r^3$

$\therefore 72\pi = \pi r^3 \qquad \therefore r^3 = 72 \qquad \therefore r = 2\sqrt[3]{9}$

$\therefore$ The height of the cylinder $= 2\sqrt[3]{9}$ cm.

29

The volume of the tank = the volume of the cuboid $+ \frac{1}{2}$ of the volume of the cylinder

$= 7 \times 7 \times 14 + \frac{1}{2} \times \frac{22}{7} \times (3.5)^2 \times 14$

$= 686 + 269.5 = 955.5$ m^3.

30

The circumference of the base of the cylinder = BC

$\therefore 2\pi r = 44 \qquad \therefore r = \frac{44 \times 7}{2 \times 22} = 7$ cm.

The height = AB = 10 cm.

The volume $= \pi r^2 h = \frac{22}{7} \times (7)^2 \times 10 = 1540$ cm^3.

31

The volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.808$ cm^3.

The surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (2.1)^2 = 55.44$ cm^2.

32

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 4188 = \frac{4}{3} \times 3.141 \times r^3$

$\therefore r^3 = \frac{4188 \times 3}{4 \times 3.141} = 1000 \qquad \therefore r = 10$ cm.

33

The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 562.5\pi = \frac{4}{3}\pi r^3 \qquad \therefore r^3 = \frac{562.5 \times 3}{4} = 421.875$

$\therefore r = \sqrt[3]{421.875} = 7.5$ cm.

$\therefore$ The surface area of the sphere

$= 4\pi r^2 = 4 \times \pi \times (7.5)^2 = 225\pi$ cm^2.

34

1. b
2. a
3. c
4. c
5. b
6. d
7. b

35

The volume of the cylinder $= \pi r^2 h$

$= \pi \times (4)^2 \times 18 = 288\pi \text{ cm}^3.$

$\because$ The volume of the cylinder = the volume of the sphere.

$\therefore$ The volume of the sphere $= 288\pi \text{ cm}^3.$

$\therefore \frac{4}{3}\pi r^3 = 288\pi$

$\therefore r^3 = \frac{288 \times 3}{4} = 216$

$\therefore$ The radius length of the sphere = 6 cm.

36

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100$

$\therefore r = \sqrt{100} = 10$ cm.

, $\because$ the radius length of the sphere

= the radius length of the cylinder base

$\therefore$ The volume of the sphere $= \frac{4}{3} \times 3.14 \times (10)^3$

$= 4186\frac{2}{3} \text{ cm}^3.$

37

The volume of the cuboid $= 77 \times 24 \times 21 = 38808 \text{ cm}^3.$

$\because$ The volume of the cuboid = the volume of the sphere

$\therefore 38808 = \frac{4}{3}\pi r^3$

$\therefore r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 9261$

$\therefore r = \sqrt[3]{9261} = 21$ cm.

38

The volume of the sphere $= \frac{4}{3}\pi (3)^3 = 36\pi \text{ cm}^3.$

$\because$ The volume of the cylinder = the volume of the sphere

$\therefore$ The volume of the cylinder $= 36\pi \text{ cm}^3.$

$\therefore \pi r^2 h = 36\pi \quad \therefore 9\pi h = 36\pi \quad \therefore h = 4$ cm.

39

$\because$ The sphere touches the six faces of the cube

$\therefore$ The edge length of the cube = 2 r

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 36\pi = \frac{4}{3}\pi r^3 \quad \therefore r^3 = \frac{36 \times 3}{4} = 27$

$\therefore r = 3$ cm.

$\therefore$ The edge length of the cube $= 2 \times 3 = 6$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 6^3 = 216 \text{ cm}^3.$

40

$\because$ The volume of the sphere

= the volume of 8 small spheres

$\therefore \frac{4}{3}\pi r_1^3 = 8 \times \frac{4}{3}\pi r_2^3$

$\therefore (16.8)^3 = 8 r_2^3 \qquad \therefore r_2^3 = \frac{(16.8)^3}{8}$

$\therefore r_2 = \frac{16.8}{2} = 8.4$ cm.

41

The volume of the sphere $= \frac{4}{3}\pi (15)^3 = 4500\pi \text{ cm}^3.$

$\because$ The volume of the cylinder

$= \frac{4}{9}$ the volume of the sphere

$\therefore \pi r^2 h = \frac{4}{9} \times 4500\pi \qquad \therefore r^2 \times 20 = 2000$

$\therefore r^2 = \frac{2000}{20} = 100 \qquad \therefore r = \sqrt{100} = 10$ cm.

42

$\because$ The sum of lengths of all edges = 52 cm.

, the sum of the 4 heights $= 3 \times 4 = 12$ cm.

$\therefore$ The sum of the remained edges $= 52 - 12 = 40$ cm.

$\because$ The base is a square.

$\therefore$ The side length of the square $= \frac{40}{8} = 5$ cm.

$\therefore$ The volume $= 5 \times 5 \times 3 = 75 \text{ cm}^3.$

43

The volume of the metal = the outer volume

– the inner volume $= \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3$

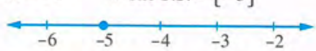
$= \frac{4}{3} \times \pi \left((3.5)^3 - (2.1)^3\right) = \frac{88}{21} \times 33.614 \approx 140.859 \text{ cm}^3.$

$\therefore$ The mass of the metal $= 140.859 \times 20 \approx 2817$ gm.

Answers of Exercise 10

1

1 $\because x = -5 \qquad \therefore$ The S.S. $= \{-5\}$

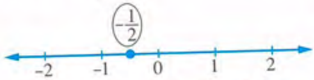


2 $\because 5x = 1 - 6 = -5 \qquad \therefore x = \frac{-5}{5} = -1$

$\therefore$ The S.S. $= \{-1\}$

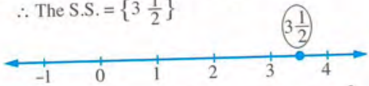
3 $\because 2x = 3 - 4 = -1 \qquad \therefore x = -\frac{1}{2}$

$\therefore$ The S.S. $= \{-\frac{1}{2}\}$



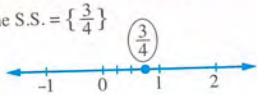
4 $\because 2x = 4 + 3 = 7$ $\therefore x = \frac{7}{2} = 3\frac{1}{2}$

$\therefore$ The S.S. $= \{3\frac{1}{2}\}$



5 $\because 4x - 1 = 2$ $\therefore 4x = 2 + 1 = 3$ $\therefore x = \frac{3}{4}$

$\therefore$ The S.S. $= \{\frac{3}{4}\}$



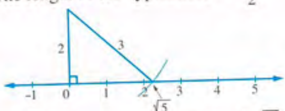
6 $\because \sqrt{5}\,x = 4 + 1 = 5$

$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5}\}$

The length of one side of the right angle $= \frac{5-1}{2} = 2$

$\therefore$ The length of the hypotenuse $= \frac{5+1}{2} = 3$

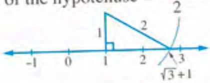


7 $\because x = \sqrt{3} + 1$ $\therefore$ The S.S. $= \{\sqrt{3} + 1\}$

The length of one side of the right angle

$= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



8 $\because 2 - \sqrt{6}\,x = 8$ $\therefore -\sqrt{6}\,x = 8 - 2 = 6$

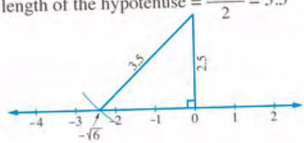
$\therefore x = -\frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{-6\sqrt{6}}{6} = -\sqrt{6}$

$\therefore$ The S.S. $= \{-\sqrt{6}\}$

The length of one side of the right angle $= \frac{6-1}{2}$

$= 2.5$

The length of the hypotenuse $= \frac{6+1}{2} = 3.5$

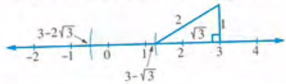


9 $\because x = 3 - 2\sqrt{3}$ $\therefore$ The S.S. $= \{3 - 2\sqrt{3}\}$

The length of one side of the right angle $= \frac{3-1}{2}$

$= 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



2

1 a 2 c 3 c 4 d 5 a 6 d

3

1 $\because x > 3$ $\therefore$ The S.S. $=]3, \infty[$

2 $\because x \le 2$ $\therefore$ The S.S. $=]-\infty, 2]$

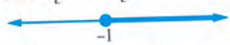
3 $\because x \le 2$ $\therefore$ The S.S. $=]-\infty, 2]$

4 $\because -x > -2$ $\therefore x < 2$

$\therefore$ The S.S. $=]-\infty, 2[$

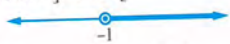
5 $\because 2x \ge -2$ $\therefore x \ge -1$

$\therefore$ The S.S. $= [-1, \infty[$



6 $\because -5x < 5$ $\therefore x > -1$

$\therefore$ The S.S. $=]-1, \infty[$

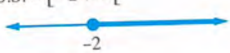


7 $\because \frac{1}{2}x \le 1$ $\therefore x \le 2$

$\therefore$ The S.S. $=]-\infty, 2]$

8 $\because -2x \le 4$ $\therefore x \ge -2$

$\therefore$ The S.S. $= [-2, \infty[$



4

1 $\because 1 < x \le 4$ $\therefore$ The S.S. $=]1, 4]$

2 $\because -8 < x < 6$ $\therefore$ The S.S. $=]-8 , 6[$

3 $\because 3 \geq x > -3$ $\therefore$ The S.S. $=]-3 , 3]$

4 $\because -4 < -x \leq -2$ $\therefore 4 > x \geq 2$

$\therefore$ The S.S. $= [2 , 4[$

5 $\because -2 \leq x + 1 \leq 3$ $\therefore -3 \leq x \leq 2$

$\therefore$ The S.S. $= [-3 , 2]$

6 $\because 2 < -x \leq 6$ $\therefore -2 > x \geq -6$

$\therefore$ The S.S. $= [-6 , -2[$

7 $\because -9 \leq 3x \leq 3$ $\therefore -3 \leq x \leq 1$

$\therefore$ The S.S. $= [-3 , 1]$

8 $\because 3 < 2x - 1 < 5$ $\therefore 4 < 2x < 6$

$\therefore 2 < x < 3$ $\therefore$ The S.S. $=]2 , 3[$

9 $\because -1 < \frac{1}{2} x \leq 2$ $\therefore -2 < x \leq 4$

$\therefore$ The S.S. $=]-2 , 4]$

10 By multiplying by 3 $\therefore 0 \leq -2x + 6 < 12$

$\therefore -6 \leq -2x < 6$ $\therefore 3 \geq x > -3$

$\therefore$ The S.S. $=]-3 , 3]$

5

Represent by yourself the S.S. on the number line :

1 $\because 3x - 2x < 4$ $\therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

2 $\because 7x - 4x \geq 9$ $\therefore 3x \geq 9$ $\therefore x \geq 3$

$\therefore$ The S.S. $= [3 , \infty[$

3 $\because 5x - 2x < 9 + 3$ $\therefore 3x < 12$ $\therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

4 $\because 7x - 5x \geq -8 + 12$ $\therefore 2x \geq 4$ $\therefore x \geq 2$

$\therefore$ The S.S. $= [2 , \infty[$

5 $\because x + x \leq 3 + 1$ $\therefore 2x \leq 4$ $\therefore x \leq 2$

$\therefore$ The S.S. $=]-\infty , 2]$

6 $\because -x + 2x \geq -3 - 1$ $\therefore x \geq -4$

$\therefore$ The S.S. $= [-4 , \infty[$

6

Represent by yourself the S.S. on the number line :

1 $\because x + 3 - x \geq 2x - x \geq x - 2 - x$

$\therefore 3 \geq x \geq -2$ $\therefore$ The S.S. $= [-2 , 3]$

2 $\because -x + x < x + x < 4 - x + x$ $\therefore 0 < 2x < 4$

$\therefore 0 < x < 2$ $\therefore$ The S.S. $=]0 , 2[$

3 $\because 4x - 4x \leq 5x + 2 - 4x < 4x + 3 - 4x$

$\therefore 0 \leq x + 2 < 3$ $\therefore -2 \leq x < 1$

$\therefore$ The S.S. $= [-2 , 1[$

4 $\because x - 1 - x < 3x - 1 - x \leq x + 1 - x$

$\therefore -1 < 2x - 1 \leq 1$ $\therefore 0 < 2x \leq 2$

$\therefore 0 < x \leq 1$ $\therefore$ The S.S. $=]0 , 1]$

5 $\because 2 + 2x - 2x \leq 3x + 3 - 2x < 5 + 2x - 2x$

$\therefore 2 \leq x + 3 < 5$ $\therefore -1 \leq x < 2$

$\therefore$ The S.S. $= [-1 , 2[$

6 By multiplying by 6

$\therefore 3x - 4 < 6x + 6 < 3x + 9$

$\therefore -4 < 3x + 6 < 9$

$\therefore -10 < 3x < 3$ $\therefore -\frac{10}{3} < x < 1$

$\therefore$ The S.S. $=]-\frac{10}{3} , 1[$

7

1 ≥ 3	2 < 3	3 < -3
4 $\geq -\frac{3}{2}$	5 $\leq 2\sqrt{2}$	6 $]2 , 4[$
7 $]-2 , 5]$	8 $]2 , \infty[$	9 6

8

1 a	2 b	3 c
4 c	5 c	

9

$\because$ The weight of one box = 45 kg.

Let the number of boxes be x

, $\because$ the maximum weight that the lift can carry is 2200 kg.

$\therefore$ The weight of boxes $\leq$ the maximum weight that the lift can carry.

$\therefore 45X \leq 2200$ $\therefore X \leq 48\frac{8}{9}$

$\therefore$ The maximum number of boxes can the lift carry in one time is 48 boxes.

10

$\because -4 < -2X < 2$ $\therefore 2 > X > -1$ $\therefore$ The S.S. $=]-1, 2[$

$\because \sqrt{3} \approx 1.7$ $\therefore \sqrt{3} \in]-1, 2[$

11

$\because a + 3 \leq X \leq b + 3$ $\therefore$ The S.S. $= [a + 3, b + 3]$

$\therefore [4, 7] = [a + 3, b + 3]$

$\therefore a + 3 = 4$ $\therefore a = 1$

$, b + 3 = 7$ $\therefore b = 4$

12

$\because \frac{1}{5} \leq \frac{2X + 1}{5} \leq 1$ $\therefore 1 \leq 2X + 1 \leq 5$

$\therefore 0 \leq 2X \leq 4$ $\therefore 0 \leq X \leq 2$

$\therefore$ The S.S. $= [0, 2]$ $\therefore m = 0, m + n = 2$ $\therefore n = 2$

13

$\because 5 \leq \frac{2X}{3} + 1 \leq 7$ $\therefore 4 \leq \frac{2X}{3} \leq 6$

$\therefore 12 \leq 2X \leq 18$ $\therefore 6 \leq X \leq 9$

$\therefore 4 \leq X - 2 \leq 7$ $\therefore$ The smallest value of $X - 2$ is 4

14

Multiply both sides by $(\sqrt{3} - \sqrt{5})$

$\therefore X \leq (\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$

"Note that the sign changed because $(\sqrt{3} - \sqrt{5})$ is a negative number because $\sqrt{3} < \sqrt{5}$ "

$\therefore X \leq -2$ $\therefore$ The S.S. $=]-\infty, -2]$

Answers of unit two

Answers of Exercise 11

1 (5 , 14) , (2 , 5) , (0 , – 1) , (– 3 , – 10)

2 The ordered pair (– 1 , 3) satisfies the relation.

3 1 (1 , – 3) , (2 , – 1) , (3 , 1) , (5 , 5)
2 (0 , 5) , (2 , 6) , (4 , 7) , (6 , 8)
3 (0 , 2) , (3 , 2) , (5 , 2) , (– 4 , 2)
4 (2.5 , 7) , (2.5 , 3) , (2.5 , – 7) , (2.5 , 4)
There are other solutions.

4 1

x	0	1	2	3
y	1	5	9	13

2

x	– 4	– 3	– 2
y	– 5	0	5

3

a	1	4	3
b	– 3	0	– 1

4

a	2	5	– 1
b	– 1	0	– 2

5 1 7 2 – 9 3 zero 4 – 1

6 ∵ (3 , 6) satisfies the relation $y = k\,x$
∴ $6 = 3\,k$ ∴ $k = 2$

7 ∵ (3 , 1) satisfies the relation $y - 3\,x = a$
∴ $1 - 3 \times 3 = a$ ∴ $a = -8$

8 ∵ (– 3 , 2) satisfies the relation $3\,x + b\,y = 1$
∴ $3 \times (-3) + b \times 2 = 1$ ∴ $2\,b = 9 + 1$
∴ $2\,b = 10$ ∴ $b = 5$

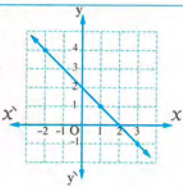
9 ∵ (3 , a) satisfies the relation $y - 2\,x = 4$
∴ $a - 2 \times 3 = 4$ ∴ $a = 10$

10 ∵ (k , 2 k) satisfies the relation $x + y = 15$
∴ $k + 2\,k = 15$ ∴ $3\,k = 15$ ∴ $k = 5$

11 1 $x = 3$ 2 y = zero

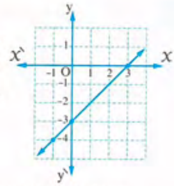
12 1

x	1	3	– 2
y	1	– 1	4



2

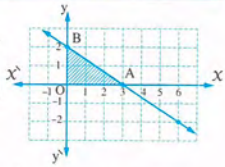
x	0	3	– 1
y	– 3	0	– 4



From 3 to 8 represent the relations graphically by yourself.

13 $y = -\frac{2}{3}x + 2$

x	0	3	6
y	2	0	– 2



From the graph :
The area of Δ OAB $= \frac{1}{2} \times 3 \times 2 = 3$ square units.

14 ∵ The straight line intersects x-axis at (3 , b)
∴ b = 0
∴ (3 , 0) satisfies the relation $2\,x - y = a$
∴ $2 \times 3 - 0 = a$ ∴ $a = 6$

15 1 b 2 c 3 b 4 b
5 d 6 a 7 b 8 a
9 d 10 c 11 c 12 c
13 c 14 c

16 Let the first number be x and the second be y
∴ $2\,x + y = 12$ ∴ $y = 12 - 2\,x$
∵ The two numbers are even natural numbers.
∴ x has the values 0 , 2 , 4 , 6 then we can register the different possibilities to the two numbers in the following table :

x	0	2	4	6
y	12	8	4	0

17 Let the length of the rectangle = x cm. and the width = y cm. ∴ $x > y$
, ∵ the perimeter of the rectangle = 14 cm.
∴ $2\,(x + y) = 14$ ∴ $x + y = 7$
We can record the different possibilities of the length and the width of the rectangle in the opposite table :

x	6	5	4
y	1	2	3

18 Let the number of bills of L.E. 5 be x , then its value = $5x$

and let the number of bills of L.E. 20 be y

, then its value = 20 y

$\therefore 5x + 20y = 65$ where x and y are natural numbers.

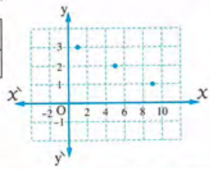
$\therefore x + 4y = 13 \qquad \therefore y = \frac{13 - x}{4}$

$\because x \le 10$, $(13 - x)$ is divisible by 4

i.e. x has the values 9 , 5 and 1

, then we can write the different possibilities in the following table :

x	1	5	9
y	3	2	1



19 Let the store sold in this week x computer's table and y chairs.

$\therefore 100x + 50y = 500$

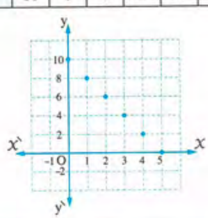
where x and y are two natural numbers.

$\therefore 2x + y = 10 \qquad \therefore y = 10 - 2x$

$\therefore x$ is not more than 5

$\therefore$ We can write the expectations which represent the number of computer's tables and the number of chairs in the following table :

x	0	1	2	3	4	5
y	10	8	6	4	2	0



20 Let the length of any of the two congruent sides in the triangle be x cm. and the length of the third side be y cm.

$\because$ the perimeter of the triangle = 19

$\therefore 2x + y = 19 \qquad \therefore y = 19 - 2x$

$\therefore x$ and y are positive integers then x is not more than 9 and from the inequality of the triangle then x has the values 5 , 6 , 7 , 8 and 9

, then we can write all the possibilities in the following table :

x	5	6	7	8	9
y	9	7	5	3	1

Answers of Exercise 12

1

figure (1) the slope is positive.

figure (2) the slope is negative.

figure (3) the slope is undefined.

figure (4) the slope equals zero.

2

1 negative 2 zero 3 undefined 4 positive

3

1 zero 2 undefined 3 x-axis 4 $\overline{BC}$ or $\overline{AC}$

4

1 the slope of $\overleftrightarrow{AB} = \frac{4-3}{3-1} = \frac{1}{2}$

2 the slope of $\overleftrightarrow{AB} = \frac{0-2}{5-1} = -\frac{1}{2}$

3 the slope of $\overleftrightarrow{AB} = \frac{5-2}{6-3} = 1$

4 the slope of $\overleftrightarrow{AB} = \frac{-1+1}{4-2} = 0$

5 the slope of $\overleftrightarrow{AB} = \frac{3-3}{2-1} = 0$

6 the slope of $\overleftrightarrow{AB} = \frac{4-2}{5-5} = \frac{2}{\text{zero}}$ undefined.

7 the slope of $\overleftrightarrow{AB} = \frac{2+1}{3-3} = \frac{3}{\text{zero}}$ undefined.

8 the slope of $\overleftrightarrow{AB} = \frac{1+2}{4-3} = \frac{3}{1} = 3$

9 the slope of $\overleftrightarrow{AB} = \frac{1-3}{2+1} = \frac{-2}{3}$

10 the slope of $\overleftrightarrow{NK} = \frac{-7+2}{-1-4} = \frac{-5}{-5} = 1$

11 the slope of $\overleftrightarrow{EO} = \frac{0+1}{0+3} = \frac{1}{3}$

12 the slope of $\overleftrightarrow{AB} = \frac{-1+9}{-1+6} = \frac{8}{5}$

5

1 Taking the two points (0 , 0) , (1 , 2) which lie on the straight line we find that :

the slope $= \frac{2-0}{1-0} = 2$

2 Taking the two points (0 , – 1) , (– 2 , 3) which lie on the straight line we find that :

the slope $= \frac{3-(-1)}{-2-0} = \frac{4}{-2} = -2$

6

$\because m(\angle M) = 45°$ $\therefore \Delta MNL$ is an isosceles triangle.

$\therefore ML = LN$

$\because$ The length of $\overline{ML} = 4$ units.

$\therefore$ The length of $\overline{LN} = 4$ units.

$\therefore N = (3 , 6)$

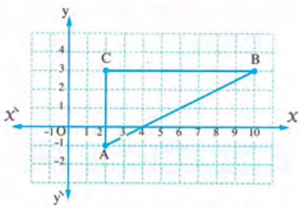
, the slope of $\overleftrightarrow{MN} = \frac{6-2}{3-7} = \frac{4}{-4} = -1$

7

the slope of $\overleftrightarrow{AB} = \frac{3+1}{10-2} = \frac{1}{2}$

the slope of $\overleftrightarrow{BC} = \frac{3-3}{2-10} = $ zero

the slope of $\overleftrightarrow{AC} = \frac{3+1}{2-2} = \frac{4}{0}$ (undefined)



from the graph we find that ΔABC is right-angled.

8

$\because$ The slope of the straight line which passes through the two points (1 , 3) and (3 , k) equals 3

$\therefore \frac{k-3}{3-1} = 3$ $\therefore \frac{k-3}{2} = 3$

$\therefore k - 3 = 6$ $\therefore k = 9$

9

$\because$ The slope of the straight line which passes through the two points (3 , c) and (5 , – 2) equals – 3

$\therefore \frac{-2-c}{5-3} = -3$ $\therefore \frac{-2-c}{2} = -3$

$\therefore -2 - c = -6$ $\therefore c = 4$

10

$-2 = \frac{2-4}{x-(-1)}$ $\therefore -2 = \frac{-2}{x+1}$

$\therefore x + 1 = 1$ $\therefore x = 0$

11

$\because \frac{-1-y}{3-(-2)} = -0.6$ $\therefore \frac{-1-y}{5} = -0.6$

$\therefore -1 - y = -3$ $\therefore y = 2$

12

$\because$ The straight line is parallel to x-axis

$\therefore$ The slope = zero $\therefore \frac{k-4}{2-3} = $ zero

$\therefore k - 4 = $ zero. $\therefore k = 4$

13

$\because$ The straight line is parallel to y-axis

$\therefore$ The slope is undefined

$\therefore x_2 - x_1 = $ zero $\therefore 6 - 2x = 0$

$\therefore -2x = -6$ $\therefore x = 3$

14

$\because$ The straight line is perpendicular to y-axis

$\therefore$ The straight line is parallel to x-axis

$\therefore$ The slope = zero $\therefore y_2 - y_1 = $ zero

$\therefore 3y - 6 = 0$ $\therefore 3y = 6$ $\therefore y = 2$

15

$\because$ The slope of the straight line passing through the two points (– 5 , 11) and (0 , 8) $= \frac{8-11}{0-(-5)} = \frac{-3}{5}$ (1)

$\because$ The slope of the straight line passing through the two points (0 , 8) and (5 , 5) $= \frac{5-8}{5-0} = \frac{-3}{5}$ (2)

from (1) and (2) we find that the three points are collinear.

(lying on the straight line whose slope $= -\frac{3}{5}$)

16

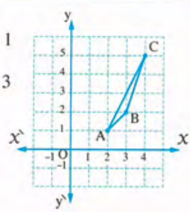
The slope of $\overleftrightarrow{AB} = \frac{2-1}{3-2} = 1$

, the slope of $\overleftrightarrow{BC} = \frac{5-2}{4-3} = 3$

and the slope of $\overleftrightarrow{AC}$

$= \frac{5-1}{4-2} = 2$

we observe that the three points are not collinear.



17

1 $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-1}{2-1} = 1$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-3-2}{-3-2} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

2 $\because$ The slope of $\overleftrightarrow{AB} = \frac{7-(-3)}{-6-4} = \frac{10}{-10} = -1$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-4-7}{5-(-6)} = \frac{-11}{11} = -1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

3 $\because$ The slope of $\overleftrightarrow{AB} = \frac{4-12}{2-(-2)} = \frac{-8}{4} = -2$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-4-4}{6-2} = \frac{-8}{4} = -2$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

18

1 $\because$ The slope of $\overleftrightarrow{AB} = \frac{0-1}{3-2} = \frac{-1}{1} = -1$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-1-0}{5-3} = -\frac{1}{2}$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

$\therefore$ The points A , B and C are not collinear.

2 $\because$ The slope of $\overleftrightarrow{AB} = \frac{1-2}{3-(-1)} = -\frac{1}{4}$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{2-1}{7-3} = \frac{1}{4}$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

$\therefore$ The points A , B and C are not collinear.

3 $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-(-3)}{2-0} = \frac{5}{2}$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-3-2}{-3-2} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

$\therefore$ The points A , B and C are not collinear.

19

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{5-1}{2-8} = -\frac{2}{3}$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

$\therefore C \notin \overleftrightarrow{AB}$

20

$\because$ The slope of the straight line which passes through the two points (4 , 1) and (– 2 , 7)

$= \frac{7-1}{-2-4} = \frac{6}{-6} = -1$

$\because$ The slope of the straight line which passes through the two points (– 2 , 7) and (3 , y)

$= \frac{y-7}{3-(-2)} = \frac{y-7}{5}$

$\because$ The three points are collinear.

$\therefore \frac{y-7}{5} = -1$ $\qquad \therefore y-7 = -5$

$\therefore y = -5+7$ $\qquad \therefore y = 2$

21

$\because$ The slope of the straight line which passes through the two points (3 , – 1) and (x , 1) equals $\frac{2}{3}$

$\therefore \frac{1-(-1)}{x-3} = \frac{2}{3}$ $\qquad \therefore \frac{2}{x-3} = \frac{2}{3}$

$\therefore x-3 = 3$ $\qquad \therefore x = 6$

$\because$ The slope of the straight line which passes through the two points (3 , – 1) and (9 , y) equals $\frac{2}{3}$

$\therefore \frac{y-(-1)}{9-3} = \frac{2}{3}$ $\qquad \therefore \frac{y+1}{6} = \frac{2}{3}$ $\qquad \therefore 3y+3 = 12$

$\therefore 3y = 9$ $\qquad \therefore y = 3$

Answers of Exercise 13

1

$\because$ The uniform velocity $= \frac{\text{the covered distance}}{\text{the taken time}}$

$= \frac{180}{3} = 60$ km./hr.

$\therefore$ The covered distance = the taken time × the uniform velocity = 60 × 5 = 300 km.

2

$\because$ The rate of consumption of fuel

$= \frac{\text{the amount of consumed fuel}}{\text{time}}$

$= \frac{2.47}{3} = \frac{247}{300}$ litre/hr.

$\therefore$ The consumed amount

= the rate of consumption × time

$= \frac{247}{300} \times 10 = 8\frac{7}{30}$ litre.

3

1 At the moment of starting the motion, the body is at a distance of 2 metres from the fixed point.

At t = 6 , the body is at a distance of 8 metres. Taking the two points (0 , 2) and (6 , 8) on the straight line which represents the relation between t and d

$\therefore$ the slope $= \frac{8-2}{6-0} = \frac{6}{6} = 1$

it represents the velocity of the body within a going trip.

2 At the moment of starting the motion , the body is at t distance of 12 metres from the fixed point.

At t = 6 the body is at a distance of 2 metres. Taking the two points (0 , 12) and (6 , 2) on the straight line representing the relation between t and d

$\therefore$ The slope $= \frac{2-12}{6-0} = \frac{-10}{6} = -\frac{5}{3}$

It represents the velocity of the body within the returning back.

3 On starting the motion , the body is at a distance of 8 metres from the fixed point.

At t = 6 the body is at a distance of 8 metres.

$\because$ The straight line representing the relation is horizontal. $\therefore$ The slope = zero

It means that the body is rest.

4

Taking two points on the straight line representing the relation between t and d say (0 , 50) and (4 , 150)

$\therefore$ The uniform velocity = the slope of the straight line

$= \frac{150-50}{4-0} = \frac{100}{4} = 25$ km./hr.

5

1 Taking two points on the straight line representing the relation between t and d say (0 , 50) and (2 , 200)

$\therefore$ The unifrom velocity = the slope of the straight line $= \frac{200-50}{2-0} = 75$ km./hr.

2 from the graph :

The car is at a distance = 275 km. from the point 0 after passing 3 hours from the moment of beginning the motion.

6

1 The velocity within the first 3 hours = the slope of the striaght line $\overleftrightarrow{OB} = \frac{125}{3} = 41\frac{2}{3}$ km./hr.

The velocity within the next 2 hours = the slope of the straight line $\overleftrightarrow{BC} = \frac{125}{2} = 62\frac{1}{2}$ km./hr.

2 The average velocity within the all trip

$= \frac{\text{total distance}}{\text{total time}} = \frac{250}{5} = 50$ km./hr.

7

1 The velocity within the first 3 hours = the slope of the striaght line $= \frac{60-20}{3-0} = \frac{40}{3} = 13\frac{1}{3}$ km./hr.

2 The velocity within the next 4 hours = the slope of the straight line $= \frac{0-60}{7-3} = \frac{-60}{4} = -15$ km./hr.

The negative sign means that the bicycle returns back with velocity 15 km./hr.

The total distance = 40 + 60 = 100 km.

8

1 The slope of the straight line $\overleftrightarrow{AB}$

$= \frac{60-20}{4-0} = \frac{40}{4} = 10$

It means the increasing of the capital within the first 4 years with rate equals 10 thousands pounds/year.

The slope of $\overleftrightarrow{BC} = \frac{60-60}{6-4} =$ zero

It means constancy of the capital within the fifth and sixth years.

The slope of $\overleftrightarrow{CD} = \frac{50-60}{8-6} = \frac{-10}{2} = -5$

It means decreasing of the capital within the 7^{th} and 8^{th} years with rate = 5 thousands/year.

2 The starting capital of the company

= 20 thousand pounds.

9

1 The slope of $\overleftrightarrow{AB} = \frac{125-50}{8-0} = \frac{75}{8} = 9\frac{3}{8}$

It means that the increase in height goes with respect to the increase in age.

The slope of $\overleftrightarrow{BC} = \frac{175-125}{18-8} = \frac{50}{10} = 5$

It means that the increase in height goes with respect to the increase in age but with a rate less than the rate within the first 8 years.

The slope of $\overleftrightarrow{CD} = \frac{175-175}{22-18} = 0$

It denotes the constancy in height inspite of the increase in age after 18 years.

2 $\because$ The height of the person at age 30 years = 175 cm. and the height of the person at age 8 years = 125 cm.

$\therefore$ The difference = 175 – 125 = 50 cm.

10

1 The greatest capacity of the tank = 70 litre.

2 The tank will be empty after 30 hours.

3 The remained fuel = 35 litre.

4 Taking the two points (0 , 70) , (30 , 0) on the straight line representing the relation.

$\therefore$ The rate of consumption of the fuel =

The slope of the straight line = $\frac{70-0}{0-30} = -2\frac{1}{3}$ litre/hr.

The negative sign means the rate of consumption.

i.e. The amount of fuel is consumpted with rate $2\frac{1}{3}$ litre/hr.

11

1 100 pages.

2 Taking the two points (0 , 100) and (3 , 40) on the straight line representing the relation.

we find that the rate of decreasing the number of pages = the slope of the straight line

$= \frac{40-100}{3-0} = \frac{-60}{3} = -20$ pages/hr.

The negative sign expresses the decreasing in the number of remained pages with rate 20 page/h.

3 $\because$ The remained pages decreases with rate 20 page/h.

$\because$ There are 100 pages.

$\therefore$ The person finishes reading the book after $\frac{100}{20} = 5$ hours.

12

1 The depth of the well before beginning digging = 5 m.

2 The depth of the well after finishing digging = 40 m.

3 The total time taken in digging = 10 hr.

4 The average of digging the well in the first 5 hours = the slope of the straight line = $\frac{27.5-5}{5-0} = 4.5$ m/hr.

5 The average of digging in the last two hours

= the slope of the straight line

$= \frac{40-27.5}{10-8} = 6.25$ m/hr.

13

1 The uniform velocity during the going trip = the slope of the straight line = $\frac{60-0}{3-0} = 20$ km./hr.

2 The average velocity during returning back

$= \frac{\text{total distance}}{\text{total time}} = \frac{60}{5} = 12$ km./hr.

3 It means that the bicycle stopped within the 6^{th} hour from the beginning.

14

Let the covered distance be d km

The amount of the remained fuel in the tank be y litre.

At the beginning the covered distance = zero

The amount of remained fuel = 40 litre

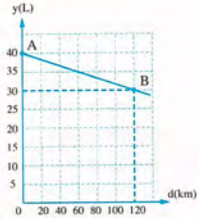
We express this by the point A (0 , 40)

$\because$ After covering distance.

The amount of remained fuel

$= \frac{3}{4} \times 40 = 30$ litre.

We expresses this by the point B (120 , 30)



$\therefore$ The rate of decreasing the amount of fuel

= the slope of $\overleftrightarrow{AB}$

$= \frac{30-40}{120-0} = \frac{-10}{120} = -\frac{1}{12}$

$\therefore$ The amount of fuel decreases with the rate one litre for every 12 km.

$\therefore$ The distance covered by the car when the tank becomes empty = $12 \times 40 = 480$ km.

15

1 100 km.

2 The train A took 2 hours

the train B took $2\frac{1}{2}$ hours.

3 The average speed = $\frac{\text{total distance}}{\text{total time}}$

with respect to the train A

The average speed = $\frac{100}{2} = 50$ km./hr.

with respect to the train B

the average speed = $\frac{100}{2.5} = 40$ km./hr.

4 It means that the train A was at rest from half past ten till half past eleven.

16

1 Tortoise

2 The velocity of the tortoise = $\frac{\text{the covered distance}}{\text{the taken time}}$

$= \frac{100}{60} = 1\frac{2}{3}$ metre / minute.

3 The average velocity of the rabbit = $\frac{\text{total distance}}{\text{total time}}$

$= \frac{100}{65} = 1\frac{7}{13}$ metre / minute.

4 It means that the rabbit was at rest from the tenth minute to 60^{th} minute.

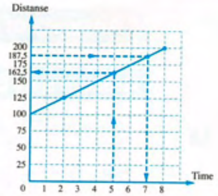
17

1 The velocity of the bicycle = the slope of the straight line $= \frac{200 - 125}{8 - 2} = 12.5$ km./h.

2 300 minutes = 5 hours ,
then the bicycle is at
distance = 162.5 km.

3 7 hours.

4 From the graph.
the starting point is
far from the fixed
point = 100 km.



Answers of unit three

Answers of Exercise 14

1

Sets	Tallies	Freq.
25 –	~~||||~~	5
30 –	~~||||~~ ~~||||~~ |||	13
35 –	~~||||~~ ~~||||~~ ~~||||~~ |	16
40 –	~~||||~~	5
45 –	|	1
Total		40

Sets	Freq.
25 –	5
30 –	13
35 –	16
40 –	5
45 –	1
Total	40

2

Sets	Tallies	Freq.
30 –	||||	4
40 –	~~||||~~	5
50 –	~~||||~~ ||	7
60 –	~~||||~~ |||	8
70 –	~~||||~~ |	6
80 –	||||	4
90 –	~~||||~~ |	6
Total		40

Sets	Freq.
30 –	4
40 –	5
50 –	7
60 –	8
70 –	6
80 –	4
90 –	6
Total	40

The set which has the highest frequency is 60 –
The sets which have the lowest frequency are 80 – , 30 –

3

1

Sets	Tallies	Freq.
20 –	|||	3
24 –	||	2
28 –	~~||||~~ |	6
32 –	~~||||~~ ||	7
36 –	~~||||~~ ~~||||~~ ||	12
Total		30

Sets	Freq.
20 –	3
24 –	2
28 –	6
32 –	7
36 –	12
Total	30

2 12 students.

4

Sets	Tallies	Freq.
0 –	||	2
4 –	~~||||~~ ||	7
8 –	~~||||~~ ~~||||~~ ||	12
12 –	~~||||~~ ~~||||~~ ~~||||~~	15
16 –	||||	4
Total		40

Sets	Freq.
0 –	2
4 –	7
8 –	12
12 –	15
16 –	4
Total	40

The percentage of those who obtained 12 marks at least $= \frac{19}{40} \times 100 = 47.5\ \%$

5

1 The least height = 112 cm. and the greatest height = 199 cm.

The range = 199 – 112 = 87 cm.

2

Sets	Tallies	Freq.
110 –	||	2
120 –	|||	3
130 –	|||	3
140 –	~~||||~~ |	6
150 –	~~||||~~ ||||	9
160 –	~~||||~~ |||	8
170 –	~~||||~~ ||	7
180 –	~~||||~~ ||	7
190 –	~~||||~~	5
Total		50

Sets	Freq.
110 –	2
120 –	3
130 –	3
140 –	6
150 –	9
160 –	8
170 –	7
180 –	7
190 –	5
Total	50

6

Sets	Tallies	Freq.
165 –	~~||||~~ |||	8
170 –	~~||||~~ ~~||||~~	10
175 –	~~||||~~ ~~||||~~ ~~||||~~	15
180 –	~~||||~~ |	6
185 –	~~||||~~ ~~||||~~	10
190 –	||||	4
195 –	|	1
200 –	|	1
Total		55

Sets	Freq.
165 –	8
170 –	10
175 –	15
180 –	6
185 –	10
190 –	4
195 –	1
200 –	1
Total	55

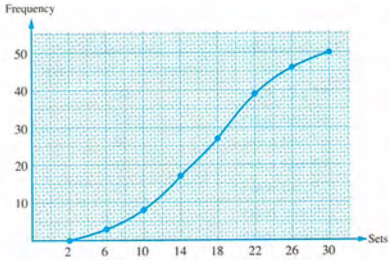
1 39 soldiers. 2 22 soldiers.

Answers of Exercise 15

First : Problems on the ascending cumulative frequency curve.

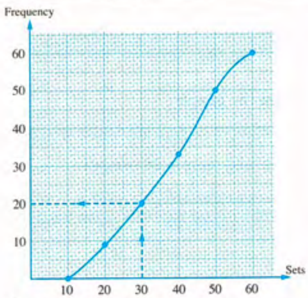
1

The upper boundaries of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



2

The upper boundaries of sets	Ascending cumulative frequency
less than 10	0
less than 20	9
less than 30	20
less than 40	33
less than 50	50
less than 60	60



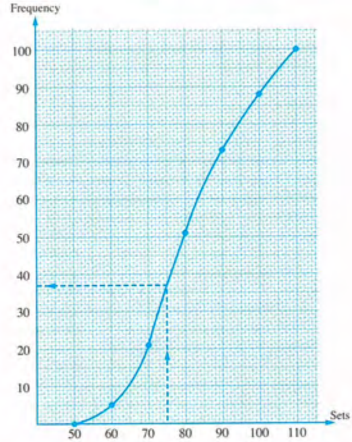
From the graph :

The number of failed pupils = 20 pupils.

3

1

The upper boundaries of sets	Ascending cumulative frequency
less than 50	0
less than 60	5
less than 70	21
less than 80	51
less than 90	73
less than 100	88
less than 110	100



2 **From the graph :** The number of factories which work less than 75 hours = 37 factories.

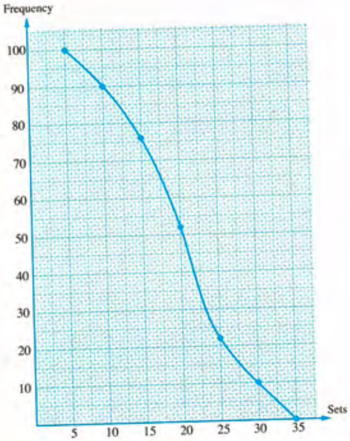
3 The percentage of the number of factories which work less than 75 hours

$$= \frac{37}{100} \times 100 \% = 37 \%$$

Second : Problems on the descending cumulative frequency curve.

4

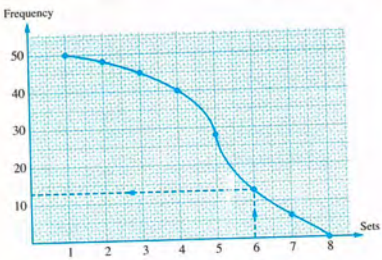
The lower limits of sets	Descending cumulative frequency
5 and more	100
10 and more	90
15 and more	76
20 and more	52
25 and more	22
30 and more	10
35 and more	0



5

1

The lower boundaries of sets	Descending cumulative frequency
1 and more	50
2 and more	48
3 and more	45
4 and more	40
5 and more	28
6 and more	13
7 and more	6
8 and more	0

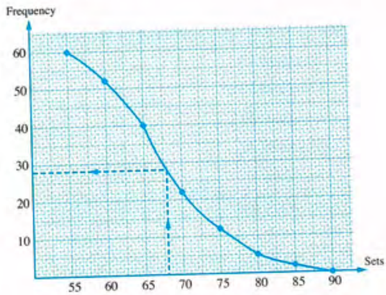


2 **From the graph :** The number of pupils who study 6 hours and more daily = 13 pupils.

3 The percentage of the number of pupils who study 6 hours and more daily = $\frac{13}{50} \times 100\ \% = 26\ \%$

6 The missed value in the table = 10

The lower limits of sets	Descending cumulative frequency
55 and more	60
60 and more	52
65 and more	40
70 and more	22
75 and more	12
80 and more	5
85 and more	2
90 and more	0



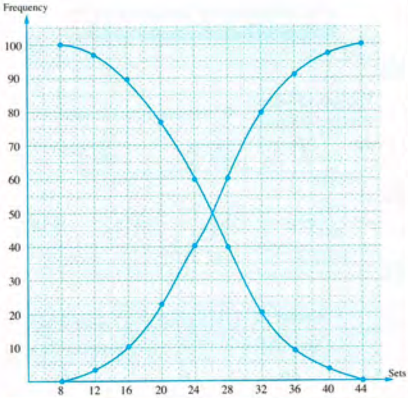
From the graph :

The number of persons whose weights are 68 kg. and more = 28 persons.

Third : Problems on the two curves together.

7

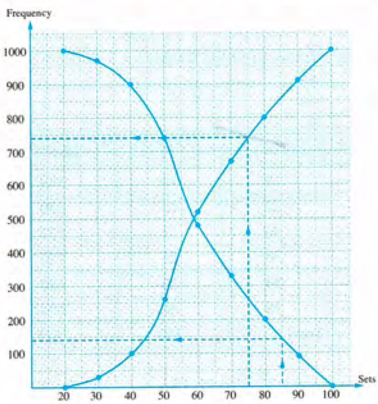
The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 8	0	8 and more	100
less than 12	4	12 and more	96
less than 16	11	16 and more	89
less than 20	23	20 and more	77
less than 24	41	24 and more	59
less than 28	61	28 and more	39
less than 32	80	32 and more	20
less than 36	91	36 and more	9
less than 40	97	40 and more	3
less than 44	100	44 and more	0



8

1

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 20	0	20 and more	1000
less than 30	30	30 and more	970
less than 40	100	40 and more	900
less than 50	260	50 and more	740
less than 60	520	60 and more	480
less than 70	670	70 and more	330
less than 80	800	80 and more	200
less than 90	910	90 and more	90
less than 100	1000	100 and more	0



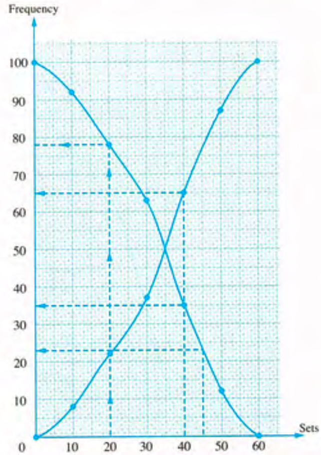
2 740 students. 3 140 students.

9

1

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 0	0	0 and more	100
less than 10	8	10 and more	92
less than 20	22	20 and more	78
less than 30	37	30 and more	63
less than 40	65	40 and more	35
less than 50	88	50 and more	12
less than 60	100	60 and more	0

2



3 **From the graph :** The number of students who got less than 40 marks = 65 students and the number of students who got 40 marks or more = 35 students.

4 The number of students who got 20 marks or more $= 78$ and their percentage $= \frac{78}{100} \times 100\ \% = 78\ \%$

5 The number of students who got 45 marks or more $= 23$ students and their percentage $= \frac{23}{100} \times 100\ \% = 23\ \%$

10

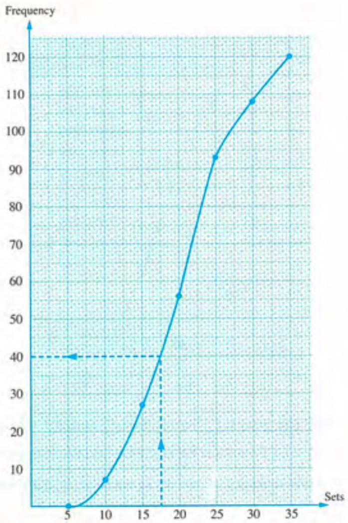
1 The frequency distribution table.

Sets	5 –	10 –	15 –	20 –	25 –	30 –	Total
Frequency	7	20	29	37	15	12	120

2 The ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	7
less than 15	27
less than 20	56
less than 25	93
less than 30	108
less than 35	120

3



4 **From the graph :** The number of workers whose experience years are less than 17.5 years = 40 workers.

Answers of Exercise 16

1

1 $\frac{\text{The sum of values}}{\text{Number of values}}$ 2 Its lower limit, its upper limit.

3 10 4 11 5 14 6 3940

2

1 c 2 d 3 c 4 b 5 a

3

Sets	Centre of Sets "x"	Frequency "f"	Center of sets × frequency "$x \times f$"
5 –	10	6	60
15 –	20	8	160
25 –	30	4	120
35 –	40	2	80
Total		20	420

$\therefore$ The mean $= \frac{420}{20} = 21$

4 1

Sets	"x"	"f"	"$x \times f$"
10 –	15	1	15
20 –	25	2	50
30 –	35	4	140
40 –	45	2	90
50 –	55	1	55
Total		10	350

$\therefore$ The mean of marks of students $= \frac{350}{10} = 35$ marks.

2 The number of failed students = 3 students.

5

Sets	"x"	"f"	"$x \times f$"
16 –	18	10	180
20 –	22	15	330
24 –	26	22	572
28 –	30	25	750
32 –	34	20	680
36 –	38	8	304
Total		100	2816

$\therefore$ The mean $= \frac{2816}{100} = 28.16$

6

Sets	"X"	"f"	"$X \times f$"
15 –	20	2	40
25 –	30	3	90
35 –	40	5	200
45 –	50	8	400
55 –	60	6	360
65 –	70	4	280
75 –	80	2	160
Total		30	1530

$\therefore$ The mean $= \frac{1530}{30} = 51$

7

Sets	"X"	"f"	"$X \times f$"
140 –	142	12	1704
144 –	146	20	2920
148 –	150	38	5700
152 –	154	22	3388
156 –	158	17	2686
160 –	162	11	1782
Total		120	18180

$\therefore$ The mean $= \frac{18180}{120} = 151.5$ cm.

8 1

Sets	"X"	"f"	"$X \times f$"
1 –	1.5	2	3
2 –	2.5	3	7.5
3 –	3.5	5	17.5
4 –	4.5	12	54
5 –	5.5	15	82.5
6 –	6.5	7	45.5
7 –	7.5	6	45
Total		50	255

$\therefore$ The mean of the number of hours of study

$= \frac{255}{50} = 5.1$ hours.

2 The number of pupils who study less than 4 hours daily $= 2 + 3 + 5 = 10$ pupils.

9

1 25 – , 10

2

Sets	"X"	"f"	"$X \times f$"
5 –	10	3	30
15 –	20	10	200
25 –	30	12	360
35 –	40	10	400
45 –	50	5	250
Total		40	1240

$\therefore$ The mean $= \frac{1240}{40} = 31$ marks.

3 The number of students whose marks are not less than 35 = 15 students.

10 The missed number is 5

Sets	"X"	"f"	"$X \times f$"
6 –	8	2	16
10 –	12	3	36
14 –	16	5	80
18 –	20	8	160
22 –	24	6	144
26 –	28	4	112
30 –	32	2	64
Total		30	612

$\therefore$ The mean $= \frac{612}{30} = 20.4$ kg.

11

1 $X = 30$

, $k + 2 = 100 - (10 + 17 + 20 + 32 + 4) = 17$

$\therefore k = 15$

2

Sets	"X"	"f"	"$X \times f$"
10 –	15	10	150
20 –	25	17	425
30 –	35	20	700
40 –	45	32	1440
50 –	55	17	935
60 –	65	4	260
Total		100	3910

$\therefore$ The mean $= \frac{3910}{100} = 39.1$

12

1 $3k + 4k = 50 - (7 + 10 + 8 + 4)$

$\therefore 7k = 21 \qquad \therefore k = \frac{21}{7} = 3$

2

Sets	"x"	"f"	"$x \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

$\therefore$ The mean $= \frac{2200}{50} = 44$ kg.

13

1 $k - 2 = 50 - (4 + 5 + 8 + 7 + 5 + 1)$

$\therefore k - 2 = 20 \qquad \therefore k = 22$

2

Sets	"x"	"f"	"$x \times f$"
2 –	4	4	16
6 –	8	5	40
10 –	12	8	96
14 –	16	20	320
18 –	20	7	140
22 –	24	5	120
26 –	28	1	28
Total		50	760

$\therefore$ The mean $= \frac{760}{50} = 15.2$ days.

14 $\because$ The total of marks of the student in 5 months $= 5 \times 23.8 = 119$ marks.

, let the required mark of the sixth month be x

$\therefore \frac{119 + x}{6} = 24 \quad \therefore 119 + x = 144$

$\therefore x = 144 - 119 = 25$ marks.

$\therefore$ The mark of the student in the 6th month is 25

15 $\because$ The total of marks of Magdi in 4 exams $= 4 \times 16 = 64$ marks.

, let the required mark be x $\therefore \frac{64 + x}{5} = 18$

$\therefore 64 + x = 90 \quad \therefore x = 90 - 64 \quad \therefore x = 26$ marks.

$\therefore$ The mark of Magdi in the 5th exam should be 26 marks.

16

1 $a = \frac{0 + 4}{2} = 2 \quad , b = \frac{90}{6} = 15 \quad , c = \frac{300}{30} = 10$

$\therefore \frac{4 + d}{2} = 6 \qquad \therefore d = 8$

$, e = \frac{16 + 12}{2} = 14 \; , f = \frac{16 + 20}{2} = 18$

$, x = 10 \times 18 = 180$

$, y = 1140 - (10 + 90 + 300 + 180) = 560$

$, z = \frac{560}{14} = 40$

$, m = 5 + 15 + 30 + 40 + 10 = 100$

2 The mean $= \frac{1140}{100} = 11.4$ marks.

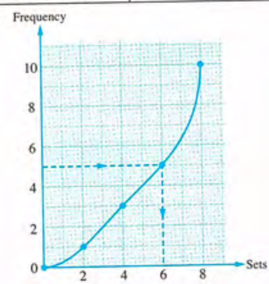
Answers of Exercise 17

1

1 b 2 c 3 a 4 d
5 b 6 d 7 d

2

The upper limits of sets	Ascending cumulative frequency
less than 0	0
less than 2	1
less than 4	3
less than 6	5
less than 8	10

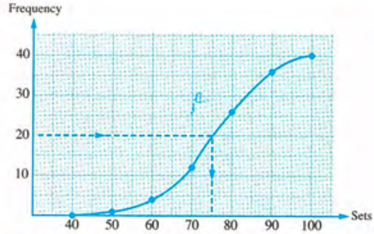


$\because$ The order of the median $= \frac{10}{2} = 5$

$\therefore$ The median $= 6$

3

The upper boundaries of sets	Ascending cumulative frequency
less than 40	0
less than 50	1
less than 60	4
less than 70	12
less than 80	26
less than 90	36
less than 100	40

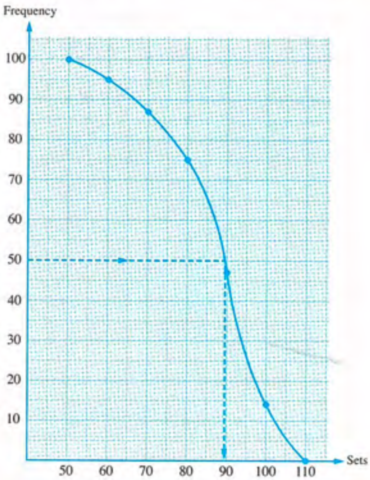


∵ The order of the median $= \frac{40}{2} = 20$

∴ The percentage of intelligence ≈ 75%

4

The lower boundaries of sets	Descending cumulative frequency
50 and more	100
60 and more	95
70 and more	87
80 and more	75
90 and more	47
100 and more	14
110 and more	0

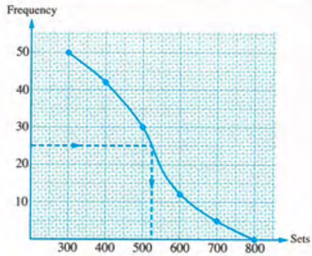


∵ The order of the median $= \frac{100}{2} = 50$

∴ The median of working hours ≈ 89.5 hours.

5

The lower boundaries of sets	Descending cumulative frequency
300 and more	50
400 and more	42
500 and more	30
600 and more	12
700 and more	5
800 and more	0

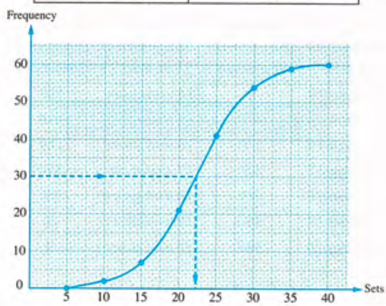


∵ The order of the median $= \frac{50}{2} = 25$

∴ The median wage = 520 pounds.

6

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	2
less than 15	7
less than 20	21
less than 25	41
less than 30	54
less than 35	59
less than 40	60



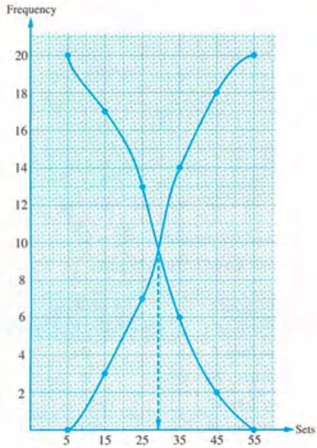
∵ The order of the median $= \frac{60}{2} = 30$

∴ The median mark ≈ 22 marks.

7

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 15	3
less than 25	7
less than 35	14
less than 45	18
less than 55	20

The lower limits of sets	Descending cumulative frequency
5 and more	20
15 and more	17
25 and more	13
35 and more	6
45 and more	2
55 and more	0

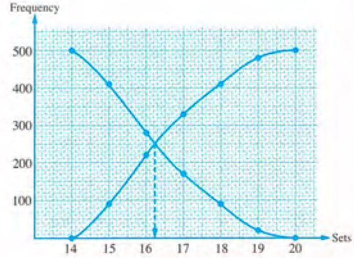


From the graph we find that the median ≈ 29 kg.

8

The upper boundaries of sets	Ascending cumulative frequency
less than 14	0
less than 15	90
less than 16	220
less than 17	330
less than 18	410
less than 19	480
less than 20	500

The lower boundaries of sets	Descending cumulative frequency
14 and more	500
15 and more	410
16 and more	280
17 and more	170
18 and more	90
19 and more	20
20 and more	0

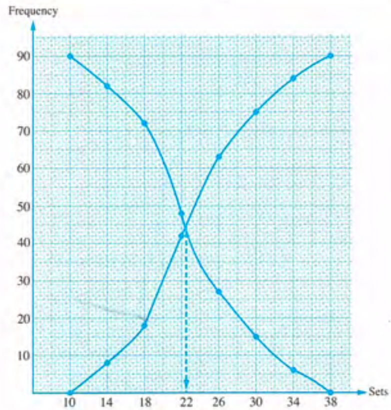


∴ The median age ≈ 16.3 years.

9

The upper limits of sets	Ascending cumulative frequency
less than 10	0
less than 14	8
less than 18	18
less than 22	42
less than 26	63
less than 30	75
less than 34	84
less than 38	90

The lower limits of sets	Descending cumulative frequency
10 and more	90
14 and more	82
18 and more	72
22 and more	48
26 and more	27
30 and more	15
34 and more	6
38 and more	0



From the graph we find that the median mark ≈ 22.5 marks

10

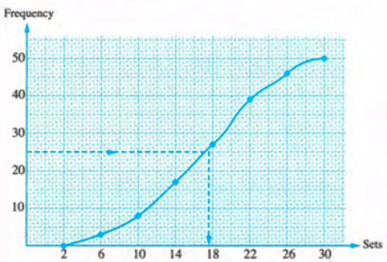
1

Sets	"x"	"f"	"$x \times f$"
2 –	4	3	12
6 –	8	5	40
10 –	12	9	108
14 –	16	10	160
18 –	20	12	240
22 –	24	7	168
26 –	28	4	112
Total		50	840

$\therefore$ The mean $= \frac{840}{50} = 16.8$

2 We form the ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ The median ≈ 17.6

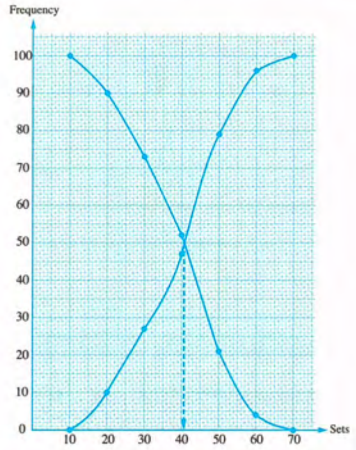
11

1 $x = 30$, $k + 2 = 100 - (10 + 17 + 20 + 32 + 4)$

$\therefore k + 2 = 17 \qquad \therefore k = 15$

2

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 10	0	10 and more	100
less than 20	10	20 and more	90
less than 30	27	30 and more	73
less than 40	47	40 and more	53
less than 50	79	50 and more	21
less than 60	96	60 and more	4
less than 70	100	70 and more	0



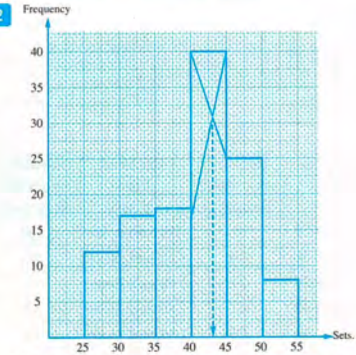
The median ≈ 41

Answers of Exercise 18

1 1 b 2 b 3 a 4 c

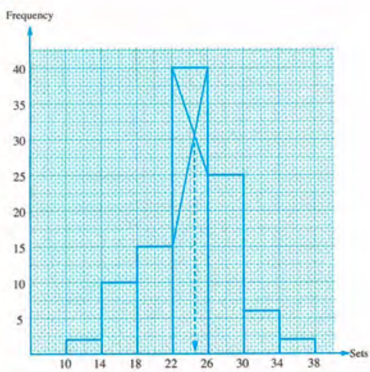
5 d 6 b 7 d

2



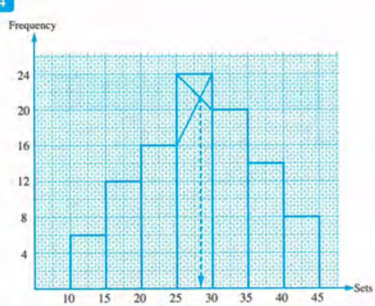
From the graph : The mode age = 43 years.

3



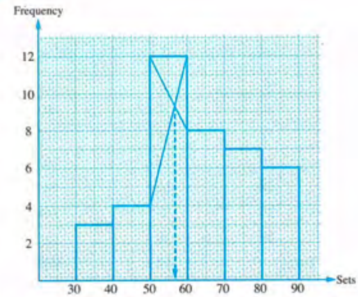
From the graph : The mode mark ≈ 24.5 marks.

4



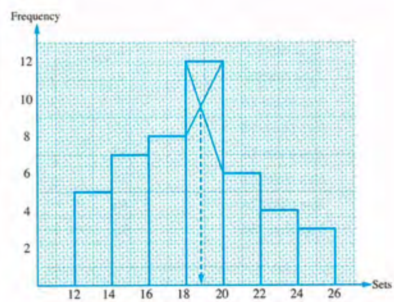
From the graph : The mode wage ≈ 28.5 pounds.

5



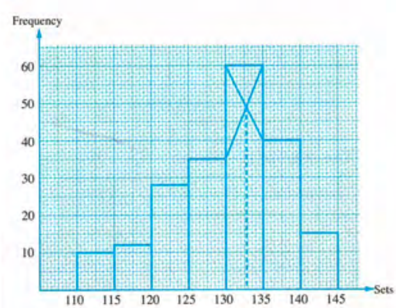
From the graph : The mode = 57

6



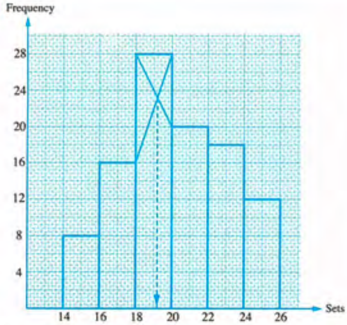
From the graph : The mode age ≈ 18.8 years.

7



From the graph : The mode height ≈ 132.75 cm.

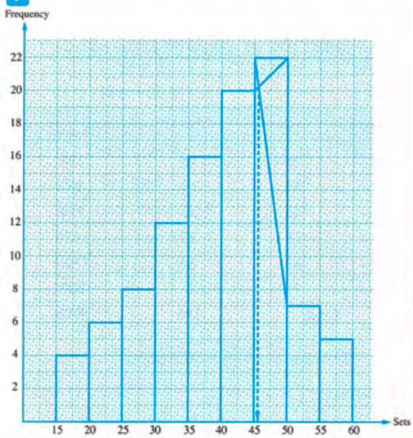
8



From the graph :

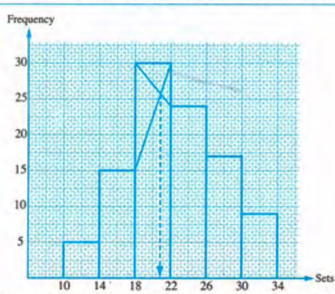
The mode of the amount of milk ≈ 19.2 galoons.

9



From the graph : The mode mark ≈ 45.5 marks.

10



From the graph : The mode weight ≈ 20.8 kg.

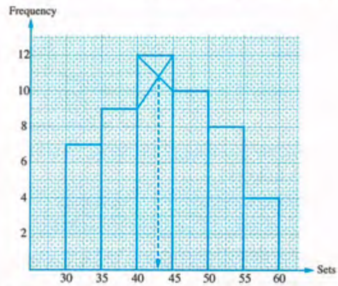
11

[1] $\because k + 4 + 3k + 4k + 3k + 1 + 3k - 1 + k + 1 = 50$

$\therefore 15k + 5 = 50 \qquad \therefore 15k = 45 \qquad \therefore k = 3$

[2]

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
number of students	7	9	12	10	8	4	50



From the graph : The mode weight ≈ 43 kg.

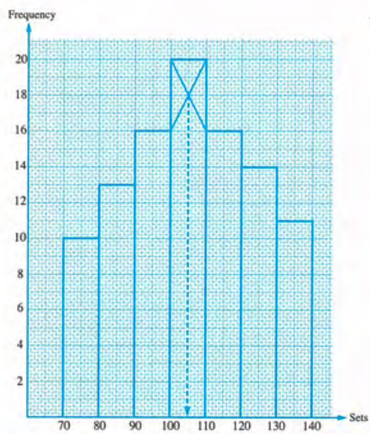
12

[1] $x = 110$

$, k - 4 = 100 - (10 + 13 + 20 + 16 + 14 + 11)$

$\therefore k - 4 = 16 \qquad \therefore k = 20$

[2]



From the graph : The mode wage = 105 pounds.

13

1

Sets	"x"	"f"	"$x \times f$"
35 –	40	15	600
45 –	50	30	1500
55 –	60	23	1380
65 –	70	20	1400
75 –	80	8	640
85 –	90	4	360
Total		100	5880

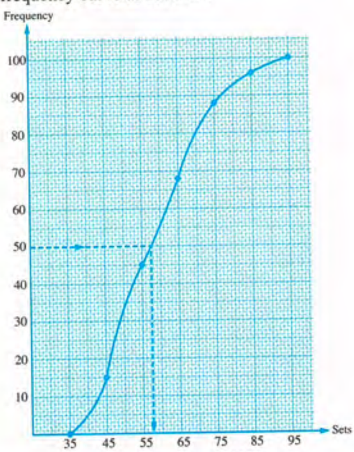
$\therefore$ The mean of working hours $= \frac{5880}{100}$

$= 58.8$ hours.

2 We form the ascending cumulative frequency table as follows :

The upper limits of sets	Ascending cmulative frequency
less than 35	0
less than 45	15
less than 55	45
less than 65	68
less than 75	88
less than 85	96
less than 95	100

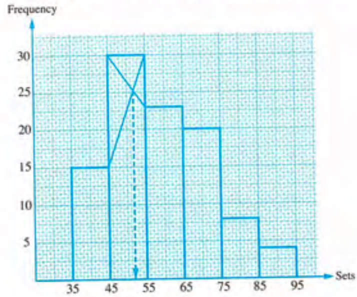
, then we draw the ascending cumulative frequency curve as follows :



$\because$ The order of the median $= \frac{100}{2} = 50$

$\therefore$ The median ≈ 57.5 hours.

3 We graph the histogram of the distribution as follows :



From the graph :

We find that the mode ≈ 52 hours.

14

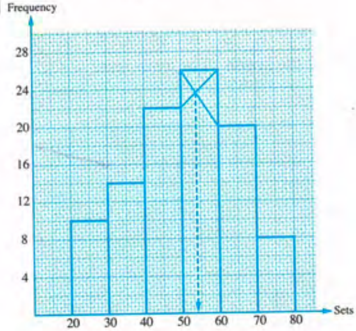
1 $k = 100 - (10 + 22 + 26 + 20 + 8) = 14$

2

Sets	"x"	"f"	"$x \times f$"
20 –	25	10	250
30 –	35	14	490
40 –	45	22	990
50 –	55	26	1430
60 –	65	20	1300
70 –	75	8	600
Total		100	5060

$\therefore$ The mean $= \frac{5060}{100} = 50.6$ pounds.

3



From the graph : The mode value = 54 pounds.

15

[1] $\because 3k + 4k = 50 - (7 + 10 + 8 + 4)$

$\therefore 7k = 21 \qquad \therefore k = \frac{21}{7} = 3$

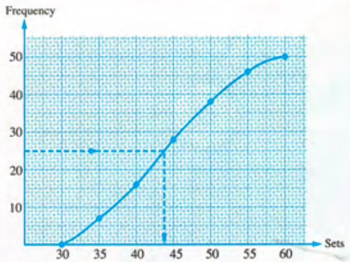
[2]

Sets	"x"	"f"	"$x \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

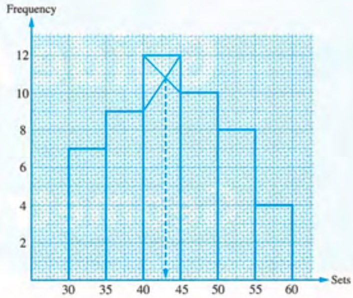
$\therefore$ The mean $= \frac{2200}{50} = 44$ kg.

[3]

The upper limits of sets	Ascending cumulative frequency
less than 30	0
less than 35	7
less than 40	16
less than 45	28
less than 50	38
less than 55	46
less than 60	50



[4]



From the graph : The mode weight = 43 kg.

[5] The order of the median $= \frac{50}{2} = 25$

$\therefore$ The median ≈ 43.5 kg.

Answers of accumulative basic skills

1

[1] 6	[2] 0	[3] 15
[4] 154	[5] 21	[6] $\frac{2}{3}$
[7] 7500	[8] 4	[9] 12
[10] 9	[11] 27	[12] 6 , 8 , 2

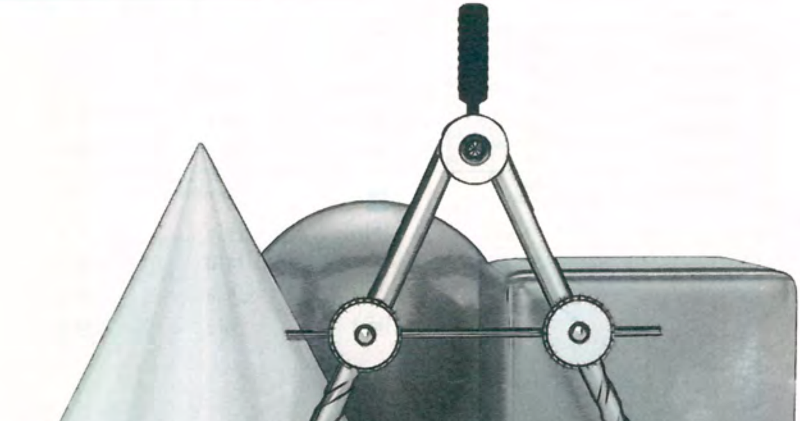
2

[1] c	[2] c	[3] a
[4] a	[5] d	[6] c
[7] d	[8] b	[9] d
[10] d	[11] c	[12] a

Guide Answers

Of

Geometry Exercises



Answers of unit four

Answers of Exercise 1

1

1 a median 2 3 3 one point 4 $1:2$

5 $2:1$ 6 4 7 16

2

1 d 2 a 3 d 4 d

5 a 6 d 7 c

3

1 8 cm. , 5 cm. 2 6 cm. , 4 cm. , $\frac{1}{3}$, $\frac{2}{3}$

3 6 cm. , 3 cm. , 4 cm. 4 5 cm. , 12 cm. , 27 cm.

4

$\because$ $\overline{AD}$, $\overline{BE}$ are two medians in Δ ABC ,

$\overline{AD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of concurrence of the medians of Δ ABC

$\therefore MD = \frac{1}{3} AD = \frac{1}{3} \times 6 = 2$ cm. (1)

, $ME = \frac{1}{3} BE = \frac{1}{3} \times 9 = 3$ cm. (2)

$\because$ D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$ in Δ ABC

$\therefore DE = \frac{1}{2} AB = \frac{1}{2} \times 9 = 4.5$ cm. (3)

From (1) , (2) and (3) :

$\therefore$ The perimeter of Δ MDE $= 2 + 3 + 4.5 = 9.5$ cm. (The req.)

5

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore BC = 2\,DE$ $\therefore BC = 8$ cm.

$\because$ M is the intersection point of medians of Δ ABC

$\therefore MC = 2\,DM$ $\therefore MC = 6$ cm.

, $BM = \frac{2}{3} BE$ $\therefore BM = 4$ cm.

$\therefore$ The perimeter of Δ BMC $= 8 + 6 + 4 = 18$ cm. (The req.)

6

$\because$ M is the intersection point of the medians of Δ ABC

$\therefore XM = \frac{1}{2} MC = 4$ cm.

$\therefore$ The perimeter of Δ MXY $= 4 + 5 + 3 = 12$ cm. (First req.)

, $AM = 2\,MY = 6$ cm.

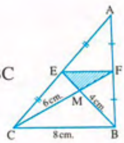
$\because$ X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{BC}$

$\therefore AC = 2\,XY = 10$ cm.

$\therefore$ The perimeter of Δ MAC $= 6 + 8 + 10 = 24$ cm. (Second req.)

7



$\because$ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BE}$, $\overline{CF}$ are two medians in Δ ABC

$\therefore$ M is the intersection point of the medians of Δ ABC

$\therefore ME = \frac{1}{2} MB = 2$ cm. (1)

, $MF = \frac{1}{2} MC = 3$ cm. (2)

$\because$ F is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore FE = \frac{1}{2} BC = 4$ cm. (3)

From (1) , (2) and (3) :

$\therefore$ The perimeter of Δ MFE $= 2 + 3 + 4 = 9$ cm. (The req.)

8

$\because$ M is the intersection point of medians of Δ ABC

$\therefore MF = \frac{1}{2} AM$ (1)

, $MD = \frac{1}{2} MC$ (2)

$\because$ D is the midpoint of $\overline{AB}$,

F is the midpoint of $\overline{BC}$ in Δ ABC

$\therefore DF = \frac{1}{2} AC$ (3)

By adding (1) , (2) and (3) :

$\therefore MF + MD + DF = \frac{1}{2} AM + \frac{1}{2} MC + \frac{1}{2} AC$

$\therefore$ The perimeter of Δ MFD

$= \frac{1}{2} (AM + MC + AC)$

$= \frac{1}{2}$ the perimeter of Δ AMC

$= \frac{1}{2} \times 36 = 18$ cm. (The req.)

9

$\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ $\overline{CD}$ is a median in Δ ABC

$\therefore DM = \frac{1}{2} MC = 3$ cm.

, $\because \Delta AMD$ is a right-angled triangle at M

$\therefore (AM)^2 = (AD)^2 - (DM)^2 = 25 - 9 = 16$

$\therefore AM = 4$ cm.

$\therefore ME = \frac{1}{2} AM = 2$ cm. (The req.)

10

$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other.

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{DM}$ is a median in ΔADC

$\because DE = 2 EM$

$\therefore$ E is the intersection point of the medians of ΔADC

$\because E \in \overline{FC}$

$\therefore \overline{CF}$ is a median in ΔACD $\therefore AF = FD$ (Q.E.D.)

11

$\because$ The two diagonals of the rectangle bisect each other.

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{BM}$ is a median in ΔABC

$\because$ E is the midpoint of $\overline{AB}$

$\therefore \overline{CE}$ is a median in ΔABC

$\because \overline{CE} \cap \overline{BM} = \{F\}$

$\therefore$ F is the intersection point of the medians of ΔABC

(First req.)

$\therefore BF = \frac{2}{3} BM$ $\therefore 4 = \frac{2}{3} BM$ $\therefore BM = 6$ cm.

$\because$ The two diagonals of the rectangle are equal in length and bisect each other.

$\therefore AM = BM = 6$ cm. (Second req.)

12

$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because AM = \frac{2}{3} AD$

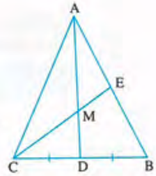
$\therefore$ M is the intersection point of the medians of ΔABC

$\therefore \overline{CF}$ is a median in ΔABC

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore BF = \frac{1}{2} AB$

$\because AC = AB$ $\therefore BF = \frac{1}{2} AC$ (Q.E.D.)

13



$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because AM = 2 MD$

$\therefore$ M is the intersection point of the medians of ΔABC

$\because M \in \overline{CE}$

$\therefore \overline{CE}$ is a median in ΔABC

$\therefore EM = \frac{1}{3} EC = \frac{1}{3} \times 12 = 4$ cm. (The req.)

14

$\because$ M is the point of concurrence of the medians of ΔABC

$\therefore \overline{CD}$ is a median in ΔABC

$\therefore$ D is the midpoint of $\overline{AB}$

In ΔAMB :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BM}$

$\therefore \overline{MD}$, $\overline{AE}$ are two medians in ΔAMB

$\therefore$ N is the point of concurrence of the medians of ΔAMB

$\therefore MN = 2 ND$ $\therefore x + 3 = 2(x - 1)$

$\therefore x + 3 = 2x - 2$ $\therefore 3 + 2 = 2x - x$

$\therefore x = 5$

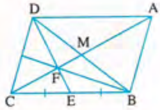
$\therefore ND = 5 - 1 = 4$ cm. , $MN = 5 + 3 = 8$ cm.

$\therefore MD = ND + MN = 12$ cm.

, $\because \overline{CD}$ is a median in ΔABC

$\therefore MC = 2 MD = 24$ cm. (The req.)

15



$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other.

$\therefore$ M is the midpoint of $\overline{BD}$

$\therefore \overline{CM}$ is a median in ΔDBC

$\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{DE}$ is a median in ΔDBC

$\therefore$ F is the intersection point of the medians of ΔDBC

$\therefore \overrightarrow{BF}$ bisects $\overline{CD}$ (Q.E.D. 1)

$\therefore CF = \frac{2}{3} CM$, $\because CM = \frac{1}{2} AC$

$\therefore CF = \frac{2}{3} \times \frac{1}{2} AC = \frac{1}{3} AC$ (Q.E.D. 2)

16

$\because \overline{AD}$ and $\overline{BE}$ are medians in ΔABC

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because$ M $\in \overline{CF}$ $\therefore \overline{CF}$ is a median in Δ ABC

$\therefore$ F is the midpoint of $\overline{AB}$

In Δ ABM :

$\because$ F is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{BM}$

$\therefore \overline{NF} // \overline{AM}$ $\therefore \overline{NF} // \overline{MD}$ (1)

In Δ BMC :

$\because$ D is the midpoint of $\overline{BC}$, N is the midpoint of $\overline{BM}$

$\therefore \overline{ND} // \overline{CM}$ $\therefore \overline{ND} // \overline{MF}$ (2)

From (1) and (2) :

$\therefore$ The figure FNDM is a parallelogram. (Q.E.D.)

17

$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in Δ ABC

, $\because$ AM = 2 MD , M $\in \overline{AD}$

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because$ M $\in \overline{BE}$

$\therefore \overline{BE}$ is a median in Δ ABC $\therefore$ BM = 2 ME

$\therefore$ BM = 4 cm. $\therefore$ BE = 2 + 4 = 6 cm.

, $\because \Delta$ BCE in which :

D is the midpoint of $\overline{BC}$, $\overline{DF} // \overline{BE}$

$\therefore$ F is the midpoint of $\overline{EC}$

$\therefore$ DF = $\frac{1}{2}$ BE = 3 cm. (The req.)

18

$\because$ D is the midpoint of $\overline{BC}$, $\overrightarrow{DF} // \overline{AC}$

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore$ DF = $\frac{1}{2}$ AC

In Δ ABD :

$\because$ E is the midpoint of $\overline{BD}$, F is the midpoint of $\overline{AB}$

$\therefore \overline{AE}$ and $\overline{DF}$ are medians in Δ ABD

$\therefore$ M is the intersection point of the medians of Δ ABD

$\therefore$ DM = $\frac{2}{3}$ DF , $\because$ DF = $\frac{1}{2}$ AC

$\therefore$ DM = $\frac{2}{3} \times \frac{1}{2}$ AC = $\frac{1}{3}$ AC (Q.E.D.)

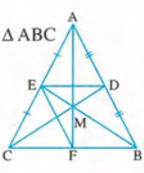
19

$\because \overline{CD}$ and $\overline{BE}$ are two medians in Δ ABC

$\therefore$ M is the intersection point of the medians of Δ ABC

$\therefore \overline{AF}$ is a median in Δ ABC

$\therefore$ F is the midpoint of $\overline{BC}$



, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{FE} // \overline{AB}$, FE = $\frac{1}{2}$ AB

$\therefore \overline{FE} // \overline{BD}$, FE = BD

$\therefore$ DBFE is a parallelogram (Q.E.D.)

Answers of Exercise 2

1

1 3 | 2 half the length of the hypotenuse
3 right | 4 half the length of the hypotenuse
5 twice | 6 twice

2

1 3 | 2 10 | 3 8 | 4 18 , 9 , $\frac{1}{3}$, 3
5 5 , 5 , 15 | 6 9 , 8 , 10 , 27

3

1 b | 2 b | 3 c
4 a | 5 b | 6 a

4

In Δ ADC :

$\because$ m ($\angle$ D) = 90° , E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ AC (1)

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC (2)

From (1) and (2) :

$\therefore$ AB = DE (Q.E.D.)

5

In Δ LXZ :

$\because$ D is the midpoint of $\overline{LX}$, E is the midpoint of $\overline{LZ}$

$\therefore$ DE = $\frac{1}{2}$ XZ (1)

From Δ XYZ :

$\because$ m ($\angle$ Y) = 90° , M is the midpoint of $\overline{XZ}$

$\therefore$ YM = $\frac{1}{2}$ XZ (2)

From (1) and (2) :

$\therefore$ DE = YM (Q.E.D.)

6

In Δ ACD :

$\because$ E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$

$\therefore EF = \frac{1}{2} AC$ $\therefore AC = 8$ cm.

In ΔABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC = 4$ cm. (The req.)

7

In ΔABC :

$\because m(\angle BAC) = 90°$, D is the midpoint of $\overline{BC}$

$\therefore BC = 2\,AD = 2 \times 3 = 6$ cm.

In ΔCBE :

$\because m(\angle CBE) = 90°$, $m(\angle E) = 30°$

$\therefore EC = 2\,BC = 2 \times 6 = 12$ cm.

, $\because$ F is the midpoint of $\overline{CE}$

$\therefore BF = \frac{1}{2} EC = \frac{1}{2} \times 12 = 6$ cm. (The req.)

8

In ΔABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 60°$

$\therefore m(\angle CAB) = 30°$ $\therefore BC = \frac{1}{2} AC$

$\because DE = BC$ $\therefore DE = \frac{1}{2} AC$

$\because \overline{DE}$ is a median in ΔACD

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

9

In ΔABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC$

, $\because AB = DE = 5$ cm. $\therefore DE = \frac{1}{2} AC$

$\because \overline{DE}$ is a median in ΔACD

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

10

In ΔDBC :

$\because$ E is the midpoint of $\overline{BC}$, $\overline{EF} \parallel \overline{BD}$

$\therefore EF = \frac{1}{2} BD$

, $\because AM = EF$ $\therefore AM = \frac{1}{2} BD$

$\because \overline{AM}$ is a median in ΔABD

$\therefore m(\angle BAD) = 90°$ (Q.E.D.)

11

$\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = 33° + 27° = 60°$

$\therefore$ In ΔADC :

$m(\angle DAC) = 180° - (60° + 90°) = 30°$

$\therefore DC = \frac{1}{2} AD$

$\therefore AD = 8$ cm. (The req.)

12

In ΔADB :

$\because m(\angle ADB) = 90°$, $AE = EB$

$\therefore DE = \frac{1}{2} AB$

Similarly in ΔACB :

$\because m(\angle ACB) = 90°$, $AE = EB$

$\therefore CE = \frac{1}{2} AB$ $\therefore DE = CE$

$\therefore \Delta CED$ is an isosceles triangle. (Q.E.D.)

13

In ΔLYE :

$\because m(\angle YLE) = 90°$, $m(\angle E) = 30°$

$\therefore LY = \frac{1}{2} YE = 5$ cm.

In ΔZYX :

$\because m(\angle ZYX) = 90°$, L is the midpoint of $\overline{ZX}$

$\therefore YL = \frac{1}{2} ZX$ $\therefore ZX = 10$ cm. (The req.)

14

In ΔABC :

$\because m(\angle ABC) = 90°$, $m(\angle C) = 30°$

$\therefore AC = 2\,AB = 14$ cm.

, $\because$ D is the midpoint of $\overline{AC}$

$\therefore BD = \frac{1}{2} AC = 7$ cm.

, in ΔDEC :

$\because m(\angle DEC) = 90°$, $m(\angle C) = 30°$

$\therefore DE = \frac{1}{2} DC$, $\because DC = \frac{1}{2} AC = 7$ cm.

$\therefore DE = 3.5$ cm. (The req.)

15

In ΔABC :

$\because m(\angle ABC) = 90°$, $m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC = 4$ cm.

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = 4$ cm.

In ΔXBY :

$\because m(\angle XBY) = 90^\circ$

, Z is the midpoint of $\overline{XY}$

$\therefore BZ = \frac{1}{2} XY = 2$ cm. (The req.)

16

In Δ MED :

$\because m(\angle MED) = 90^\circ$ $\therefore (MD)^2 = 3^2 + 4^2 = 25$

$\therefore MD = \sqrt{25} = 5$ cm.

, $\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore AD = 3\ MD = 15$ cm.

, $\because m(\angle BAC) = 90^\circ$

, $\overline{AD}$ is a median in Δ ABC

$\therefore BC = 2\ AD = 30$ cm. (The req.)

17

In Δ ABC :

$\because m(\angle BAC) = 90^\circ$

$\therefore (BC)^2 = (12)^2 + (9)^2 = 225$

$\therefore BC = \sqrt{225} = 15$ cm.

, $\because \overline{AD}$ is a median in Δ ABC , $m(\angle BAC) = 90^\circ$

$\therefore AD = \frac{1}{2} BC = 7\frac{1}{2}$ cm.

, $\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore AM = \frac{2}{3} AD = 5$ cm. (The req.)

18

$\because$ ABCD is a parallelogram

$\therefore m(\angle C) = m(\angle A) = 60^\circ$

$\therefore$ In Δ DEC :

$m(\angle EDC) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$

$\therefore CE = \frac{1}{2} DC$ $\therefore DC = 8$ cm.

$\therefore$ The perimeter of the parallelogram ABCD

$= (12 + 8) \times 2 = 40$ cm. (The req.)

19

$\because m(\angle BAD) = 90^\circ$, $m(\angle BAE) = 30^\circ$

$\therefore m(\angle DAF) = 60^\circ$

From Δ AFD :

$\because m(\angle AFD) = 90^\circ$, $m(\angle DAF) = 60^\circ$

$\therefore m(\angle ADF) = 30^\circ$ $\therefore AD = 2\ AF = 8$ cm.

$\therefore$ The area of the square $= 64\ cm^2$. (The req.)

20

In Δ BCE :

$\because m(\angle EBC) = 30^\circ$

$\therefore CE = \frac{1}{2} BE$ (1)

, $\because m(\angle EBC) = 30^\circ$ $\therefore m(\angle ABE) = 60^\circ$

$\therefore$ In Δ ABE :

$\because m(\angle EAB) = 30^\circ$

, $\because m(\angle AEB) = 90^\circ$

$\therefore BE = \frac{1}{2} AB$ (2)

From (1) and (2) :

$\therefore CE = \frac{1}{2} \times \frac{1}{2} AB = \frac{1}{4} AB$ (Q.E.D.)

21

In Δ ABC :

$\because m(\angle ABC) = 90^\circ$, $m(\angle A) = 30^\circ$

$\therefore AC = 2\ BC = 16$ cm.

, $m(\angle C) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

In Δ BCD :

$\because m(\angle BDC) = 90^\circ$, $m(\angle C) = 60^\circ$

$\therefore m(\angle CBD) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

$\therefore CD = \frac{1}{2} BC = 4$ cm.

$\therefore AD = AC - CD = 16 - 4 = 12$ cm. (The req.)

22

In Δ ABC :

$\because m(\angle B) = 30^\circ$, $m(\angle C) = 90^\circ$

$\therefore AC = \frac{1}{2} AB$

, $\because$ E is the midpoint of $\overline{BC}$

, O is the midpoint of $\overline{AC}$

$\therefore EO = \frac{1}{2} AB$ $\therefore EO = AC$

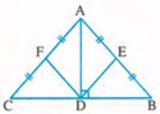
In Δ DEO :

$\because$ X is the midpoint of $\overline{DE}$

, Y is the midpoint of $\overline{DO}$

$\therefore XY = \frac{1}{2} EO$ $\therefore XY = \frac{1}{2} AC$ (Q.E.D)

23



In Δ ADB

$\because m(\angle ADB) = 90^\circ$

, E is the midpoint of $\overline{AB}$

$\therefore DE = \frac{1}{2} AB$

In Δ ADC :

$\because$ m ($\angle$ ADC) = 90°

, F is the midpoint of $\overline{AC}$ $\quad\therefore$ DF = $\frac{1}{2}$ AC

$\therefore$ DE + DF = $\frac{1}{2}$ AB + $\frac{1}{2}$ AC but AB = AC (Given)

$\therefore$ DE + DF = $\frac{1}{2}$ AB + $\frac{1}{2}$ AB = AB (Q.E.D.)

24

In Δ ABC :

$\because$ $\overline{EO}$ // $\overline{AC}$, E is the midpoint of $\overline{AB}$

$\therefore$ O is the midpoint of $\overline{BC}$

, $\because$ BC = 4 + 12 = 16 cm.

$\therefore$ BO = $\frac{1}{2}$ BC = 8 cm.

$\therefore$ DO = 8 − 4 = 4 cm.

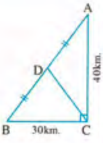
$\therefore$ BD = DO

, $\because$ $\overline{EO}$ // $\overline{AC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore$ m ($\angle$ BEO) = m ($\angle$ A) = 90° (corresponding angles)

$\therefore$ ED = $\frac{1}{2}$ BO = 4 cm. (The req.)

25



Let the service station lie at the point D which is the midpoint of $\overline{AB}$

$\therefore$ The road length = the length of $\overline{CD}$

In Δ ACB :

$\because$ m ($\angle$ ACB) = 90°

$\therefore$ $(AB)^2 = (AC)^2 + (BC)^2 = 1600 + 900 = 2500$

$\therefore$ AB = 50 km.

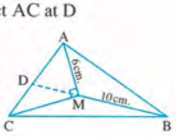
, $\because$ D is the midpoint of $\overline{AB}$

$\therefore$ CD = $\frac{1}{2}$ AB = $\frac{1}{2} \times 50$ = 25 km.

$\therefore$ The length of the road 25 km. (The req.)

26

Constr : Draw $\overrightarrow{BM}$ to intersect $\overline{AC}$ at D



Proof : $\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ MD = $\frac{1}{2}$ BM = 5 cm.

In Δ AMC : $\because$ m ($\angle$ AMC) = 90°

, $\overline{MD}$ is a median

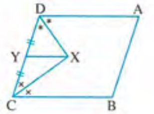
$\therefore$ MD = $\frac{1}{2}$ AC $\quad\therefore$ AC = 10 cm. (First req.)

In Δ AMC : $\because$ m ($\angle$ AMC) = 90°

$\therefore$ $(MC)^2 = (10)^2 - (6)^2 = 64$

$\therefore$ MC = $\sqrt{64}$ = 8 cm. (Second req.)

27



$\because$ $\overline{DA}$ // $\overline{CB}$

, $\overleftrightarrow{DC}$ is a transversal.

$\therefore$ m ($\angle$ ADC) + m ($\angle$ DCB) = 180°

$\therefore$ $\frac{1}{2}$ m ($\angle$ ADC) + $\frac{1}{2}$ m ($\angle$ DCB) = 90°

$\therefore$ m ($\angle$ XDC) + m ($\angle$ DCX) = 90°

but the sum of the measures of the interior angles of a triangle XDC = 180°

$\therefore$ m ($\angle$ DXC) = 90° , $\because$ DY = YC

$\therefore$ XY = $\frac{1}{2}$ DC $\quad\therefore$ XY = YC (Q.E.D.)

Answers of Exercise 3

1

1 x = 50°	2 x = 56°	3 y = 63°
4 ℓ = 65° , z = 50°	5 x = 54° , y = 117°	
6 x = 69° , y = 111°		7 x = 120°
8 x = 68° , y = 127°		

2

1 congruent	2 60°	3 F
4 50°	5 70°	6 20°

3

1 b	2 c	3 c	4 b	5 a
6 b	7 d	8 a	9 b	10 b

4

In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$= \frac{180° - 40°}{2} = 70°$ (First req.)

$\because$ m ($\angle$ ABC) = m ($\angle$ ACB) ,

$\angle$ ABD supplements $\angle$ ABC

, $\angle$ ACE supplements $\angle$ ACB

$\because$ The supplementaries of the congruent angles are congruent

$\therefore$ $\angle$ ABD $\equiv$ $\angle$ ACE (Second req.)

5

From Δ ABC :

$\because$ AB = AC $\quad\therefore$ m ($\angle$ B) = m ($\angle$ ACB) = 70°

$\therefore$ m ($\angle$ BAC) = 180° − (2 × 70°) = 40°

In Δ ACD :

$\because$ AC = CD $\therefore$ m ($\angle$ CAD) = m ($\angle$ D)

$\because$ $\angle$ ACB is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) + m ($\angle$ D)

$\therefore$ m ($\angle$ CAD) = $\frac{70°}{2}$ = 35°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ BAC) + m ($\angle$ CAD)

= 40° + 35° = 75° (The req.)

6

$\because$ $\angle$ ACD is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ ACD) = 30° + 40° = 70°

From Δ ACD :

$\because$ AC = AD

$\therefore$ m ($\angle$ D) = m ($\angle$ ACD) = 70° (First req.)

$\therefore$ m ($\angle$ CAD) = 180° – (70° + 70°) = 40° (Second req.)

7

$\because$ Δ ACD is an equilateral triangle

$\therefore$ m ($\angle$ CAD) = 60° (1)

From Δ ABC :

$\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BCA) = $\frac{180° - 40°}{2}$ = 70° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ BAD) = 60° + 70° = 130° (The req.)

8

In Δ ABD :

$\because$ AB = AD

$\therefore$ m ($\angle$ ADB) = m ($\angle$ ABD) = $\frac{180° - 120°}{2}$ = 30° (First req.)

$\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{DC}$ is a transversal to them

$\therefore$ m ($\angle$ C) + m ($\angle$ ADC) = 180°

$\therefore$ m ($\angle$ C) = 180° – (65° + 30°) = 85° (Second req.)

9

$\because$ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30° (alternate angles)

In Δ ABC :

$\because$ AC = BC

$\therefore$ m ($\angle$ CAB) = m ($\angle$ B) = $\frac{180° - 30°}{2}$ = 75° (The req.)

10

$\because$ Δ DEC is an equilateral triangle.

$\therefore$ m ($\angle$ ECD) = 60° (1)

From Δ ABC :

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ ACB)

, $\because$ m ($\angle$ B) + m ($\angle$ ACB) = 180° – 80° = 100°

$\therefore$ m ($\angle$ B) = m ($\angle$ ACB) = $\frac{100°}{2}$ = 50° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ BCD) = 50° + 60° = 110° (The req.)

11

In Δ ABC :

$\because$ BA = BC , m ($\angle$ B) = 80°

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BCA) = $\frac{180° - 80°}{2}$ = 50°

$\therefore$ m ($\angle$ DAC) = 114° – 50° = 64°

In Δ ADC :

$\because$ DA = DC , m ($\angle$ DAC) = 64°

$\therefore$ m ($\angle$ ADC) = 180° – (64° × 2) = 52° (The req.)

12

From Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ BCA)

$\because$ m ($\angle$ B) + m ($\angle$ BCA) = 180° – 48° = 132°

$\therefore$ m ($\angle$ B) = m ($\angle$ BCA) = $\frac{132°}{2}$ = 66° (First req.)

$\because$ $\overrightarrow{CD}$ bisects $\angle$ ACB

$\therefore$ m ($\angle$ BCD) = $\frac{66°}{2}$ = 33° (Second req.)

13

$\because$ Δ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 60°

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB) = 30°

$\because$ $\overrightarrow{BD}$ bisects $\angle$ ABC , $\overrightarrow{CD}$ bisects $\angle$ ACB

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) = 30°

$\therefore$ From Δ DBC :

m ($\angle$ D) = 180° – (2 × 30°) = 120° (The req.)

14

$\because$ Δ ABC is an equilateral triangle.

$\therefore$ m ($\angle$ ABC) = 60° (1)

From Δ DBC :

$\because DB = DC$, $m(\angle D) = 100°$

$\therefore m(\angle DBC) = m(\angle DCB) = \frac{180° - 100°}{2} = 40°$ (2)

From (1) and (2) :

$\therefore m(\angle ABD) = m(\angle ABC) - m(\angle DBC)$

$= 60° - 40° = 20°$ (The req.)

15

$\because \Delta ABC$ is an equilateral triangle.

$\therefore m(\angle ACB) = m(\angle B) = m(\angle BAC) = 60°$

$\therefore m(\angle ACD) = 120°$

In ΔACD :

$\because AC = CD$ $\therefore m(\angle CAD) = m(\angle D)$

$\because m(\angle CAD) + m(\angle D) = 180° - 120° = 60°$

$\therefore m(\angle CAD) = \frac{60°}{2} = 30°$

$\therefore m(\angle BAD) = 60° + 30° = 90°$

$\therefore \overline{BA} \perp \overline{AD}$ (Q.E.D.)

16

From ΔABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore \Delta\Delta$ ABD , ACE in them :

$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BD = EC \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$ then we deduce that $AD = AE$

$\therefore \Delta ADE$ is an isosceles triangle. (Q.E.D. 1)

$\therefore m(\angle ADE) = m(\angle AED)$

$\therefore \angle ADE \equiv \angle AED$ (Q.E.D. 2)

17

$\because \Delta\Delta$ ADE , BCE in them :

$$\begin{cases} AD = CB \\ AE = EB \\ m(\angle A) = m(\angle B) \end{cases}$$

$\therefore \Delta ADE \equiv \Delta BCE$, then we deduce that $DE = CE$

In ΔDEC :

$\because DE = CE$ $\therefore m(\angle EDC) = m(\angle ECD)$

$\because m(\angle DEC) = 40°$

$\therefore m(\angle EDC) + m(\angle ECD) = 180° - 40° = 140°$

$\therefore m(\angle EDC) = \frac{140°}{2} = 70°$ (The req.)

18

$\because \angle LZX$ is an exterior angle of ΔXYZ

$\therefore m(\angle X) + m(\angle Y) = 130°$

$\because ZX = ZY$ $\therefore m(\angle X) = m(\angle Y)$

$\therefore m(\angle Y) = \frac{130°}{2} = 65°$

$\because \overrightarrow{LM} \parallel \overline{XY}$, $\overleftrightarrow{LY}$ is a transversal to them

$\therefore m(\angle MLY) = m(\angle Y) = 65°$ (The req.)

19

$\because \overrightarrow{AE} \parallel \overline{BC}$ and $\overleftrightarrow{BD}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle DAE)$ (corresponding angles)

, $\because \overrightarrow{AE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle C) = m(\angle EAC)$ (alternate angles)

but $m(\angle B) = m(\angle C)$ because $AB = AC$

$\therefore m(\angle DAE) = m(\angle EAC)$

i.e. $\overrightarrow{AE}$ bisects $\angle DAC$ (Q.E.D.)

20

$\because B \in \overline{AD}$

$\therefore m(\angle ABC) + m(\angle CBE) + m(\angle EBD) = 180°$ (1)

$\because$ The sum of measures of the angles of the triangle $= 180°$

$\therefore m(\angle ABC) + m(\angle A) + m(\angle C) = 180°$ (2)

From (1) and (2) :

$\therefore m(\angle CBE) + m(\angle EBD) = m(\angle A) + m(\angle C)$

, $\because m(\angle CBE) = m(\angle EBD)$ (Given)

, $m(\angle A) = m(\angle C)$ (because $BA = BC$)

$\therefore m(\angle CBE) = m(\angle C)$ and they are alternate angles.

$\therefore \overrightarrow{BE} \parallel \overline{AC}$ (Q.E.D.)

21

In ΔDEC :

$\because DE = DC$

$\therefore m(\angle DEC) = m(\angle C) = \frac{180° - 40°}{2} = 70°$

$\because \overline{AD} \parallel \overline{EC}$, $\overleftrightarrow{DE}$ is a transversal to them

$\therefore m(\angle ADE) = m(\angle DEC) = 70°$ (alternate angles)

$\because AD = AE$

$\therefore m(\angle AED) = m(\angle ADE) = 70°$ (First req.)

In ΔAED :

$\because m(\angle EAD) = 180° - (70° + 70°) = 40°$

$\because m(\angle BAD) = m(\angle C) = 70°$

(from properties of the parallelogram)

$\therefore m(\angle BAE) = 70° - 40° = 30°$ (Second req.)

22

In Δ DBC : $\because$ DB = DC

$\therefore m(\angle DCB) = m(\angle DBC) = \frac{180° - 140°}{2} = 20°$

, $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{DC}$ is a transversal.

$\therefore m(\angle EDC) = m(\angle DCB) = 20°$ (alternate angles)

In Δ DCE : $\because$ DE = EC

$\therefore m(\angle DCE) = m(\angle EDC) = 20°$

$\therefore m(\angle ACB) = 20° + 20° = 40°$

From Δ ABC :

$m(\angle A) = 180° - (20° + 40°) = 120°$ (The req.)

23

From Δ ABC :

$\because$ AB = AC $\quad \therefore m(\angle B) = m(\angle C)$

$\therefore 2x + 13 = 3x - 17 \quad \therefore x = 30°$

$\therefore m(\angle B) = m(\angle C) = 2 \times 30 + 13 = 73°$

$\therefore m(\angle A) = 180° - (73° + 73°) = 34°$ (The req.)

24

1 $x = 60°$, $y = 121°$ 2 $x = 45°$, $y = 105°$

3 $x = 44°$

4 $x = 75°$, $y = 15°$ 5 $x = 25°$, $y = 92°$

6 $y = 110°$, $\ell = 40°$, $z = 70°$

7 $x = 30°$, $y = 40°$ 8 $x = 70°$, $y = 50°$

9 $x = 120°$ 10 $x = 40°$

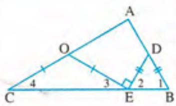
11 $x = 100°$ 12 $x = 15°$

25

1 3 cm. 2 5° 3 5°

4 66.5° 5 7° 6 22°

26



In Δ DBE :

$\because$ DB = DE

$\therefore m(\angle 1) = m(\angle 2)$

In Δ OEC :

$\because$ OE = OC $\quad \therefore m(\angle 3) = m(\angle 4)$

, $\because E \in \overline{BC}$, $m(\angle DEO) = 90°$

$\therefore m(\angle 2) + m(\angle 3) = 90°$

$\therefore m(\angle 1) + m(\angle 4) = 90°$

In Δ ABC :

$\therefore m(\angle A) = 180° - 90° = 90°$ (The req.)

27

In $\Delta\Delta$ EBD and CBD :

$\left\{\begin{array}{l} \overline{BD} \text{ is a common side} \\ m(\angle EBD) = m(\angle CBD) \\ m(\angle EDB) = m(\angle CDB) \end{array}\right.$

$\therefore \Delta EBD \equiv \Delta CBD$, then we deduce that :

BE = BC , $m(\angle BED) = m(\angle C)$

, $\because$ BA = BC $\quad \therefore$ BA = BE

$\therefore m(\angle A) = m(\angle BEA)$

, $\because m(\angle BEA) + m(\angle BED) = 180°$

$\therefore m(\angle A) + m(\angle C) = 180°$ (Q.E.D.)

28

$\Delta\Delta$ XYM , MZL in them :

$\left\{\begin{array}{l} XY = MZ \\ YM = LZ \\ m(\angle Y) = m(\angle Z) = 90° \end{array}\right.$

$\therefore \Delta XYM \equiv \Delta MZL$, then we deduce that :

XM = ML , $m(\angle XMY) = m(\angle MLZ)$

$\because \angle MLZ$ complements $\angle LMZ$

$\therefore \angle XMY$ complements $\angle LMZ$

$\therefore m(\angle XML) = 90°$

$\therefore$ From Δ XLM :

$\because$ MX = ML , $m(\angle XML) = 90°$

$\therefore m(\angle MXL) = m(\angle MLX) = \frac{180° - 90°}{2} = 45°$

(The req.)

29

From Δ BDC :

$\because$ BD = CD

$\therefore m(\angle DBC) = m(\angle BCD)$ (1)

$\because \angle ADB$ is an exterior angle of Δ CBD

$\therefore m(\angle ADB) = m(\angle DBC) + m(\angle BCD)$

from (1) : $m(\angle ADB) = 2\, m(\angle BCD)$ (2)

In Δ ABD :

$\because$ AB = AD

$\therefore m(\angle ABD) = m(\angle ADB)$

from (2) :

$\therefore m(\angle ABD) = m(\angle ADB) = 2\, m(\angle BCD)$

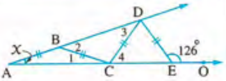
$\because \angle BAE$ is an exterior angle of Δ ABD

$\therefore$ m ($\angle$ BAE) = m ($\angle$ ABD) + m ($\angle$ ADB)

= 2 m ($\angle$ BCD) + 2 m ($\angle$ BCD)

= 4 m ($\angle$ BCD) (Q.E.D.)

30



In Δ ABC :

$\because$ BC = BA

$\therefore$ m ($\angle$ A) = m ($\angle$ 1) = x

, $\because$ $\angle$ 2 is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ 2) = m ($\angle$ A) + m ($\angle$ 1) = $x + x = 2x$

In Δ DBC : $\because$ CB = CD

$\therefore$ m ($\angle$ 3) = m ($\angle$ 2) = 2 x

, $\because$ $\angle$ 4 is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ 4) = m ($\angle$ A) + m ($\angle$ 3) = $x + 2x = 3x$ (1)

, $\because$ m ($\angle$ DEC) = 180° – 126° = 54°

In Δ CDE : $\because$ DC = DE

$\therefore$ m ($\angle$ 4) = m ($\angle$ DEC) = 54° (2)

From (1) and (2) : $\therefore$ 3 x = 54°

$\therefore x = \frac{54°}{3} = 18°$ (The req.)

Answers of Exercise 4

1

1 AB = AC	2 YX = YZ	3 XY = XZ
4 AB = AC = BC	5 ML = MN	6 BA = BC
7 ZX = ZY	8 CB = CA	9 AC = AB

2

1 congruent , isosceles
2 equilateral
3 isosceles
4 isosceles
5 equilateral
6 6
7 60°

3

$\because$ B $\in \overline{DC}$ $\therefore$ m ($\angle$ ABC) = 180° – 125° = 55°

In Δ ABC : m ($\angle$ C) = 180° – (55° + 70°) = 55°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

4

$\because$ Y $\in \overline{ZL}$ $\therefore$ m ($\angle$ XYZ) = 180° – 120° = 60°

, $\because$ XY = XZ

$\therefore$ Δ XYZ is an equilateral triangle. (Q.E.D.)

5

$\because$ B $\in \overrightarrow{AD}$ $\therefore$ m ($\angle$ ABC) = 180° – 120° = 60°

Similarly : m ($\angle$ ACB) = 60°

$\therefore$ m ($\angle$ A) = 180° – (60° + 60°) = 60°

$\therefore$ m ($\angle$ A) = m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore$ Δ ABC is an equilateral triangle. (Q.E.D.)

6

$\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore$ m ($\angle$ DBC) = m ($\angle$ ADB) = 40° (alternate angles)

In Δ DBC : m ($\angle$ C) = 180° – (100° + 40°) = 40°

$\therefore$ m ($\angle$ DBC) = m ($\angle$ C) $\therefore$ DB = DC

$\therefore$ Δ DBC is an isosceles triangle. (Q.E.D.)

7

$\because$ $\overline{XY}$ // $\overline{AC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ A) = m ($\angle$ ABX) = 62° (alternate angles)

$\therefore$ m ($\angle$ ABC) = 180° – (62° + 56°) = 62°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ A)

$\therefore$ CA = CB (Q.E.D.)

8

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ C) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (corresponding angles) (2)

Similarly m ($\angle$ AYX) = m ($\angle$ C) (3)

From (1) , (2) and (3) :

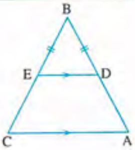
$\therefore$ m ($\angle$ AXY) = m ($\angle$ AYX) $\therefore$ AX = AY

$\therefore$ Δ AXY is an isosceles triangle. (Q.E.D. 1)

$\because$ AB = AC , AX = AY subtracting

$\therefore$ XB = YC (Q.E.D. 2)

9



From Δ EBD : $\because$ DB = EB

$\therefore$ m ($\angle$ BDE) = m ($\angle$ BED) (1)

$\because$ $\overline{DE}$ // $\overline{AC}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore$ m ($\angle$ A) = m ($\angle$ BDE)

(corresponding angles) (2)

Similarly m ($\angle$ C) = m ($\angle$ BED) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ A) = m ($\angle$ C)

$\therefore$ AB = BC (Q.E.D.)

10

$\because MB = MC \quad \therefore m(\angle B) = m(\angle C)$ (1)

$\because \overline{AD} \,//\, \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle A) = m(\angle C)$ (alternate angles) (2)

similarly $m(\angle D) = m(\angle B)$ (3)

from (1), (2) and (3) $\quad \therefore m(\angle A) = m(\angle D)$

$\therefore MA = MD$ (Q.E.D.)

11

$\because B \in \overline{AE} \quad \therefore \angle ABC$ supplements $\angle EBC$

similarly $\angle ACB$ supplements $\angle ACD$

, $\because m(\angle EBC) = m(\angle ACD)$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore AB = AC = 8$ cm.

$\therefore$ The perimeter of $\Delta ABC = 8 + 8 + 10 = 26$ cm.

(The req.)

12

$\because AB = AC \quad \therefore m(\angle B) = m(\angle C)$ (1)

$\because \overline{AB} \,//\, \overline{DE}$, $\overleftrightarrow{BE}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle DEF)$ (corresponding angles) (2)

similarly $m(\angle C) = m(\angle DFE)$ (3)

from (1), (2) and (3)

$\therefore m(\angle DEF) = m(\angle DFE)$

$\therefore DE = DF$ (Q.E.D. 1)

In $\Delta\Delta$ ABC, DEF :

$\because m(\angle B) = m(\angle DEF)$, $m(\angle C) = m(\angle DFE)$

$\therefore m(\angle BAC) = m(\angle EDF)$ (Q.E.D. 2)

13

$\because \overline{ED} \,//\, \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore m(\angle EDB) = m(\angle DBC)$ (alternate angles)

but $m(\angle EBD) = m(\angle DBC)$

$\therefore m(\angle EDB) = m(\angle EBD) \quad \therefore EB = ED$

$\therefore \Delta EBD$ is an isosceles triangle. (Q.E.D.)

14

$\because \overrightarrow{AE} \,//\, \overline{BC}$ and $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore m(\angle DAE) = m(\angle B)$ (corresponding angles)

$\because \overrightarrow{AE} \,//\, \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle EAC) = m(\angle C)$ (alternate angles)

but $m(\angle DAE) = m(\angle EAC)$

$\therefore m(\angle B) = m(\angle C)$

$\therefore AB = AC$ (Q.E.D.)

15

$\because m(\angle ABC) = m(\angle ACB) \quad \therefore AB = AC$

$\therefore \Delta\Delta$ ADB, AEC in them :

$$\begin{cases} AB = AC \\ DB = EC \\ m(\angle D) = m(\angle E) = 90^\circ \end{cases}$$

$\therefore \Delta ADB \equiv \Delta AEC$

$\therefore m(\angle DAB) = m(\angle CAE)$ (Q.E.D.)

16

In Δ YZX :

$\because YZ = YX \quad \therefore m(\angle Z) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

$\because m(\angle ZMX) = 50^\circ + 15^\circ = 65^\circ$

$\therefore$ In ΔMZX : $m(\angle Z) = m(\angle ZMX) \quad \therefore MX = ZX$

$\therefore \Delta MZX$ is an isosceles triangle. (Q.E.D.)

17

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$

$\because m(\angle MCA) = 25^\circ \quad \therefore m(\angle MCB) = 55^\circ - 25^\circ = 30^\circ$

$\therefore m(\angle MBC) = m(\angle MCB) \quad \therefore MB = MC$

$\therefore \Delta MBC$ is an isosceles triangle. (Q.E.D.)

18

$\because \angle ADC$ is an exterior angle of ΔADB

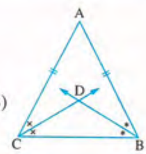
$\therefore m(\angle ADC) = 40^\circ + 30^\circ = 70^\circ$

$\because AD = AC \quad \therefore m(\angle C) = m(\angle ADC) = 70^\circ$

$\therefore$ In ΔABC : $m(\angle BAC) = 180^\circ - (40^\circ + 70^\circ) = 70^\circ$

$\therefore m(\angle BAC) = m(\angle C) \quad \therefore AB = BC$ (Q.E.D.)

19



$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$

$\because m(\angle DBC) = \frac{1}{2} m(\angle ABC)$

$m(\angle DCB) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle DBC) = m(\angle DCB) \quad \therefore DB = DC$

$\therefore \Delta DBC$ is an isosceles triangle. (Q.E.D.)

20

$\because$ Δ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ACB) = 60°

$\because$ $\angle$ ACB is an exterior angle of Δ DCF

$\therefore$ m ($\angle$ D) = 60° – 30° = 30°

$\therefore$ m ($\angle$ D) = m ($\angle$ F) $\therefore$ CD = CF

$\therefore$ Δ DCF is an isosceles triangle. (Q.E.D.)

21

$\because$ DA = DC $\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30°

$\because$ $\angle$ ADB is an exterior angle of Δ ADC

$\therefore$ m ($\angle$ ADB) = 30° + 30° = 60°

$\because$ DA = DB

$\therefore$ Δ ABD is an equilateral triangle. (Q.E.D. 1)

$\because$ m ($\angle$ BAD) = 60° , m ($\angle$ DAC) = 30°

$\therefore$ m ($\angle$ BAC) = 90°

$\therefore$ Δ ABC is a right-angled triangle. (Q.E.D. 2)

22

$\because$ $\overline{ED}$ // $\overline{AC}$, $\overleftrightarrow{EC}$ is a transversal to them.

$\therefore$ m ($\angle$ DEC) = m ($\angle$ ACE) (alternate angles)

$\because$ m ($\angle$ DEC) = m ($\angle$ AEC)

$\therefore$ m ($\angle$ ACE) = m ($\angle$ AEC)

$\therefore$ AE = AC (1)

, $\because$ $\overline{DE}$ // $\overline{AC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ A) = m ($\angle$ BED) = 60°

(corresponding angles) (2)

from (1) and (2) :

$\therefore$ Δ AEC is an equilateral triangle. (Q.E.D.)

23

In Δ ABC : m ($\angle$ ACB) = 180° – (60° + 90°) = 30°

In Δ ECD : m ($\angle$ ECD) = 180° – (30° + 90°) = 60°

$\because$ C $\in$ $\overline{BD}$

$\therefore$ m ($\angle$ ACE) = 180° – (30° + 60°) = 90°

In Δ ACE : m ($\angle$ CAE) = 180° – (90° + 45°) = 45°

$\therefore$ m ($\angle$ CAE) = m ($\angle$ CEA) = 45° $\therefore$ CA = CE

In Δ ECD : $\because$ m ($\angle$ D) = 90° , m ($\angle$ CED) = 30°

$\therefore$ CE = 2 CD = 6 cm. but AC = CE

$\therefore$ AC = 6 cm. (The req.)

24

In Δ ADE : $\because$ $\angle$ ADE $\equiv$ $\angle$ AED $\therefore$ AD = AE

D $\in$ $\overline{BC}$, E $\in$ $\overline{BC}$

$\because$ $\angle$ ADB supplements $\angle$ ADE ,

$\angle$ AEC supplements $\angle$ AED

but m ($\angle$ ADE) = m ($\angle$ AED)

$\therefore$ m ($\angle$ ADB) = m ($\angle$ AEC)

(supplementaries of the congruent angles are congruent)

$\therefore$ $\Delta\Delta$ ADB , AEC in them :

$\left\{\begin{array}{l}\text{m}(\angle \text{ADB}) = \text{m}(\angle \text{AEC}) \\ \text{AD} = \text{AE} \\ \text{BD} = \text{CE}\end{array}\right.$

$\therefore$ Δ ADB $\equiv$ Δ AEC

We deduce that AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

25

$\because$ BY = ZC then adding YC to both sides

$\therefore$ BC = ZY

$\therefore$ $\Delta\Delta$ ABC , XZY in them :

$\left\{\begin{array}{l}\text{AB} = \text{XZ} \\ \text{BC} = \text{ZY} \\ \text{m}(\angle \text{B}) = \text{m}(\angle \text{Z})\end{array}\right.$

$\therefore$ Δ ABC $\equiv$ Δ XZY

We deduce that m ($\angle$ ACB) = m ($\angle$ XYZ)

$\therefore$ EC = EY

$\therefore$ Δ EYC is an isosceles triangle. (Q.E.D.)

26

In Δ BMC :

$\because$ m ($\angle$ MBC) = m ($\angle$ MCB)

$\therefore$ MB = MC

, $\because$ m ($\angle$ ABM) = m ($\angle$ MCD)

(complementaries of equal angles in measure are equal in measure)

$\therefore$ $\Delta\Delta$ ABM , DCM in them :

$\left\{\begin{array}{l}\text{AB} = \text{DC (two sides in a square)} \\ \text{BM} = \text{CM (proved)} \\ \text{m}(\angle \text{ABM}) = \text{m}(\angle \text{DCM}) \text{ (proved)}\end{array}\right.$

$\therefore \Delta ABM \equiv \Delta DCM$ we deduce that AM = DM

$\therefore \Delta AMD$ is an isosceles triangle. (Q.E.D.)

27

In $\Delta\Delta$ ABF , AME : m ($\angle$ B) = m ($\angle$ AME) = 90°

m ($\angle$ BAF) = m ($\angle$ MAE) ($\overrightarrow{AE}$ bisects $\angle$ BAC)

$\therefore$ m ($\angle$ AFB) = m ($\angle$ E) (1)

, $\because$ $\overline{AD}$ // $\overline{BF}$, $\overleftrightarrow{AF}$ is a transversal to them.

$\therefore$ m ($\angle$ DAE) = m ($\angle$ AFB) (alternate angles) (2)

from (1) and (2) :

$\therefore$ m ($\angle$ E) = m ($\angle$ DAE)

$\therefore$ DA = DE (Q.E.D.)

28

$\because$ m ($\angle$ EAM) = m ($\angle$ EMA) $\therefore$ EA = EM

, $\because$ $\overline{AE}$ is a median in Δ ABD , m ($\angle$ BAD) = 90°

$\therefore AE = \frac{1}{2}$ BD $\therefore EM = \frac{1}{2}$ BD (1)

, $\because$ E is the midpoint of $\overline{BD}$, $\overline{EM}$ // $\overline{BC}$

$\therefore EM = \frac{1}{2}$ BC (2)

from (1) and (2) :

$\therefore \frac{1}{2} BD = \frac{1}{2} BC$ $\therefore$ BD = BC (Q.E.D.)

29

$\because$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

$\therefore 2x - 1 = x + 3$ $\therefore 2x - x = 3 + 1$ $\therefore x = 4$

$\therefore$ AB = AC = 2 × 4 − 1 = 7 cm. , BC = 9 − 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (The req.)

30

[1] $\because 3x + x + 50° + 30° = 180°$

$\therefore 4x + 80° = 180°$

$\therefore 4x = 180° - 80° = 100°$ $\therefore x = \frac{100°}{4} = 25°$

$\therefore$ m ($\angle$ A) = 3 × 25° = 75° ,

m ($\angle$ B) = 25° + 50° = 75°

$\therefore$ m ($\angle$ A) = m ($\angle$ B) $\therefore$ CB = CA

[2] $\because$ 2 z + 3 z − 10° + z + 40° = 180°

$\therefore$ 6 z + 30° = 180°

$\therefore$ 6 z = 180° − 30° = 150° $\therefore z = \frac{150°}{6} = 25°$

$\therefore$ m ($\angle$ B) = 3 × 25° − 10° = 65°

, m ($\angle$ C) = 25° + 40° = 65°

$\therefore$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

[3] $\because$ $\angle$ DBC is an exterior angle of Δ ABC

$\therefore 3x = x - 20° + x + 70°$

$\therefore 3x = 2x + 50°$ $\therefore x = 50°$

$\therefore$ m ($\angle$ A) = 50° − 20° = 30° ,

m ($\angle$ C) = 50° + 70° = 120°

$\therefore$ m ($\angle$ ABC) = 180° − (30° + 120°) = 30°

$\therefore$ m ($\angle$ A) = m ($\angle$ ABC) $\therefore$ CB = CA

31

[1] c [2] b

Answers of Exercise 5

1

[1] An axis of symmetry. [2] 3 [3] 1 [4] zero

[5] Bisects it and it is perpendicular to the base.

[6] Bisects the base and is perpendicular to it.

[7] Bisects each of the base and the vertex angle.

[8] The straight line perpendicular to it at its middle.

[9] at equal distances [10] 3 [11] 1 [12] 3

2

[1] 35° [2] 70° [3] 55° [4] 2 [5] $\overleftrightarrow{AD}$

3

[1] a [2] c [3] b [4] b [5] a

[6] a [7] b [8] a

4

$\because$ BA = BC , $\overline{BD} \perp \overline{AC}$

$\therefore$ $\overrightarrow{BD}$ bisects each of $\angle$ ABC , $\overline{AC}$

$\therefore$ AC = 2 AD = 40 cm.

, m ($\angle$ DBC) = $\frac{1}{2}$ m ($\angle$ ABC) = 45° (1) (First req.)

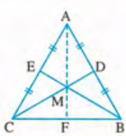
$\because$ ΔABC in which m ($\angle$B) = 90° , BA = BC

$\therefore$ m ($\angle$ C) = 45° (2)

From (1) and (2) : $\therefore$ DB = DC

$\therefore \Delta$ DBC is an isosceles triangle. (Second req.)

5



In ΔABC : AB = AC ,

M is the point of

intersection of its medians

$\therefore$ $\overline{AF}$ is a median of ΔABC

$\therefore \overrightarrow{AM} \perp \overline{BC}$ (Q.E.D. 1)

, $\overrightarrow{AM}$ bisects $\angle$ BAC (Q.E.D. 2)

6

In ΔABC : $\because AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore BC = 2 \times 5 = 10$ cm. (First req.)

In the right-angled triangle ADB at D

$AD = \sqrt{(13)^2 - (5)^2} = 12$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 10 \times 12 = 60$ cm^2.

(Second req.)

7

$\because AB = AC$, $\overline{AD} \perp \overline{BC}$ $\therefore BD = \frac{1}{2} BC = 5$ cm.

, $m(\angle BAC) = 2 \times 30^\circ = 60^\circ$

$\therefore \Delta ABC$ is an equilateral triangle.

$\because AB = 10$ cm.

$\because$ In ΔADB which is right-angled at D

$\therefore AD = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3}$ cm. (First req.)

The number of axes of symmetry of $\Delta ABC = 3$

(Second req.)

The area of $\Delta ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$ cm^2.

(Third req.)

8

In ΔABC : $\because AB = AC$, $\overrightarrow{AE}$ bisects $\angle$ BAC

$\therefore BE = \frac{1}{2} BC$ (Q.E.D.1)

, $\overline{AE} \perp \overline{BC}$

$\therefore \overleftrightarrow{AE}$ is the axis of symmetry of $\overline{BC}$, $D \in \overleftrightarrow{AE}$

$\therefore BD = CD$ (Q.E.D.2)

9

$\because C \in \overline{BD}$, $m(\angle ACD) = 130^\circ$

$\therefore m(\angle ACB) = 180^\circ - 130^\circ = 50^\circ$

From ΔABC : $m(\angle B) = 180^\circ - (80^\circ + 50^\circ) = 50^\circ$

$\therefore m(\angle B) = m(\angle ACB)$

$\therefore \Delta ABC$ is an isosceles triangle.

, $\overrightarrow{AE}$ bisects $\angle$ BAC

$\therefore \overline{AE} \perp \overline{BC}$, E is the midpoint of $\overline{BC}$ (Q.E.D.)

10

$\because m(\angle ABX) = m(\angle ACY)$

$\therefore m(\angle ABC) = m(\angle ACB)$

(The supplementaries of congruent angles are congruent)

$\therefore AB = AC$

$\because \overline{AD}$ is a median of ΔABC which is isosceles.

$\therefore \overline{AD} \perp \overline{BC}$ (Q.E.D.)

11

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them

$\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)

but $m(\angle ABD) = m(\angle DBC)$

$\therefore m(\angle ADB) = m(\angle ABD)$

$\therefore$ In ΔABD : $AB = AD$ (Q.E.D.1)

, $\because \overrightarrow{AE}$ bisects $\angle$ BAD $\therefore \overline{AE} \perp \overline{BD}$ (Q.E.D.2)

, $BE = ED$ (Q.E.D.3)

12

In ΔACD :

$\because$ E is the midpoint of $\overline{AD}$

, $\overline{CE} \perp \overline{AD}$ $\therefore DC = AC$

$\therefore \Delta ACD$ is an isosceles triangle.

, $\because \angle ADC$ is an exterior angle of ΔADB

$\therefore m(\angle ADC) = 20^\circ + 30^\circ = 50^\circ$

From ΔCDE :

$m(\angle DCE) = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$

, $\because \overrightarrow{CE}$ bisects $\angle$ ACD

$\therefore m(\angle ACE) = m(\angle DCE) = 40^\circ$ (The req.)

13

In ΔADC :

$\because$ E is the midpoint of $\overline{DC}$

, $\overline{AE} \perp \overline{DC}$ $\therefore AD = AC$

$\therefore \Delta ADC$ is an isosceles triangle.

$\therefore m(\angle ADC) = m(\angle C) = 70^\circ$

, $\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

, $\because BD = AC$, $AD = AC$

$\therefore m(\angle B) = m(\angle BAD) = \frac{70^\circ}{2} = 35^\circ$ (The req.)

14

In ΔXYL : $\because XL = XY$, M is the midpoint of $\overline{LY}$

$\therefore \overleftrightarrow{XM}$ is the axis of $\overline{LY}$

similarly in ΔZYL , $\overleftrightarrow{ZM}$ is the axis of $\overline{LY}$

$\therefore$ X , M and Z are on the same straight line. (Q.E.D.)

15

$\because AB = AC$ $\quad\therefore A \in$ the axis of $\overline{BC}$ (1)

, $m(\angle ABC) = m(\angle ACB)$,

$\because m(\angle ABD) = m(\angle ACD)$

by subtracting :

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore DB = DC$ $\quad\therefore D \in$ the axis of $\overline{BC}$ (2)

From (1) and (2) :

$\therefore \overleftrightarrow{AD}$ is the axis of $\overline{BC}$ (Q.E.D.)

16

$\because \overline{AD}$ bisects the base of ΔABC which is an isosceles triangle.

$\therefore \overline{AD} \perp \overline{BC}$ $\quad\therefore m(\angle ADB) = 90^\circ$

$\because \overleftrightarrow{XY} \parallel \overline{BC}$, $\overleftrightarrow{AD}$ is a transversal to them.

$\therefore m(\angle YAD) = m(\angle ADB) = 90^\circ$ (alternate angles)

$\therefore \overline{AD} \perp \overleftrightarrow{XY}$ (Q.E.D.)

17

$\because AB = AC$, $EB = EC$ $\quad\therefore \overleftrightarrow{AE}$ is the axis of $\overline{BC}$

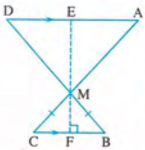
$\therefore BD = DC$ (First req.)

$\therefore DC = 3$ cm.

In ΔADC which is right-angled at D

$\therefore AD = \sqrt{(10)^2 - (3)^2} = \sqrt{100 - 9} = \sqrt{91}$ cm. (Second req.)

18 **Constr. :**



Draw $\overleftrightarrow{MF} \perp \overline{BC}$ to meet $\overline{BC}$ at F and $\overline{AD}$ at E

Proof : $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle A) = m(\angle C)$ similarly $m(\angle B) = m(\angle D)$

$\because MB = MC$ $\quad\therefore m(\angle B) = m(\angle C)$

$\therefore m(\angle A) = m(\angle D)$ $\quad\therefore AM = DM$

$\therefore \Delta AMD$ is an isosceles triangle. (Q.E.D.1)

In ΔMBC : $\because MB = MC$, $\overleftrightarrow{MF} \perp \overline{BC}$

$\therefore \overleftrightarrow{MF}$ is the axis of symmetry of ΔMBC

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{FE}$ is a transversal to them.

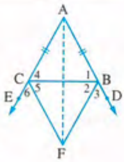
$\therefore m(\angle AEM) = m(\angle BFM) = 90^\circ$

$\therefore \overline{ME} \perp \overline{AD}$

, $\because MA = MD$ $\quad\therefore \overleftrightarrow{ME}$ is the axis of ΔAMD

$\therefore \overleftrightarrow{EF}$ is the axis of symmetry of each of ΔAMD , ΔBMC (Q.E.D.2)

19



$\because AB = AC$

$\therefore m(\angle 1) = m(\angle 4)$ (1)

$\because m(\angle DBC) = 180^\circ - m(\angle 1)$ (2)

, $m(\angle BCE) = 180^\circ - m(\angle 4)$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle DBC) = m(\angle BCE)$

$\therefore \frac{1}{2} m(\angle DBC) = \frac{1}{2} m(\angle BCE)$

$\therefore m(\angle 2) = m(\angle 5)$ $\quad\therefore FB = FC$

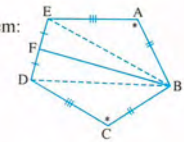
$\therefore \Delta BFC$ is an isosceles triangle. (Q.E.D.1)

$\because AB = AC$, $FB = FC$

$\therefore \overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$ (Q.E.D.2)

20

Constr. : Draw $\overline{BD}$, $\overline{BE}$



Proof: $\Delta\Delta ABE$, CBD in them:

$$\begin{cases} m(\angle A) = m(\angle C) \\ AB = CB \text{ ,} \\ AE = CD \end{cases}$$

$\therefore \Delta ABE \equiv \Delta CBD$

, then we deduce that: $BE = BD$

, $\because \overline{BF}$ is a median of ΔBED which is isosceles

$\therefore \overline{BF} \perp \overline{DE}$ (Q.E.D.)

21

1 c	2 b	3 c	4 b	5 a	6 b

22

In ΔABD :

$\because$ E is the midpoint of $\overline{AB}$

, $\overline{DE} \perp \overline{AB}$ $\quad\therefore DA = DB$

$\therefore m(\angle A) = m(\angle ABD)$ (1)

, in ΔDBC :

$\because$ O is the midpoint of $\overline{BC}$

, $\overline{DO} \perp \overline{BC}$ $\quad\therefore DB = DC$

$\therefore m(\angle DBC) = m(\angle C)$ (2)

, $\because m(\angle ABD) + m(\angle DBC) = 130°$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle A) + m(\angle C) = 130°$

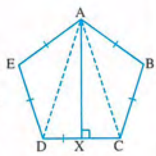
From the quadrilateral ABCD

$\therefore m(\angle ADC) = 360° - (130° + 130°) = 100°$ (The req.)

23

Constr. :

Draw $\overline{AD}$, $\overline{AC}$



Proof :

$\because$ ABCDE is a regular pentagon

$\therefore$ The measure of each interior angle = $108°$

, AB = BC = CD = DE = EA

In ΔABC : $\because AB = BC$, $m(\angle ABC) = 108°$

$\therefore m(\angle BAC) = \frac{180° - 108°}{2} = 36°$

$\therefore \Delta ABC$, ΔAED in them :

$$\begin{cases} AB = AE \\ BC = ED \\ m(\angle ABC) = m(\angle AED) = 108° \end{cases}$$

$\therefore \Delta ABC \equiv \Delta AED$

Then we deduce that : $m(\angle EAD) = m(\angle BAC) = 36°$

, $AC = AD$ $\therefore \Delta ADC$ is an isosceles triangle.

, $\because \overline{AX} \perp \overline{CD}$

$\therefore m(\angle CAX) = m(\angle DAX)$

$= \frac{108° - (36° + 36°)}{2} = 18°$ (The req.)

Answers of unit five

Answers of Exercise 6

1

1 $>$ 2 $>$ 3 $<$ 4 $<$

5 $<$ 6 $>$ 7 $>$, $<$

2

1 $m(\angle 1) < m(\angle 3)$ 2 $m(\angle 4) < m(\angle 2)$

3 $m(\angle 3) < m(\angle 5)$ 4 $m(\angle 6) < m(\angle 2)$

5 $m(\angle 1) < m(\angle 3) < m(\angle 5)$

6 $m(\angle 3) < m(\angle 5) < m(\angle 7)$

7 $m(\angle 1) < m(\angle 3) < m(\angle 5) < m(\angle 7)$

3

$\because \overleftrightarrow{AD} \perp \overline{BC}$ from its midpoint.

$\therefore \overleftrightarrow{AD}$ is the axis of symmetry of $\overline{BC}$

$\therefore AB = AC$

$\because E \in \overline{AB}$ $\therefore AB > AE$

$\therefore AC > AE$ (Q.E.D.)

4

$\because \overline{AB} \,//\, \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle BCD) = m(\angle ABC)$ (alternate angles)

$\therefore m(\angle BCD) + m(\angle ACB) > m(\angle ABC)$

$\therefore m(\angle ACD) > m(\angle ABC)$ (1) (Q.E.D.1)

$\because E \in \overrightarrow{CD}$ $\therefore \angle ADE$ is an exterior angle of ΔACD

$\therefore m(\angle ADE) > m(\angle ACD)$ (2)

From (1) and (2) :

$\therefore m(\angle ADE) > m(\angle ABC)$ (Q.E.D.2)

5

$\because E \in \overrightarrow{CB}$ $\therefore \angle ABE$ is an exterior angle of ΔABC

$\therefore m(\angle ABE) > m(\angle A)$ (1)

$\Delta\Delta$ ABM , CDM in them :

$$\begin{cases} AM = MC \\ MB = MD \\ m(\angle AMB) = m(\angle DMC) \end{cases}$$ (V.O.A)

$\therefore \Delta ABM \equiv \Delta CDM$, then we deduce that :

$m(\angle A) = m(\angle ACD)$ and from (1) :

$\therefore m(\angle ABE) > m(\angle ACD)$ (Q.E.D.)

6

$\because$ The figure is a parallelogram.

$\therefore AD = BC$, $AB = CD$

$\because DX < BY$ $\therefore AX > CY$

$\therefore AX + AB > CY + CD$ (Q.E.D.)

7

$\because D \in \overline{AB}$ $\therefore \angle ADC$ is an exterior angle of ΔDBC

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle ADC) = m(\angle ACD)$

because ΔADC in which $AD = AC$

$\therefore m(\angle ACD) > m(\angle B)$

$\therefore m(\angle ACD) + m(\angle DCB) > m(\angle B)$

$\therefore m(\angle ACB) > m(\angle B)$ (Q.E.D.)

8

In ΔAXY : $\because m(\angle AXY) = m(\angle AYX)$

$\therefore AX = AY$ (1)

$\because AC > AB$ $\therefore AY + YC > AX + XB$ (2)

From (1) and (2) :

$\therefore YC > XB$ (Q.E.D.)

9

$\because \angle ADC$ is an exterior angle of ΔDBC

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle B) = m(\angle ACB)$

(because $AB = AC$ in ΔABC)

$\therefore m(\angle ADC) > m(\angle ACB)$ (Q.E.D.)

10

$\because m(\angle ACB) > m(\angle ABC)$

$\therefore$ The supplement

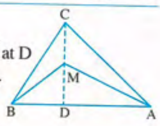
of $\angle ABC >$ the supplement of $\angle ACB$

$\therefore m(\angle ABD) > m(\angle ACE)$

$\therefore \frac{1}{2} m(\angle ABD) > \frac{1}{2} m(\angle ACE)$

i.e. $m(\angle ABX) > m(\angle ACY)$ (Q.E.D.)

11



Const : Draw $\overrightarrow{CM}$ to intersect $\overline{BA}$ at D

Proof : $\because \angle AMD$ is an exterior angle of ΔAMC

$\therefore m(\angle AMD) > m(\angle ACM)$ (1)

$\because \angle BMD$ is an exterior angle of ΔCMB

$\therefore m(\angle BMD) > m(\angle BCM)$ (2)

Adding (1) and (2)

$\therefore$ m ($\angle$ AMD) + m ($\angle$ BMD) > m ($\angle$ ACM) + m ($\angle$ BCM)

$\therefore$ m ($\angle$ AMB) > m ($\angle$ C) (Q.E.D.)

12

$\because$ m ($\angle$ B) > m ($\angle$ C)

$\therefore$ m ($\angle$B) + $\frac{1}{2}$m ($\angle$ BAC) > m ($\angle$ C) + $\frac{1}{2}$m ($\angle$ BAC)

$\therefore$ m ($\angle$ B) + m ($\angle$ BAD) > m ($\angle$C) + m ($\angle$ CAD)

but m ($\angle$ B) + m ($\angle$ BAD) = m ($\angle$ CDA)

(an exterior angle of ΔABD),

m ($\angle$ C) + m ($\angle$ CAD) = m ($\angle$ BDA)

(an exterior angle of ΔACD)

$\therefore$ m ($\angle$ ADC) > m ($\angle$ ADB)

$\because$ Their sum = 180° $\therefore$ m ($\angle$ ADC) > $\frac{180^\circ}{2}$

i.e. m ($\angle$ ADC) > 90°

i.e. $\angle$ ADC is an obtuse angle. (Q.E.D.)

13

$\because$ AC = AD $\therefore$ m ($\angle$ D) = m ($\angle$ ACD)

$\because$ m ($\angle$ ACB) > m ($\angle$ ABC)

$\therefore$ m ($\angle$ ACB) + m ($\angle$ ACD) > m ($\angle$ ABC) + m ($\angle$ D)

$\therefore$ m ($\angle$ BCD) > m ($\angle$ B) + m ($\angle$ D)

but the sum of measures of the interior angles of Δ BCD = 180°

$\therefore$ m ($\angle$ BCD) > $\frac{180^\circ}{2}$ **i.e.** m ($\angle$ BCD) > 90°

i.e. $\angle$ BCD is an obtuse angle. (Q.E.D.)

Answers of Exercise 7

1

1 The angle of the greater measure

2 $\angle$ A 3 m ($\angle$ D)

4 m ($\angle$ A) < m ($\angle$ B) < m ($\angle$ C)

2

1 > , > , < 2 < , < , > 3 > , > , >

3

1 $\because$ $\overline{BC}$ is the longest side

$\therefore$ $\angle$ A is the greatest angle in measure

$\because$ $\overline{AC}$ is the shortest side

$\therefore$ $\angle$ B is the smallest angle in measure

$\therefore$ The ascending order of measures of the angles is : m ($\angle$ B) , m ($\angle$ C) and m ($\angle$ A)

2 $\because$ $\overline{BC}$ is the longest side.

$\therefore$ $\angle$ A is the greatest angle in measure.

$\because$ $\overline{AB}$ is the shortest side.

$\therefore$ $\angle$ C is the smallest angle in measure.

$\therefore$The ascending order of the measures of the angles is : m ($\angle$ C) , m ($\angle$ B) and m ($\angle$ A)

4

In ΔABC : $\because$ AC > AB

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ACB) (1)

In Δ BDC : $\because$ DB = DC

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) (2)

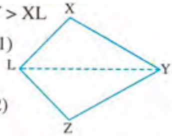
Adding (1) and (2) :

$\therefore$ m ($\angle$ ABC) + m ($\angle$ DBC) > m ($\angle$ ACB) + m ($\angle$ DCB)

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ACD) (Q.E.D.)

5

Construction : Draw $\overline{YL}$



Proof : In ΔXYL $\because$ XY > XL

$\therefore$ m ($\angle$ XLY) > m ($\angle$ XYL) (1)

In ΔZYL : $\because$ YZ > ZL

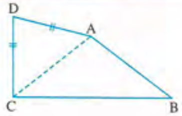
$\therefore$ m ($\angle$ ZLY) > m ($\angle$ ZYL) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ XLY) + m ($\angle$ ZLY) > m ($\angle$ XYL) + m ($\angle$ ZYL)

$\therefore$ m ($\angle$ XLZ) > m ($\angle$ XYZ) (Q.E.D.)

6



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ BC > AB

$\therefore$ m ($\angle$ BAC) > m ($\angle$ ACB) (1)

In ΔDAC : $\because$ DA = DC

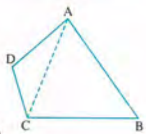
$\therefore$ m ($\angle$ DAC) = m ($\angle$ DCA) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BAC) + m ($\angle$ DAC) > m ($\angle$ ACB) + m ($\angle$ DCA)

$\therefore$ m ($\angle$ BAD) > m ($\angle$ BCD) (Q.E.D.)

7



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ AB > BC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ BAC) (1)

In ΔADC : $\because AD > DC$

$\therefore m(\angle ACD) > m(\angle CAD)$ (2)

Adding (1) and (2) :

$\therefore m(\angle BCD) > m(\angle BAD)$ (Q.E.D.)

8

In ΔMBC : $\because MC > MB$

$\therefore m(\angle MBC) > m(\angle MCB)$

$\because m(\angle MBC) = \frac{1}{2} m(\angle ABC)$

, $m(\angle MCB) = \frac{1}{2} m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) > \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle ABC) > m(\angle ACB)$ (Q.E.D.)

9

In ΔDBC : $\because DB > DC$

$\therefore m(\angle DCB) > m(\angle DBC)$

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC)$

$\therefore m(\angle ACB) - m(\angle DCB) < m(\angle ABC) - m(\angle DBC)$

$\therefore m(\angle ACD) < m(\angle ABD)$

i.e. $m(\angle ABD) > m(\angle ACD)$ (Q.E.D.)

10

In ΔABC : $\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$ (1)

$\because \overline{XY} \parallel \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles) (2)

Similarly : $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore m(\angle AXY) = m(\angle B)$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle AYX) > m(\angle AXY)$ (Q.E.D.)

11

$\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$

But $m(\angle C) = m(\angle AED)$ (corresponding angles)

, $m(\angle B) = m(\angle ADE)$ (corresponding angles)

$\therefore m(\angle AED) > m(\angle ADE)$

In ΔADE :

$\because m(\angle A) = 90°$

$\therefore m(\angle AED) + m(\angle ADE) = 90°$

, $\because m(\angle AED) > m(\angle ADE)$

$\therefore m(\angle AED) > \frac{90°}{2}$

$\therefore m(\angle AED) > 45°$ (Q.E.D.)

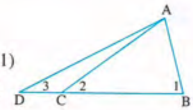
12

$\because AB > AC$, $BD = CE$ Subtracting $\therefore AD > AE$

$\therefore$ In ΔADE : $\because AD > AE$

$\therefore m(\angle AED) > m(\angle ADE)$ (Q.E.D.)

13



In ΔABC : $\because AC > AB$

$\because m(\angle 1) > m(\angle 2)$ (1)

But $\angle 2$ is an exterior angle of ΔACD

$\because m(\angle 2) > m(\angle 3)$ (2)

From (1) and (2) : $\therefore m(\angle 1) > m(\angle 3)$

$\therefore m(\angle ABD) > m(\angle D)$ (Q.E.D.)

14

$\because \Delta ABC$ is an equilateral triangle.

$\therefore m(\angle ABC) = m(\angle ACB) = 60°$

$\because m(\angle EBC) < m(\angle ECB)$ Subtracting

$\therefore m(\angle ABC) - m(\angle EBC) > m(\angle ACB) - m(\angle ECB)$

$\therefore m(\angle ABE) > m(\angle ACE)$ (1) (Q.E.D.1)

$\because m(\angle A) = m(\angle B)$

$\therefore m(\angle A) = m(\angle ABE) + m(\angle EBC)$

$\therefore m(\angle A) > m(\angle ABE)$ and from (1) :

$\therefore m(\angle A) > m(\angle ABE) > m(\angle ACE)$ (Q.E.D.2)

15

In ΔXBC : $\because XC > XB$

$\therefore m(\angle XBC) > m(\angle XCB)$

$\because$ ABCD is a rectangle.

i.e. $m(\angle ABC) = m(\angle DCB) = 90°$

$\therefore 90° - m(\angle XBC) < 90° - m(\angle XCB)$

$\therefore m(\angle ABX) < m(\angle XCD)$ (Q.E.D.)

16

In ΔADE : $\because AD = 5$ cm. , $AE = 3$ cm. $\therefore AD > AE$

$\therefore m(\angle AED) > m(\angle ADE)$

From the equilateral triangle ABC we find that :

$m(\angle A) = 60°$

$\because m(\angle AED) + m(\angle ADE) = 180° - 60° = 120°$

$\therefore m(\angle AED) > 60°$ (Q.E.D.)

17

In ΔABC : $\because AB > AC$ $\therefore m(\angle ACB) > m(\angle ABC)$

$\because 180° - m(\angle ACB) < 180° - m(\angle ABC)$

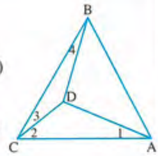
$\because D \in \overrightarrow{AB}$, $E \in \overrightarrow{AC}$ $\therefore m(\angle BCE) < m(\angle DBC)$

$\because \overrightarrow{BF}$ bisects $\angle DBC$, $\overrightarrow{CF}$ bisects $\angle BCE$

$\therefore m(\angle BCF) < m(\angle FBC)$

$\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.)

18



In ΔDBC : $\because DB > DC$

$\therefore m(\angle 3) > m(\angle 4)$ (1)

In ΔDAC : $\because DA > DC$

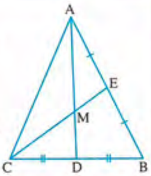
$\therefore m(\angle 2) > m(\angle 1)$ (2)

From (1) and (2) and adding :

$\therefore m(\angle 3) + m(\angle 2) > m(\angle 4) + m(\angle 1)$

$\therefore m(\angle ACB) > m(\angle DBC) + m(\angle DAC)$ (Q.E.D.)

19



$\because \overline{AD}$, $\overline{CE}$ are two medians of ΔABC intersecting at M

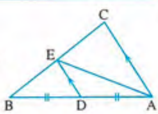
$\because AM = 2MD$, $MC = 2ME$

$\because MD > ME$ $\therefore AM > MC$

, thus in ΔAMC

$\therefore m(\angle CAM) < m(\angle MCA)$ (Q.E.D.)

20



In ΔABC : $\because$ D is the midpoint of $\overline{AB}$, $\overline{DE} \parallel \overline{AC}$

$\therefore DE = \frac{1}{2} AC$

, $\because AD = \frac{1}{2} AB$, $\because AB > AC$

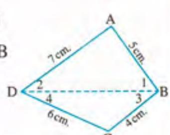
$\therefore \frac{1}{2} AB > \frac{1}{2} AC$

$\therefore AD > DE$ $\therefore m(\angle AED) > m(\angle DAE)$

But $m(\angle AED) = m(\angle CAE)$ (alternate angles)

$\therefore m(\angle CAE) > m(\angle DAE)$ (Q.E.D.)

21



First construction : Draw $\overline{BD}$

Proof : In ΔABD : $\because AD > AB$

$\therefore m(\angle 1) > m(\angle 2)$ (1)

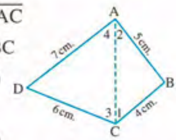
In ΔCBD : $\because CD > CB$

$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.1)



Second construction : Draw $\overline{AC}$

Proof : In ΔABC : $\because BA > BC$

$\therefore m(\angle 1) > m(\angle 2)$ (3)

In ΔADC : $\because AD > DC$

$\therefore m(\angle 3) > m(\angle 4)$ (4)

Adding (3) and (4) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$\because m(\angle BCD) > m(\angle BAD)$ (Q.E.D.2)

$\because$ The sum of measure of the interior angles of the quadrilateral = 360°

and from the two preceding requirements.

$\therefore m(\angle B) + m(\angle C) > \frac{360^\circ}{2}$

$\therefore m(\angle B) + m(\angle C) > 180^\circ$ (Q.E.D.3)

22

$\because \overline{AE}$ is a median in ΔABD , $m(\angle A) = 90^\circ$

$\therefore AE = \frac{1}{2} BD$

, $\because$ E is the midpoint of $\overline{BD}$, $\overline{EX} \parallel \overline{AC}$

$\therefore EX = \frac{1}{2} DC$

, $\because AE > EX$

$\therefore \frac{1}{2} BD > \frac{1}{2} DC$

$\therefore BD > DC$

$\therefore m(\angle C) > m(\angle DBC)$ (Q.E.D.)

23

In ΔABM : $\because AM > BM$ $\therefore m(\angle ABM) > m(\angle A)$ (1)

$\because AM = CM$ $\therefore$ In ΔCBM : $MC > MB$

$\therefore m(\angle MBC) > m(\angle C)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ABM) + m(\angle MBC) > m(\angle A) + m(\angle C)$

$\therefore m(\angle ABC) > m(\angle A) + m(\angle C)$

$\therefore \angle ABC$ is an obtuse angle. (Q.E.D)

24

In ΔABD : $\because$ $m(\angle B) = 90^\circ - m(\angle BAD)$ (1)

From ΔACD: $m(\angle C) = 90^\circ - m(\angle CAD)$ (2)

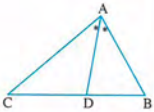
From ΔABC : $AC > AB$ $\therefore m(\angle B) > m(\angle C)$ (3)

From (1) , (2) and (3) :

$\therefore 90^\circ - m(\angle BAD) > 90^\circ - m(\angle CAD)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

25



In ΔABC : $\because AC > AB$

$\therefore m(\angle B) > m(\angle C)$

$\because m(\angle BAD) = m(\angle DAC)$

$(\overrightarrow{AD}$ bisects $\angle A)$

$\therefore m(\angle B) + m(\angle BAD) > m(\angle C) + m(\angle DAC)$

$\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

$\because \angle ADB$ is an exterior angle of ΔADC

$\therefore m(\angle ADB) = m(\angle C) + m(\angle DAC)$

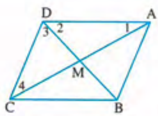
$\therefore m(\angle ADC) > m(\angle ADB)$

$\because m(\angle ADC) + m(\angle ADB) = 180°$

$\therefore m(\angle ADC) > \frac{180°}{2}$ *i.e.* $m(\angle ADC) > 90°$

i.e. $\angle ADC$ is an obtuse angle. (Q.E.D.)

26



Let $\overline{CA} \cap \overline{DB} = \{M\}$

$\because$ The two diagonals in the parallelogram bisects each other

, $\because AC > BD$ $\therefore MA > MD$,

$MC > MD$,

From ΔAMD : $\because AM > MD$

$\therefore m(\angle 2) > m(\angle 1)$ (1)

From ΔDMC : $\because MC > MD$

$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 2) + m(\angle 3) > m(\angle 1) + m(\angle 4)$

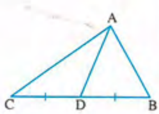
$\therefore m(\angle D) > m(\angle 1) + m(\angle 4)$

$\therefore$ In ΔADC :

$\because m(\angle D) > m(\angle CAD) + m(\angle ACD)$

$\therefore \angle D$ is an obtuse angle. (Q.E.D.)

27



$\because$ The perimeter of ΔACD

$= CD + DA + AC$

The perimeter of ΔABD

$= BD + DA + AB$

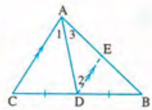
$\because$ The perimeter of $\Delta ACD >$ the perimeter of ΔABD

$\therefore CD + DA + AC > BD + DA + AB$

But $CD = BD$

$\therefore AC > AB$ $\therefore m(\angle B) > m(\angle C)$ (Q.E.D.)

28



Construction : Draw $\overrightarrow{DE} \parallel \overline{AC}$ to intersect $\overline{AB}$ at E

Proof : In ΔABC : $\because \overline{DE} \parallel \overline{AC}$,

D is the midpoint of $\overline{BC}$

$\therefore$ E is the midpoint of $\overline{BA}$

$\therefore AE = \frac{1}{2}AB$, $DE = \frac{1}{2}AC$

, $\because AB > AC$ $\therefore AE > DE$

$\because m(\angle 2) > m(\angle 3)$ (1)

$\because \overline{DE} \parallel \overline{AC}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore m(\angle 1) = m(\angle 2)$ (Alternate angles)

From (1) : $\therefore m(\angle 1) > m(\angle 3)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

Answers of Exercise 8

1

1 A side greater in length than that opposite to the other angle , greater in measure than the measure of the angle opposite to the other side.

2 The shortest side.

3 The hypotenuse.

4 The length of the line segment drawn from the given point perpendicular to the given straight line.

5 $\overline{AB}$ 6 $\overline{AC}$ 7 $\overline{BC}$

2

1 c 2 a 3 d 4 a

3

1 $>$, $>$, $<$ 2 $>$, $>$, $>$

3 $>$, $>$, $>$, $>$ 4 $>$, $<$, $<$, $>$

4 $YZ < XY < XZ$

5 $AC > AB > BC$

6

$\because \overline{AE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle C) = m(\angle EAC) = 30°$ (alternate angles) (1)

, $\because \overline{AE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore m(\angle B) = m(\angle DAE) = 70°$

(corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle B) > m(\angle C)$

$\therefore AC > AB$ (Q.E.D.)

7

$\because C \in \overrightarrow{AE}$ $\quad \therefore m(\angle ACB) = 180° - 120° = 60°$

$\because B \in \overrightarrow{CD}$ $\quad \therefore m(\angle ABC) = 180° - 110° = 70°$

$\therefore m(\angle A) = 180° - (60° + 70°) = 50°$

$\therefore m(\angle ACB) > m(\angle A)$ $\quad \therefore AB > BC$ (Q.E.D.)

8

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle B) = 65°$

$\therefore m(\angle DCB) = 65° + 20° = 85°$

In ΔDBC : $\therefore m(\angle D) = 180° - (65° + 85°) = 30°$

$\therefore$ In ΔDAC : $m(\angle D) > m(\angle ACD)$

$\therefore AC > AD$ but $AB = AC$

$\therefore AB > AD$ (Q.E.D.)

9

In ΔDBC : $\because DB = DC$

$\therefore m(\angle B) = m(\angle DCB) = \frac{180° - 100°}{2} = 40°$

$\because \overrightarrow{CD}$ bisects $\angle ACB$ $\quad \therefore m(\angle ACD) = 40°$

$\because D \in \overline{AB}$

$\therefore m(\angle ADC) = 180° - 100° = 80°$

$\therefore$ In ΔADC : $m(\angle A) = 180° - (40° + 80°) = 60°$

$\therefore m(\angle ADC) > m(\angle A)$

$\therefore AC > DC$ but $DC = DB$

$\therefore AC > DB$ (Q.E.D.)

10

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle ACB) = m(\angle DAC) = 30°$ (alternate angles)

In ΔABC : $\because m(\angle BAC) > m(\angle ACB)$

$\therefore BC > AB$ (Q.E.D.)

11

In ΔACM : $\because m(\angle C) = 90°$ $\quad \therefore AM > CM$ (1)

In ΔBDM : $\because m(\angle D) = 90°$ $\quad \therefore BM > DM$ (2)

Adding (1) and (2) : $\therefore AM + MB > CM + MD$

$\therefore AB > CD$ (Q.E.D.)

12

In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\because m(\angle ABM) < m(\angle ACM)$

$\therefore m(\angle ABC) - m(\angle ABM) > m(\angle ACB) - m(\angle ACM)$

$\therefore m(\angle MBC) > m(\angle MCB)$

From ΔMBC : $\therefore MC > MB$ (Q.E.D.)

13

In ΔABC : $\because \angle B$ is an obtuse angle.

$\therefore m(\angle B) > m(\angle C)$ (1)

$\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal.

$\therefore m(\angle ADE) = m(\angle B)$ (corresponding angles) (2)

$\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{EC}$ is a transversal.

$\therefore m(\angle AED) = m(\angle C)$ (corresponding angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle ADE) > m(\angle AED)$

$\therefore AE > AD$ (Q.E.D.)

14

In ΔABC : $\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$ (1)

$\because \overline{DE} \parallel \overline{BC}$ and $\overleftrightarrow{DC}$ is a transversal.

$\therefore m(\angle D) = m(\angle C)$ (alternate angles) (2)

$\therefore \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{BE}$ is a transversal.

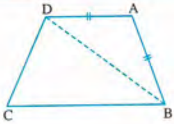
$\therefore m(\angle E) = m(\angle B)$ (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle D) > m(\angle E)$ and from ΔADE

$\therefore AE > AD$ (Q.E.D.)

15



Const. : Draw $\overline{BD}$

Proof : In ΔADB

$\because AD = AB$

$\therefore m(\angle ADB) = m(\angle ABD)$

, $\because m(\angle ADC) > m(\angle ABC)$

$\therefore m(\angle ADC) - m(\angle ADB) > m(\angle ABC) - m(\angle ABD)$

$\therefore m(\angle BDC) > m(\angle DBC)$

$\therefore$ In ΔBDC : $BC > CD$ (Q.E.D.)

16

In ΔABC : $\because AB > AC$

$\therefore m(\angle ABC) < m(\angle ACB)$

, $\because B \in \overline{AD}$, $C \in \overline{AE}$

$\therefore 180° - m(\angle ABC) > 180° - m(\angle ACB)$

$\therefore m(\angle CBD) > m(\angle BCE)$

$\because \overrightarrow{BF}$ bisects $\angle DBC$, $\overrightarrow{CF}$ bisects $\angle BCE$

$\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.1)

$\therefore CF > BF$ (Q.E.D.2)

17

In ΔABD : $\because BD = AD$

$\therefore m(\angle BAD) = m(\angle B)$

$\therefore m(\angle BAD) + m(\angle DAC) > m(\angle B)$

$\therefore m(\angle BAC) > m(\angle B)$ $\therefore BC > AC$ (Q.E.D.)

18

In ΔDBC : $\because m(\angle B) > m(\angle DCB)$

$\therefore DC > DB$ but $DB = AD$ $\therefore DC > AD$

$\therefore$ In ΔADC : $m(\angle A) > m(\angle ACD)$ (Q.E.D.1)

$\because m(\angle BDC) = 180° - (70° + 50°) = 60°$

$\because \angle BCD$ is an exterior angle of ΔADC

$\therefore m(\angle BDC) = m(\angle A) + m(\angle ACD) = 60°$

$\because m(\angle A) > m(\angle ACD)$ $\therefore m(\angle ACD) < 30°$

$\therefore m(\angle ACD) + m(\angle DCB) < 30° + 50°$

$\therefore m(\angle ACB) < 80°$

$\therefore \angle ACB$ is an acute angle. (Q.E.D.2)

19

In ΔAFB : $\because FA = FB$

$\therefore m(\angle FBA) = m(\angle FAB) = 50°$ (1)

$\because \angle AFD$ is an exterior angle of ΔAFB

$\therefore m(\angle AFD) = 50° + 50° = 100°$ (2)

$\therefore$ In ΔAFD :

$\because FA = FD$ $\therefore m(\angle FDA) = \frac{180° - 100°}{2} = 40°$

From (1) and (2) : $\therefore$ In ΔABD

$m(\angle ABD) > m(\angle ADB)$

$\therefore AD > AB$ (Q.E.D.1)

In ΔABD : $\because \overline{AF}$ is a median , $AF = \frac{1}{2}BD$

$\therefore m(\angle DAB) = 90°$

$\therefore \overline{BC}$ is a hypotenuse of ΔBAC

$\therefore BC > AC$ (Q.E.D.2)

20

$\because \angle ADB$ is an exterior angle of ΔADC

$\therefore m(\angle ADB) > m(\angle C)$

, $\because m(\angle C) = m(\angle B)$, ($AB = AC$ in ΔABC)

$\therefore m(\angle ADB) > m(\angle B)$

And from ΔABD : $AB > AD$ (Q.E.D.)

21

$\Delta\Delta ABD$, AED in them

$m(\angle B) = m(\angle AED)$, $m(\angle BAD) = m(\angle DAE)$

$\therefore m(\angle ADB) = m(\angle ADE)$

$\therefore \Delta\Delta ABD$, AED

In them $\begin{cases} m(\angle BAD) = m(\angle EAD) \\ m(\angle ADB) = m(\angle ADE) \\ \overline{AD} \text{ is a common side} \end{cases}$

$\therefore \Delta ABD \equiv \Delta AED$, then we deduce that :

$BD = DE$ (Q.E.D.1)

$\therefore$ In ΔDEC

$\because m(\angle DEC) = 90°$ $\therefore DC > DE$

$\because DE = DB$ $\therefore DC > DB$ (Q.E.D.2)

22

$\because m(\angle ADC) = 180° - 110° = 70°$

$\therefore \Delta ACD$ in which $m(\angle ADC) > m(\angle C)$

$\therefore AC > AD$ (1)

$\because \Delta ADB$ is an obtuse-angled at D

$\therefore AB > AD$ (2)

By adding (1) and (2) : $\therefore AB + AC > 2 AD$ (Q.E.D.)

23

In ΔABC : $\because m(\angle B) = 90°$

$\therefore$ The hypotenuse $\overline{AC}$ is the longest side.

$\therefore AB < AC$, $BC < AC$

By adding : $\therefore AB + BC < 2 AC$ (Q.E.D.)

24

$\because \overline{AD} // \overline{CE}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle DAC) = m(\angle ACE)$

$\therefore m(\angle BCE) > m(\angle DAC)$

$\therefore m(\angle BCE) > m(\angle BAD)$, ($\overrightarrow{AD}$ bisects $\angle BAC$) (1)

$\because \overline{AD} // \overline{CE}$ and $\overleftrightarrow{BE}$ is a transversal.

$\therefore m(\angle BAD) = m(\angle E)$ (corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle BCE) > m(\angle E)$ and from ΔBCE

$\therefore BE > BC$ (Q.E.D.)

25

$\because \Delta$ XYM is right-angled at Y

$\therefore \angle$ XMY is an acute angle.

$\therefore \angle$ XMZ is an obtuse angle.

$\because \Delta$ XMZ is an obtuse-angled at M

$\therefore$ XZ > XM (Q.E.D.)

26

In Δ ABC : $\because$ AB > BC

$\therefore$ m ($\angle$ C) > m ($\angle$ A) (1)

, $\because$ m ($\angle$ ABC) = 90°

$\therefore \angle$ A complements $\angle$ C (2)

, in Δ ABD : $\because$ m ($\angle$ ADB) = 90°

$\therefore \angle$ A complements $\angle$ ABD (3)

From (2) and (3) :

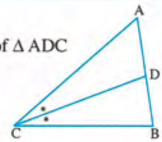
$\therefore$ m ($\angle$ C) = m ($\angle$ ABD)

, from (1) :

$\therefore$ m ($\angle$ ABD) > m ($\angle$ A)

$\therefore$ In Δ ABD : AD > BD (Q.E.D.)

27



$\because \angle$ BDC is an exterior angle of Δ ADC

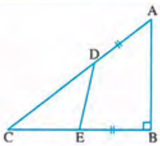
$\therefore$ m ($\angle$ BDC) > m ($\angle$ ACD)

$\because$ m ($\angle$ BCD) = m ($\angle$ ACD)

$\therefore$ m ($\angle$ BDC) > m ($\angle$ BCD)

In Δ DBC : $\therefore$ BC > BD (Q.E.D.)

28



In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore$ AC > BC

$\because$ AD = BE

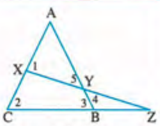
$\therefore$ AC – AD > BC – BE

$\therefore$ DC > EC

$\therefore$ In Δ DEC : $\because$ DC > EC

$\therefore$ m ($\angle$ CED) > m ($\angle$ CDE) (Q.E.D.)

29



$\because \angle$ 1 is an exterior angle of Δ XZC

$\therefore$ m ($\angle$ 1) > m ($\angle$ 2)

But m ($\angle$ 3) = m ($\angle$ 2) , (AB = AC in Δ ABC)

$\therefore$ m ($\angle$ 1) > m ($\angle$ 3)

But $\angle$ 3 is an exterior angle of Δ YZB

$\therefore$ m ($\angle$ 3) > m ($\angle$ 4) $\therefore$ m ($\angle$ 1) > m ($\angle$ 4)

, $\because$ m ($\angle$ 4) = m ($\angle$ 5) (V.O.A.)

$\therefore$ m ($\angle$ 1) > m ($\angle$ 5)

and from Δ AYX : $\therefore$ AY > AX (Q.E.D.)

30

$\because$ m ($\angle$ A) + m ($\angle$ B) + m ($\angle$ C) = 180°

$\therefore 5x + 2° + 6x - 10° + x + 20° = 180°$

$\therefore 12x + 12° = 180°$ $\therefore 12x = 180° - 12° = 168°$

$\therefore x = \frac{168°}{12°} = 14°$ $\therefore$ m ($\angle$ A) = $5 \times 14° + 2° = 72°$

, m ($\angle$ B) = $6 \times 14° - 10° = 74°$

m ($\angle$ C) = $14° + 20° = 34°$

$\therefore$ AB < BC < AC (The req.)

31

In Δ ABC : $\because$ AB < AC

$\therefore$ m ($\angle$ ACB) < m ($\angle$ ABC)

$\therefore \frac{1}{2}$ m ($\angle$ ACB) < $\frac{1}{2}$ m ($\angle$ ABC)

$\therefore$ m ($\angle$ MCB) < m ($\angle$ MBC) (1)

And from Δ MBC : MB < MC (2)

$\because \overline{XY} \parallel \overline{CB}$, $\overleftrightarrow{XB}$ is a transversal.

$\therefore$ m ($\angle$ X) = m ($\angle$ MBC) (alternate angles) (3)

$\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{CY}$ is a transversal.

$\therefore$ m ($\angle$ Y) = m ($\angle$ MCB) (alternate angles)

$\because$ In Δ XMY from (1) , (2) and (3) :

$\because$ m ($\angle$ Y) < m ($\angle$ X) $\therefore$ XM < MY (4)

By adding (2) and (4) : $\therefore$ MB + MX < MC + MY

$\therefore$ BX < CY (Q.E.D.)

Answers of Exercise 9

1

1. $\because 3 + 4 < 9$ $\therefore$ lengths are not suitable
2. $\because 5 + 7 > 8$ $\therefore$ lengths are suitable
3. $\because 4 + 6 = 10$ $\therefore$ lengths are not suitable
4. $\because 6 + 8 > 13$ $\therefore$ lengths are suitable
5. $\because 3 + 4 > 5$ $\therefore$ lengths are suitable
6. $\because 9 + 9 < 19$ $\therefore$ lengths are not suitable

2

Let the length of the third side be ℓ

1 $\because 9-6<\ell<9+6 \quad \therefore 3<\ell<15$

$\therefore \ell \in]3, 15[$

2 $\because 3-3<\ell<3+3 \quad \therefore 0<\ell<6$

$\therefore \ell \in]0, 6[$

3 $\because 3.2-2.9<\ell<3.2+2.9$

$\therefore 0.3<\ell<6.1 \quad \therefore \ell \in]0.3, 6.1[$

4 $\because 7.3-5.7<\ell<7.3+5.7$

$\therefore 1.6<\ell<13 \quad \therefore \ell \in]1.6, 13[$

3

1 b	2 b	3 c	4 d	5 a
6 b	7 d	8 b	9 a	10 a

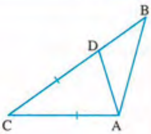
4

In Δ XLY : XL + LY > XY (The triangle inequality)

But XL = LZ $\quad \therefore$ LZ + LY > XY

$\therefore$ YZ > XY (Q.E.D.)

5



In Δ ABC :

$\because$ CA + AB > BC

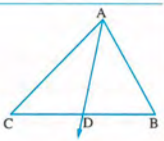
(triangle inequality)

$\therefore$ CA + AB > BD + DC

But CA = DC

$\therefore$ AB > BD (Q.E.D.)

6



From Δ ABD :

AD + DB > AB

(triangle inequality) (1)

From Δ ADC : AD + DC > AC

(Triangle inequality) (2)

Adding (1) and (2) :

$\therefore$ BD + DC + 2 AD > AB + AC (Q.E.D.)

7

From Δ ABM : MA + MB > AB

(Triangle inequality) (1)

From Δ BMC : MB + MC > BC

(Triangle inequality) (2)

From Δ AMC : MA + MC > AC (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 MA + 2 MB + 2 MC > AB + BC + AC

$\therefore$ MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

(Q.E.D.)

8

From Δ AEZ : AE + AZ > EZ (Triangle inequality) (1)

From Δ EBF :

EB + BF > EF (Triangle inequality) (2)

From Δ ZFC : ZC + CF > ZF (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ AB + AC + BC > EZ + EF + ZF

$\therefore$ The perimeter of Δ ABC > the perimeter of Δ EFZ

(Q.E.D.)

9

In Δ DAC : DA + DC > AC (1)

In Δ DBC : DB + DC > BC (2)

In Δ DBA : DB + DA > AB (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 (DA + DB + DC) > AC + BC + AB

$\therefore$ AC + BC + AB < 2 (DA + DB + DC)

$\therefore$ The perimeter of Δ ABC < 2 (DA + DB + DC)

(Q.E.D.)

10

$\because 7-3<AC<7+3 \quad \therefore 4<AC<10$

$\therefore AC \in]4, 10[\quad \therefore AB<AC$

$\therefore m(\angle C) < m(\angle B)$ (The req.)

11

Assuming that ABC is a triangle

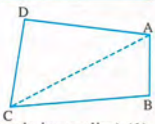
$\because$ AB < AC + BC adding AB to both sides

$\therefore$ 2 AB < AC + BC + AB

$\therefore$ AB < $\frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in the triangle is less than the half of the perimeter of the triangle (Q.E.D.)

12



Construction : Draw $\overline{AC}$

Proof : From Δ ABC

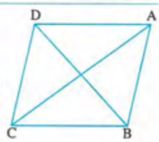
AB + BC > AC (Triangle inequality) (1)

From Δ ADC :

AC + CD > AD (Triangle inequality) (2)

From (1) and (2) : $\therefore$ AB + BC + CD > AD (Q.E.D.)

13



Let ABCD be a quadrilateral

In Δ ABC : AB + BC > AC (1)

In Δ BCD : BC + CD > BD (2)

In Δ ACD : AD + CD > AC (3)

In Δ ABD : AB + AD > BD (4)

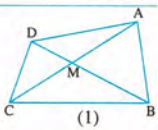
Adding (1) , (2) , (3) , and (4) :

$\because$ 2 AB + 2 BC + 2 CD + 2 AD > 2 AC + 2 BD

$\therefore$ AB + BC + CD + AD > AC + BD

$\therefore$ The sum of lengths of the two diagonals in any convex quadrilateral is less than the perimeter of the quadrilateral. (Q.E.D.)

14



Let ABCD be a quadrilateral ,

$\overline{AC} \cap \overline{BD} = \{M\}$

From Δ ABM : AB < MA + MB (1)

From Δ BMC : BC < MB + MC (2)

From Δ CMD : CD < MC + MD (3)

From Δ AMD : AD < MA + MD (4)

Adding (1) , (2) , (3) and (4)

$\therefore$ AB + BC + CD + AD

< 2 MA + 2 MC + 2 MB + 2 MD

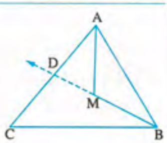
$\therefore$ AB + BC + CD + DA

< 2 (MA + MC) + 2 (MB + MD)

$\therefore$ AB + BC + CD + DA < 2 (AC + BD)

$\therefore$ The perimeter of the quadrilateral ABCD < twice the sum of lengths of the two diagonals. (Q.E.D.)

15



Construction :

Draw $\overrightarrow{BM}$ to cut $\overline{AC}$ at D

Proof :

In Δ BDC :

BC + DC > BD (Triangle inequality)

$\therefore$ BC + DC > BM + MD (1)

$\because$ In Δ AMD : AD + MD > AM (Triangle inequality)

$\therefore$ AD > AM – MD (2)

Adding (1) , (2) :

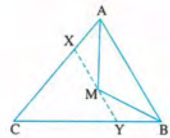
$\therefore$ BC + AD + DC > BM + MD + AM – MD

$\therefore$ BC + AC > BM + AM

$\therefore$ AM + MB < BC + AC (Q.E.D.)

Another solution :

Construction :



Draw $\overline{XY}$ Passing through the point M where

$X \in \overline{AC}$, $Y \in \overline{BC}$

Proof : In Δ CXY

$\because$ CY + CX > XM + MY adding BY and AX to both sides.

$\therefore$ CY + BY + CX + AX > XM + AX + MY + BY

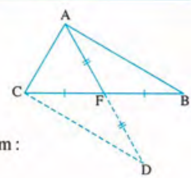
$\therefore$ BC + AC > XM + AX + MY + BY

$\because$ XM + AX > AM , MY + BY > MB

$\therefore$ BC + AC > AM + MB

$\therefore$ AM + MB < AC + BC (Q.E.D.)

16



Construction :

Extend $\overline{AF}$ as its length to D then draw $\overline{CD}$

Proof : $\Delta\Delta$ AFB , DFC in them :

$$\begin{cases} AF = DF \text{ const.} \\ BF = FC \text{ (given)} \\ m(\angle AFB) = m(\angle DFC) \quad \text{(V.O.A.)} \end{cases}$$

$\therefore$ The two triangles are congruent

then we deduce that AB = DC

but in Δ ACD we find that :

AC + CD > AD (triangle inequality)

$\therefore$ AC + AB > AD

$\because$ AD = 2 AF

$\therefore$ AC + AB > 2 AF (1) (Q.E.D.1)

From Δ ABC : $\because$ AB + AC > BC

i.e. AB + AC > 2 BF (2)

Adding (1) and (2) : 2 AB + 2 AC > 2 AF + 2 BF

Dividing by 2 : $\therefore$ AB + AC > AF + BF (Q.E.D.2)

Answers of accumulative basic skills

1

1 $2\sqrt{10}$	2 $2:3$	3 5
4 150°	5 18	6 54
7 $\frac{1}{2}P-y$	8 $5\sqrt{3}$	9 108°
10 60	11 9	12 19

2

1 (b)	2 (c)	3 (b)
4 (d)	5 (a)	6 (c)
7 (a)	8 (c)	9 (d)
10 (a)	11 (c)	12 (b)
13 (d)	14 (c)	

Guide Answers Of The Notebook

- Accumulative Tests.
- Important Questions.
- Final Examinations.

Answers of the accumulative tests on Algebra and Statistics

Accumulative test 1

1 $\boxed{1}$ c $\boxed{2}$ c $\boxed{3}$ c $\boxed{4}$ c
$\boxed{5}$ c $\boxed{6}$ b $\boxed{7}$ d $\boxed{8}$ d

2 $\boxed{1}$ $\{-1\}$ $\boxed{2}$ $\left\{\frac{1}{2}\right\}$

Accumulative test 2

1 $\boxed{1}$ c $\boxed{2}$ a $\boxed{3}$ a $\boxed{4}$ c
$\boxed{5}$ b $\boxed{6}$ c $\boxed{7}$ c $\boxed{8}$ a

2 **[a]** Prove by yourself.
[b] Prove by yourself.

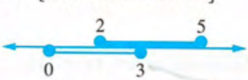
Accumulative test 3

$\boxed{1}$ c $\boxed{2}$ d $\boxed{3}$ c $\boxed{4}$ c
$\boxed{5}$ a $\boxed{6}$ c $\boxed{7}$ a $\boxed{8}$ d
$\boxed{9}$ d $\boxed{10}$ d

Accumulative test 4

1 $\boxed{1}$ d $\boxed{2}$ c $\boxed{3}$ b $\boxed{4}$ c
$\boxed{5}$ c $\boxed{6}$ c $\boxed{7}$ c $\boxed{8}$ d

2 $\boxed{1}$ $X = \{x : x \in \mathbb{R}, 2 \leq x \leq 5\}$

$\boxed{2}$ 

$\boxed{3}$ $X - Y =]3, 5]$ $\because \sqrt{29} \simeq 5.39$
$\therefore \sqrt{29} \notin X - Y$

3 

$X \cup Y = [-1, \infty[$, $X \cap Y = [-1, 3[$
, $Y - X =]4, \infty[$

Accumulative test 5

1 $\boxed{1}$ c $\boxed{2}$ c $\boxed{3}$ a $\boxed{4}$ b
$\boxed{5}$ c $\boxed{6}$ b $\boxed{7}$ c $\boxed{8}$ b

2 $4 + 2\sqrt{3}$

3 $\boxed{1}$ 22 $\boxed{2}$ 56

Accumulative test 6

1 $\boxed{1}$ b $\boxed{2}$ d $\boxed{3}$ b $\boxed{4}$ a
$\boxed{5}$ a $\boxed{6}$ b $\boxed{7}$ c $\boxed{8}$ b

2 

$\boxed{1}$ $A \cap B = [-2, 3[$ $\boxed{2}$ $B - A = [3, 5]$

3 $\boxed{1}$ $2\sqrt{6}$ $\boxed{2}$ zero

Accumulative test 7

1 $\boxed{1}$ b $\boxed{2}$ c $\boxed{3}$ b $\boxed{4}$ a
$\boxed{5}$ c $\boxed{6}$ b $\boxed{7}$ c $\boxed{8}$ b

2 **[a]** 7
[b] $10 - 4\sqrt{5}$

3 Prove by yourself , $\frac{4}{5}$

Accumulative test 8

1 $\boxed{1}$ b $\boxed{2}$ a $\boxed{3}$ a $\boxed{4}$ b
$\boxed{5}$ c $\boxed{6}$ b $\boxed{7}$ c $\boxed{8}$ d

2 $\boxed{1}$ $6\sqrt[3]{2}$ $\boxed{2}$ -1

3 **[a]** 

$\boxed{1}$ $X \cap Y = [-2, 1]$
$\boxed{2}$ $X - Y =]1, 3]$

[b] 1

Accumulative test 9

1 1 c 2 c 3 a 4 a
5 d 6 b 7 b 8 d

2 **[a]** 4 cm.
[b] Prove by yourself , 4

3 **[a]** 36π cm^2
[b] $7\sqrt{5}-4\sqrt[3]{2}$

Accumulative test 10

1 1 b 2 a 3 b 4 d
5 c 6 c 7 b 8 a

2 **[a]** 15 cm.
[b] $[-2, 3[$, represent by yourself.

3 **[a]**

1 $X \cup Y = [-1, 6]$
2 $X \cap Y = [2, 4[$

[b] $[-1, 2[$, represent by yourself.

Accumulative test 11

1 1 d 2 b 3 c 4 c
5 a 6 b 7 a 8 c

2 **[a]** Find by yourself.
[b] $3\sqrt{3}$

3 **[a]** $[3, \infty[$, represent by yourself.
[b] Represent by yourself , 2 square units.

Accumulative test 12

1 1 c 2 a 3 c 4 c
5 d 6 c 7 a 8 d

2 **[a]** Represent by yourself , 1
[b] $[0, 2]$, represent by yourself.

3 **[a]** Prove by yourself.
[b] zero.

Accumulative test 13

1 1 a 2 c 3 d 4 b 5 c

2 1 $\frac{75}{8}$, 5 , zero
2 50 cm.

3 **[a]** 440 cm^2 , 1540 cm^3
[b] Represent by yourself , $\left(\frac{-1}{2}, 0\right)$, $(0, 1)$

4 1 $]0, 3]$ 2 $[-2, \infty[$
3 $]-\infty, 0]$

Accumulative test 14

1 1 c 2 c 3 c 4 b

2 Form by yourself.

3 **[a]** $]1, 5[$
[b] Find and represent by yourself.

4 **[a]** zero
[b] $y = \sqrt{5} - 2$, prove by yourself.

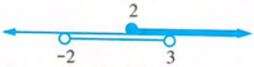
Accumulative test 15

1 1 b 2 b 3 b 4 b

2 1 20 workers.
2 Graph by yourself.

3 1 $\{\sqrt{2}\}$
2 $[-1, 5[$, represent by yourself.

4 [a] $\frac{2}{3}$, the point $C \notin \overleftrightarrow{AB}$

[b] 

1 $X \cap Y = [2 , 3[$

2 $X \cup Y =]-2 , \infty[$

3 $X - Y = [3 , \infty[$

Accumulative test 16

1 1 a 2 d 3 b 4 a
5 c 6 a 7 b 8 b

2 1 $x = 60$, $k = 14$

2 The arithmetic mean = 50.6

3 [a] 14 cm.

[b] $x = \sqrt{7} + \sqrt{3}$, $y = \sqrt{7} - \sqrt{3}$, $x^2 y^2 = 16$

Accumulative test 17

1 1 b 2 c 3 a 4 c
5 b 6 c 7 d 8 c

2 [a] Graph by yourself.

[b] 1 $k = 15$

2 The median = 52 approximately.

3 [a] $[-2 , 1]$

[b] 

1 $X \cap Y = [5 , 7]$

2 $X \cup Y =]3 , \infty[$

3 $X - Y =]3 , 5[$

Accumulative test 18

1 1 c 2 b 3 d 4 d
5 a 6 c 7 a 8 d

2 [a] $\{\sqrt{7}\}$

[b] 7

3 [a] 4

[b] 1 $x = 110$, $k = 20$

2 The mode of wages ≃ L.E. 105

Answers of important questions on Algebra and Statistics

Unit one

First Answers of multiple choice questions

1 (a)	2 (c)	3 (a)	4 (d)	5 (b)
6 (c)	7 (c)	8 (a)	9 (b)	10 (a)
11 (c)	12 (b)	13 (b)	14 (c)	15 (c)
16 (d)	17 (c)	18 (c)	19 (b)	20 (d)
21 (a)	22 (a)	23 (c)	24 (b)	25 (d)
26 (c)				

Second Answers of complete questions

1 $\mathbb{R}$ 2 $\varnothing$ 3 12 4 $\{0\}$

5 $]3 , 4]$ 6 $\sqrt{3}-\sqrt{2}$ 7 $\sqrt{5}+2$

8 $]0 , \infty[$ 9 12 10 3 , 4 11 $\sqrt{50}$

12 64 cm^3. 13 $\frac{4}{3}$ 14 3 15 12 cm.

16 20π 17 2 18 2 19 $(-1 , 2\sqrt{3})$

20 5 21 $]-2 , 3]$ 22 $\{-1 , 0 , 1\}$

23 $[-3 , 0[$ 24 2

Third Answers of essay questions

1

1 $X \cup Y = [-3 , 5]$

2 $X \cap Y = [-1 , 2[$

3 $X - Y = [-3 , -1[$

2

• $B - A = [3 , 5]$ • $A \cap B = [-2 , 3[$

• $A \cup B =]-\infty , 5]$ • $\grave{A} = [3 , \infty[$

3 $\because y = \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{2(\sqrt{3}-1)}{3-1} = \sqrt{3}-1$

$\therefore \frac{x y}{x - y} = \frac{(\sqrt{3}+1)(\sqrt{3}-1)}{\sqrt{3}+1-\sqrt{3}+1} = \frac{3-1}{2} = 1$

4 $\because x = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

, $y = \sqrt{5}+2$ $\therefore x , y$ are conjugate.

$\therefore x^2 y^2 = (x y)^2 = [(\sqrt{5}-2)(\sqrt{5}+2)]^2$

$= [5-4]^2 = 1$

5 $\because y = \frac{2}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$= \sqrt{7}+\sqrt{5}$

$\therefore x^2 + 2 x y + y^2 = (x + y)^2$

$= (\sqrt{7}-\sqrt{5}+\sqrt{7}+\sqrt{5})^2$

$= (2\sqrt{7})^2 = 28$

6 $\because y = \frac{2}{x}$

$= \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2(\sqrt{7}-\sqrt{5})}{7-5}$

$= \sqrt{7}-\sqrt{5}$

, $x = \sqrt{7}+\sqrt{5}$

$\therefore x , y$ are two conjugate numbers

$\therefore x^2 + x y + y^2 = (x + y)^2 - x y$

$= (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2$

$- (\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})$

$= (2\sqrt{7})^2 - 2 = 26$

7 $(x + y)^2 = x^2 + 2 x y + y^2 = \left(\sqrt{4+\sqrt{7}}\right)^2$

$+ 2\left(\sqrt{4+\sqrt{7}}\right)\left(\sqrt{4-\sqrt{7}}\right)$

$+ \left(\sqrt{4-\sqrt{7}}\right)^2$

$= 4+\sqrt{7}+2\sqrt{(4+\sqrt{7})(4-\sqrt{7})}$

$+ 4-\sqrt{7}$

$= 8 + 2\sqrt{16-7} = 8 + 2\sqrt{9} = 8 + 6 = 14$

8 $\because x = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{11+2\sqrt{30}}{6-5}$

$= 11 + 2\sqrt{30}$

, $\frac{1}{x} = \frac{1}{11+2\sqrt{30}} \times \frac{11-2\sqrt{30}}{11-2\sqrt{30}}$

$= \frac{11-2\sqrt{30}}{121-120} = 11 - 2\sqrt{30}$

$\therefore x + \frac{1}{x} = 11 + 2\sqrt{30} + 11 - 2\sqrt{30} = 22$

9 $\because 5x-3<2x+9$ $\therefore 5x-2x<9+3$

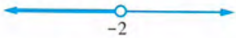
$\therefore 3x<12$ $\therefore x<4$

$\therefore$ The S.S. $=]-\infty , 4[$

10 $\because 5-3x>11$ $\therefore -3x>6$

$\therefore x<-2$

$\therefore$ The S.S. $=]-\infty , -2[$



11 $\because 1<2x+3\le 9$ $\therefore -2<2x\le 6$

$\therefore -1<x\le 3$

$\therefore$ The S.S. $=]-1 , 3]$

12 $\because \frac{x}{3}+2>3$ $\therefore \frac{x}{3}>1$

$\therefore x>3$

$\therefore$ The S.S. $=]3 , \infty[$

13 $\because 4x+3>5x+2>4x$ $\therefore 3>x+2>0$

$\therefore 1>x>-2$

$\therefore$ The S.S. $=]-2 , 1[$

14 The expression $=\sqrt[3]{2\times 4}+2\sqrt[3]{2}-2\sqrt[3]{2}-2$

$=2-2=$ zero

15 The expression $=\sqrt{25\times 3}-2\sqrt{9\times 3}+\sqrt{9\times \frac{1}{3}}$

$=5\sqrt{3}-6\sqrt{3}+\sqrt{3}=$ zero

16 The expression $=\sqrt[3]{27\times 3}+\sqrt[3]{8\times 3}-\sqrt[3]{27\times \frac{1}{9}}$

$=3\sqrt[3]{3}+2\sqrt[3]{3}-\sqrt[3]{3}=4\sqrt[3]{3}$

17 The expression $=\sqrt{4\times 3}+\sqrt[3]{27\times 2}-2\sqrt{3}$

$-\sqrt[3]{8\times 2}$

$=2\sqrt{3}+3\sqrt[3]{2}-2\sqrt{3}-2\sqrt[3]{2}$

$=\sqrt[3]{2}$

18 The expression $=\sqrt{25\times 2}+\sqrt[3]{27\times 2}-5\sqrt{4\times \frac{1}{2}}$

$-\sqrt[3]{8\times 2}$

$=5\sqrt{2}+3\sqrt[3]{2}-5\sqrt{2}-2\sqrt[3]{2}$

$=\sqrt[3]{2}$

19 $\because (x-2)^3=125$ $\therefore x-2=5$

$\therefore x=7$

$\therefore$ The S.S. $=\{7\}$

20 [1] $\because$ The volume of the sphere $=\frac{4}{3}\pi r^3$

$\therefore 36\pi=\frac{4}{3}\times\pi\times r^3$ $\therefore r^3=\frac{36\times 3}{4}=27$

$\therefore r=3$ cm.

[2] The area of the sphere $=4\pi r^2$

$=4\times 3^2\times\pi=36\pi$ cm^2.

21 $\because$ The volume of the cylinder $=\pi r^2 h$

, $\because h=r$

$\therefore$ The volume of the cylinder $=\pi h^3$

$\therefore 27\pi=\pi h^3$ $\therefore h^3=27$ $\therefore h=3$ cm.

22 The volume of the cylinder

$=\pi r^2 h=\pi\left(4\sqrt{2}\right)^2\times 9=288\pi$ cm^3.

23 The volume of the cylinder

$=\pi r^2 h=\frac{22}{7}\times 7^2\times 10=1540$ cm^3.

, The lateral area $=2\pi r h$

$=2\times\frac{22}{7}\times 7\times 10=440$ cm^2.

24 $\because$ The volume of the sphere

$=\frac{4}{3}\pi r^3=\frac{4}{3}\times 3^3\times\pi=36\pi$ cm^3.

, $\because$ The volume of the cylinder

= The volume of the sphere

$\therefore$ The volume of the cylinder $=36\pi$ cm^3.

$\therefore \pi r^2 h=36\pi$ $\therefore 9\pi h=36\pi$

$\therefore h=\frac{36}{9}=4$ cm.

25 $\because$ The volume of the cylinder

$=\pi r^2 h=3^2\times 4\times\pi=36\pi$ cm^3.

, ∵ the volume of the sphere

= The volume of the cylinder

∴ The volume of the sphere = 36π cm^3.

∴ $\frac{4}{3}\pi r^3 = 36\pi$

∴ $r^3 = \frac{36 \times 3}{4} = 27$ ∴ r = 3 cm.

---

26 ∵ The lateral area = $2\pi r h$

∴ $440 = 2 \times \frac{22}{7} \times 10 \times r$

∴ $r = \frac{440 \times 7}{2 \times 22 \times 10} = 7$ cm.

∴ The volume of the cylinder

$= \pi r^2 h = \frac{22}{7} \times 7^2 \times 10 = 1540$ cm^3.

Unit two

First Answers of multiple choice questions

1 (b) **2** (c) **3** (b) **4** (a) **5** (c)

6 (b) **7** (c) **8** (c) **9** (a) **10** (b)

Second Answers of complete questions

1 1 **2** undefined **3** zero

4 4 **5** x-axis , zero **6** – 11

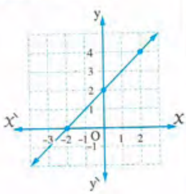
7 $\overleftrightarrow{BC}$ or $\overleftrightarrow{AC}$ **8** – 1 , zero **9** 3

10 (0 , 8)

Third Answers of essay questions

1 $y = x + 2$

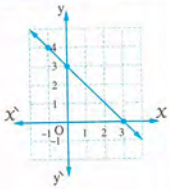
x	– 2	0	2
y	0	2	4



---

2 $y = 3 - x$

x	– 1	0	3
y	4	3	0

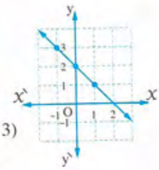


From the graph :

The point of intersection with x-axis is : (3 , 0)

3 $y = 2 - x$

x	– 1	0	1
y	3	2	1



Taking the two points (– 1 , 3) and (0 , 2) which lie on the straight line we find that :

the slope $= \frac{2-3}{0+1} = -1$

---

4 At $x = 0$ ∴ $2 \times 0 + 3y = 12$

∴ $3y = 12$ ∴ $y = 4$

∴ The straight line intersects y-axis at (0 , 4)

At $y = 0$ ∴ $2x + 3 \times 0 = 12$

∴ $2x = 12$ ∴ $x = 6$

∴ The straight line intersects x-axis at (6 , 0)

---

5 ∵ The slope of $\overleftrightarrow{AB} = \frac{4-3}{2+1} = \frac{1}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{5-4}{5-2} = \frac{1}{3}$

∴ The slope of $\overleftrightarrow{AB}$ = The slope of $\overleftrightarrow{BC}$

and the point B is a common point

∴ The points A , B and C are collinear.

---

6 ∵ The slope of $\overleftrightarrow{AB} = \frac{3+1}{-1-2} = \frac{-4}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{3-3}{2+1}$ = zero

∴ The slope of $\overleftrightarrow{AB} \neq$ The slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear.

---

7 ∵ $\frac{k-17}{6-4} = 4$ ∴ $\frac{k-17}{2} = 4$

∴ $k - 17 = 8$ ∴ $k = 25$

---

8 ∵ (k , 2 k) satisfies the relation : $3x + y = 30$

∴ $3k + 2k = 30$ ∴ $5k = 30$ ∴ $k = 6$

---

9 The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

, ∵ the slope of $\overleftrightarrow{BC} = \frac{1-5}{8-2} = \frac{-4}{6} = \frac{-2}{3}$

∴ The slope of $\overleftrightarrow{AB} \neq$ The slope of $\overleftrightarrow{BC}$

∴ $C \notin \overleftrightarrow{AB}$

10 $\because$ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-3} = \frac{2}{\text{zero}}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ is undefined

$\therefore \overleftrightarrow{AB}$ // y-axis

Unit three

First Answers of multiple choice questions

1 (b)	2 (d)	3 (c)	4 (a)	5 (d)
6 (c)	7 (a)	8 (b)	9 (c)	10 (c)
11 (c)	12 (c)	13 (b)	14 (a)	15 (c)

Second Answers of complete questions

1 5	2 3	3 7	4 the third
5 central tendency	6 4	7 40	8 3
9 10	10 median		

Third Answers of essay questions

1

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		**20**	**540**

$\therefore$ The mean $= \frac{540}{20} = 27$

2 1 n = 60

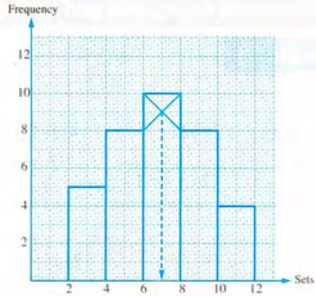
k = 100 – (10 + 22 + 25 + 20 + 8) = 15

2

Sets	x	f	$x \times f$
20 –	25	10	250
30 –	35	15	525
40 –	45	22	990
50 –	55	25	1375
60 –	65	20	1300
70 –	75	8	600
Total		**100**	**5040**

$\therefore$ The mean $= \frac{5040}{100} = 50.4$

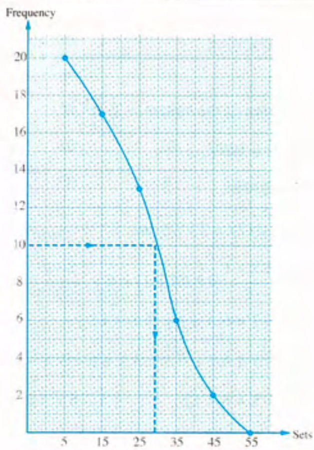
3



From the graph : The mode mark = 7 marks

4

The lower boundaries of sets	Descending cumulative frequency
5 and more	20
15 and more	17
25 and more	13
35 and more	6
45 and more	2
55 and more	0



$\because$ The order of the median $= \frac{20}{2} = 10$

$\therefore$ The median $\simeq 29$

Answers of the school book model examinations on Algebra and Statistics

Model 1

1

1 $\{-1\}$　　2 20　　3 $[-2, 2]$

4 24　　5 $\sqrt{3}-\sqrt{2}$

2

1 d　　2 c　　3 c

4 c　　5 a　　6 b

3

[a] $\sqrt{2\times 9}+\sqrt[3]{2\times 27}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{2\times 8}$

$=3\sqrt{2}+3\sqrt[3]{2}-3\sqrt{2}-\frac{1}{2}\times 2\sqrt[3]{2}$

$=2\sqrt[3]{2}$

[b] $\because x=\frac{3}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{3\left(\sqrt{5}+\sqrt{2}\right)}{5-2}$

$=\sqrt{5}+\sqrt{2}$

$\therefore$ x and y are two conjugate numbers.

4

[a] $\because$ The area of the square $=\frac{1}{2}\,d^2$

$\therefore \frac{1}{2}\,d^2=1089$

$\therefore d^2=2178$

$\therefore d=\sqrt{2178}=33\sqrt{2}$ cm.

[b] $\because 6\times\frac{3x+1}{6}<6(x+1)<6\times\frac{x+4}{2}$

$\therefore 3x+1<6x+6<3x+12$

$\therefore 3x-3x+1<6x-3x+6<3x-3x+12$

$\therefore 1<3x+6<12$　　$\therefore 1-6<3x<12-6$

$\therefore -5<3x<6$　　$\therefore -\frac{5}{3}<x<2$

$\therefore$ The S.S. $=\left]-\frac{5}{3}, 2\right[$

5

[a] The volume of the cylinder $=\pi r^2 h$

$=\left(4\sqrt{2}\right)^2\times 9\times\pi$

$=288\,\pi$ cm.^{3}

, $\because$ the volume of the cylinder

= the volume of the sphere

$\therefore$ The volume of the sphere $=288\,\pi$ cm.^{3}

$\therefore \frac{4}{3}\pi r^3=288\,\pi$

$\therefore r^3=288\times\frac{3}{4}=216$

$\therefore r=\sqrt[3]{216}=6$ cm.

[b]

Sets	x	f	$x\times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		**50**	**1550**

$\therefore$ The mean $=\frac{1550}{50}=31$

Model 2

1

1 $\sqrt{3}+\sqrt{5}$　　2 6　　3 $3+\sqrt{10}$

4 3　　5 $]3, 4]$

2

1 b　　2 a　　3 b

4 c　　5 c　　6 d

3

[a] $\frac{\sqrt{3}\left(\sqrt{5}+\sqrt{3}\right)+\sqrt{5}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$

$=\frac{\sqrt{15}+3+5-\sqrt{15}}{5-3}=\frac{8}{2}=4$

[b] The left hand side

$=\sqrt[3]{2\times 64}+\sqrt[3]{2\times 8}-2\sqrt[3]{2\times 27}$

$=4\sqrt[3]{2}+2\sqrt[3]{2}-2\times 3\sqrt[3]{2}=6\sqrt[3]{2}-6\sqrt[3]{2}=0$

= the right hand side

4

[a] $\because -2-7 < 3\mathcal{X}+7-7 \le 10-7$

$\therefore -9 < 3\mathcal{X} \le 3$

$\therefore -3 < \mathcal{X} \le 1$

$\therefore$ The S.S. $=]-3 , 1]$

[b] $\mathcal{X}^4 - 2\mathcal{X}^2 + 1 = (\mathcal{X}^2-1)^2$

$= \left(\left(\sqrt{2}+\sqrt{3}\right)^2-1\right)^2 = \left(2+2\sqrt{6}+3-1\right)^2$

$= \left(4+2\sqrt{6}\right)^2$

$= 16 + 16\sqrt{6} + 24$

$= 40 + 16\sqrt{6}$

5

[a] 20

[b]

Sets	$\mathcal{X}$	f	$\mathcal{X} \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		**20**	**540**

$\therefore$ The mean $= \frac{540}{20} = 27$

Model for the merge students

1

1 $\sqrt{3}-\sqrt{2}$ 2 $3\sqrt{6}$ 3 3

4 5 5 $\varnothing$

2

1 a 2 b 3 a 4 a 5 a

3

1 $\{5 , -5\}$ 2 $[0 , 2]$ 3 7

4 irrational

5

4

1 ✔ 2 ✔ 3 ✔ 4 ✘ 5 ✔

5

[a] The centre $= \frac{8+4}{2} = 6$

[b]

Sets	The centre of the set « $\mathcal{X}$ »	Frequency « f »	$\mathcal{X} \times f$
5 –	10	7	$10 \times 7 = 70$
15 –	20	10	$20 \times 10 =$ **200**
25 –	**30**	**12**	**$30 \times 12 = 360$**
35 –	**40**	**13**	**$40 \times 13 = 520$**
45 –	**50**	**8**	**$50 \times 8 = 400$**
Total		50	**1550**

$\therefore$ The arithmetic mean $= \frac{\sum(\mathcal{X} \times f)}{\sum(f)}$

$= \frac{\mathbf{1550}}{\mathbf{50}} = \mathbf{31}$

Answers of the schools examinations on Algebra and Statistics

1 Cairo

1

1 d	2 b	3 c
4 b	5 a	6 c

2

1 $\sqrt{6}-\sqrt{2}$	2 $[-3, 4[$	3 $5-\sqrt{3}$
4 36π	5 $\{-1, 5\}$	

3

[a] L.H.S. $=\sqrt{9\times 2}+\sqrt{16\times 2}-3\sqrt{2}-\frac{1}{2}\sqrt{4\times 2}$

$=3\sqrt{2}+4\sqrt{2}-3\sqrt{2}-\sqrt{2}$

$=3\sqrt{2}=$ R.H.S.

[b] $\because y=\frac{3}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{3\left(\sqrt{5}+\sqrt{2}\right)}{5-2}$

$=\sqrt{5}+\sqrt{2}$

$, x=\sqrt{5}-\sqrt{2}$

$\therefore$ x and y are conjugate to each other.

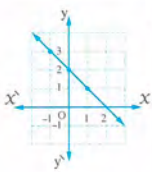
1 $x+y=\sqrt{5}-\sqrt{2}+\sqrt{5}+\sqrt{2}=2\sqrt{5}$

2 $xy=\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=5-2=3$

4

[a] $y=2-x$

x	-1	0	1
y	3	2	1



[b] $\because -2<3x+7<10$

$\therefore -9<3x<3$

$\therefore -3<x<1$

$\therefore$ The S.S. $=]-3, 1[$

5

[a] The volume $=\pi r^2 h=\frac{22}{7}\times 7^2\times 10=1540\ \text{cm}^3$.

[b]

Sets	x	f	$x\times f$
0 –	5	4	20
10 –	15	5	75
20 –	25	6	150
30 –	35	3	105
40 –	45	2	90
Total		20	440

$\therefore$ The mean $=\frac{440}{20}=22$

2 Cairo

1

1 d	2 c	3 c
4 d	5 c	6 d

2

1 zero	2 16	3 6
4 $]0, 5[$	5 $\sqrt{10}+3$	

3

[a]

1 $X\cup Y=]-1, \infty[$

2 $X-Y=]-1, 3[$

3 $X\cap Y=[3, 4]$

[b] 1 $\because y=\frac{2}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{2\left(\sqrt{3}-1\right)}{3-1}=\sqrt{3}-1$

$, x=\sqrt{3}+1$

$\therefore$ x and y are conjugate.

2 $\frac{x+y}{xy}=\frac{\sqrt{3}+1+\sqrt{3}-1}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\frac{2\sqrt{3}}{3-1}=\sqrt{3}$

4

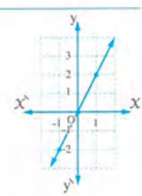
[a] The volume $=\pi r^2 h=\pi\times 4^2\times 9=144\pi\ \text{cm}^3$.

[b] The slope of $\overleftrightarrow{AB}=\frac{6-5}{2+1}=\frac{1}{3}$

5

[a] $y=2x$

x	-1	0	1
y	-2	0	2



[b]

Sets	x	f	$x\times f$
10 –	15	7	105
20 –	25	10	250
30 –	35	8	280
40 –	45	6	270
50 –	55	9	495
Total		40	1400

$\therefore$ The mean $=\frac{1400}{40}=35$

3 Cairo

1

1 b   2 b   3 a
4 c   5 d   6 a

2

1 9   2 $-\sqrt{7}+\sqrt{3}$ 3 5
4 14   5 $\{\sqrt{5}, -\sqrt{5}\}$

3

[a] L.H.S. $= \sqrt[3]{64\times 2}+\sqrt[3]{8\times 2}-2\sqrt[3]{27\times 2}$

$= 4\sqrt[3]{2}+2\sqrt[3]{2}-6\sqrt[3]{2}=0=$ R.H.S.

[b] The lateral area $= 2\pi r h = 2\times 3.14\times 6\times 10$

$= 376.8 \text{ cm}^2$.

, the volume $= \pi r^2 h = 3.14\times 6^2\times 10$

$= 1130.4 \text{ cm}^3$.

4

[a] $\because y = \frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{5-3}$

$= \sqrt{5}-\sqrt{3}$

$\therefore (x+y)^2 = \left(\sqrt{5}+\sqrt{3}+\sqrt{5}-\sqrt{3}\right)^2$

$= \left(2\sqrt{5}\right)^2 = 20$

[b] $\because 3x+7\le 10$ $\therefore 3x\le 3$

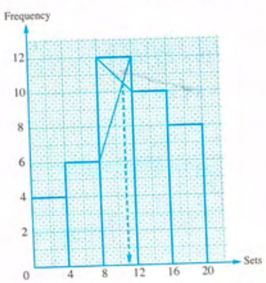
$\therefore x\le 1$ $\therefore$ The S.S. $=]-\infty, 1]$

5

[a]

1 $X\cap Y = [1, 3]$ 2 $X\cup Y =]-\infty, 5[$

[b]



$\therefore$ The mode $\simeq 11$

4 Giza

1

1 b   2 a   3 d
4 c   5 c   6 a

2

1 $\sqrt{10}+3$ 2 $\{3, 5\}$ 3 20
4 $2\sqrt{2}$ 5 -3

3

[a] $\sqrt{9\times 2}+\sqrt{16\times 2}-3\sqrt{2}-\frac{1}{2}\sqrt{4\times 2}$

$= 3\sqrt{2}+4\sqrt{2}-3\sqrt{2}-\sqrt{2}=3\sqrt{2}$

[b] $\because y = \frac{3}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$= \sqrt{5}+\sqrt{2}$

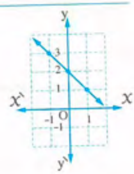
, $x = \sqrt{5}-\sqrt{2}$

$\therefore x$ and y are two conjugate numbers.

4

[a] $y = 2 - x$

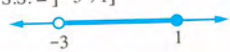
x	-1	0	1
y	3	2	1



[b] $\because -2 < 3x+7\le 10$

$\therefore -9 < 3x \le 3$ $\therefore -3 < x \le 1$

$\therefore$ The S.S. $=]-3, 1]$



5

[a] The volume $= \pi r^2 h = \pi\times 4^2\times 9 = 144\pi \text{ cm}^3$.

[b]

Sets	x	f	$x\times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		50	1550

$\therefore$ The mean $= \frac{1550}{50} = 31$

5 Giza

1 1 c 2 b 3 d
4 d 5 d 6 c

2 1 $[-2 , 2]$ 2 1 3 288
4 $\mathbb{Q}'$ 5 13

3

[a] $\because (2x+3)(5x^2-10)=0$

$\therefore 2x+3=0 \qquad \therefore 2x=-3$

$\therefore x=\frac{-3}{2} \quad$ or $\quad 5x^2-10=0$

$\therefore 5x^2=10 \qquad \therefore x^2=2$

$\therefore x=\sqrt{2} \quad$ or $\quad -\sqrt{2}$

$\therefore$ The S.S. $=\left\{\frac{-3}{2} , \sqrt{2} , -\sqrt{2}\right\}$

[b] $\sqrt{36\times 2}+2\sqrt{16\times 2}-3\sqrt{2}$

$=6\sqrt{2}+8\sqrt{2}-3\sqrt{2}=11\sqrt{2}$

[c]

1 $A\cup B=]-2 , \infty[$

2 $A\cap B=[0 , 1]$

4

[a] The volume $= x\times y\times z=5\times 3\times 2=30$ cm^3.

[b] $\frac{x+y}{xy-1}=\frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})-1}$

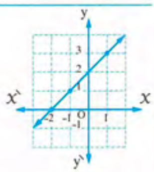
$=\frac{2\sqrt{5}}{5-2-1}=\sqrt{5}$

[c] The slope $=\frac{7-5}{3-2}=2$

5

[a] $y-x=2$

x	-2	-1	1
y	0	1	3



[b]

Sets	x	f	$x\times f$
8 –	10	4	40
12 –	14	10	140
16 –	18	16	288
20 –	22	12	264
24 –	26	8	208
Total		**50**	**940**

$\therefore$ The mean $=\frac{940}{50}=18.8$

6 Alexandria

1 1 b 2 b 3 b
4 c 5 c 6 a

2 1 11 2 2 3 $\sqrt{5}+2$
4 10 5 $\{2\sqrt{2}\}$

3

[a] $\because -1<2x+1\le 5$

$\therefore -2<2x\le 4$

$\therefore -1<x\le 2$

$\therefore$ The S.S. $=]-1 , 2]$

[b] $\because x=\frac{4}{\sqrt{7}-\sqrt{3}}\times\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}=\frac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$=\sqrt{7}+\sqrt{3}$

$\therefore (x-y)^2=\left(\sqrt{7}+\sqrt{3}-\sqrt{7}+\sqrt{3}\right)^2$

$=\left(2\sqrt{3}\right)^2=12$

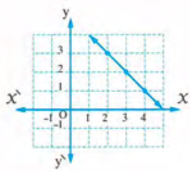
4

[a] $\sqrt{64\times 2}+\sqrt{16}-2\sqrt{16\times 2}$

$=8\sqrt{2}+4-8\sqrt{2}=4$

[b] $x+y=5$

x	2	3	4
y	3	2	1



5

[a]

1 $X \cap Y = [1 , 3]$ 2 $X \cup Y = [-2 , 5[$

[b] 1 $k = 14$

2

Sets	x	f	$x \times f$
20 –	25	10	250
30 –	35	14	490
40 –	45	22	990
50 –	55	26	1430
60 –	65	20	1300
70 –	75	8	600
Total		**100**	**5060**

$\therefore$ The mean $= \frac{5060}{100} = 50.6$

7 Alexandria

1 1 a 2 d 3 a
4 b 5 c 6 a

2 1 $8x^2$ 2 36 3 3
4 2 5 7

3

[a] $\because 2x^3 - 5 = 11$ $\therefore 2x^3 = 16$

$\therefore x^3 = 8$ $\therefore x = \sqrt[3]{8} = 2$

$\therefore$ The S.S. $= \{2\}$

[b] $\frac{y + x}{x y} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{7}-\sqrt{3}}{\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{7}+\sqrt{3}\right)} = \frac{2\sqrt{7}}{7-3} = \frac{\sqrt{7}}{2}$

4

[a]

1 $X \cup Y = [-2 , 5]$

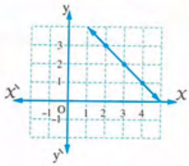
2 $X \cap Y = [1 , 3]$

[b] The volume $= \pi r^2 h = \frac{22}{7} \times 5^2 \times 7 = 550 \text{ cm}^3$.

5

[a] $x + y = 5$

x	2	3	4
y	3	2	1



[b]

Sets	x	f	$x \times f$
1 –	2	4	8
3 –	4	6	24
5 –	6	8	48
7 –	8	7	56
9 –	10	5	50
Total		**30**	**186**

$\therefore$ The mean $= \frac{186}{30} = 6.2$

8 El-Kalyoubia

1 1 c 2 d 3 d
4 b 5 c 6 a

2 1 512 2 3
3 undefined 4 zero 5 thrid

3

[a] $\because (x-2)^3 = 125$ $\therefore x - 2 = \sqrt[3]{125}$

$\therefore x - 2 = 5$ $\therefore x = 7$

$\therefore$ The S.S. $= \{7\}$

[b] $\because$ The circumference of the base $= 2\pi r$

$\therefore 2\pi r = 10\pi$ $\therefore r = 5$ cm.

$\therefore$ The volume $= \pi r^2 h = \pi \times 5^2 \times 6 = 150\pi \text{ cm}^3$.

[c] $\because 5 \le 3x - 1 < 14$ $\therefore 6 \le 3x < 15$

$\therefore 2 \le x < 5$

$\therefore$ The S.S. $= [2 , 5[$

4

[a] $\sqrt[3]{27 \times 2} + 5\sqrt[3]{8 \times 2} - 2\sqrt[3]{125 \times 2}$

$= 3\sqrt[3]{2} + 10\sqrt[3]{2} - 10\sqrt[3]{2} = 3\sqrt[3]{2}$

[b]

1 $X \cup Y =]-\infty , \infty[= \mathbb{R}$

2 $X - Y =]-\infty , -1]$

3 $\grave{X} =]3 , \infty[$

[c] $\because X = \frac{3}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} = \frac{3\left(\sqrt{6}+\sqrt{3}\right)}{6-3} = \sqrt{6}+\sqrt{3}$

$\therefore (X - y)(X + y)$

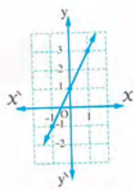
$= \left(\sqrt{6}+\sqrt{3}-\sqrt{6}+\sqrt{3}\right)\left(\sqrt{6}+\sqrt{3}+\sqrt{6}-\sqrt{3}\right)$

$= 2\sqrt{3} \times 2\sqrt{6} = 12\sqrt{2}$

5

[a] $y = 2X + 1$

X	-1	0	1
y	-1	1	3



Taking the two points (0 , 1) and (1 , 3)

$\therefore$ The slope $= \frac{3-1}{1-0} = 2$

[b]

Sets	X	f	$X \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

9 El-Sharkia

1 1 a 2 d 3 c
4 b 5 c 6 c

2 1 $\frac{3}{2}$ 2 3 3 9
4 5 5 $\sqrt{3}+\sqrt{2}$

3

[a] $\sqrt{9\times 2}+\sqrt{25\times 2}-2\sqrt{4\times 2}$

$= 3\sqrt{2}+5\sqrt{2}-4\sqrt{2} = 4\sqrt{2}$

[b]

1 $X \cap Y =]1 , 5]$

2 $Y - X =]5 , 7]$

4

[a] $\because -8 < 3X + 1 \le 4$

$\therefore -9 < 3X \le 3$ $\quad \therefore -3 < X \le 1$

$\therefore$ The S.S. $=]-3 , 1]$

[b] $\because y = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$

$\therefore \frac{X+y}{Xy} = \frac{\left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)} = \frac{2\sqrt{3}}{3-2} = 2\sqrt{3}$

5

[a] $\because (-4 , a)$ satisfies the relation.

$\therefore a = -4 + 2 = -2$

[b]

Sets	X	f	$X \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

10 El-Monofia

1 1 a 2 b 3 c
4 d 5 d 6 c

2 1 4 2 undefined 3 3
4 2 5 6

3

[a] 1 The volume $= \pi r^2 h = \pi \times 3^2 \times 9 = 81\pi \text{ cm}^3$.

2 The lateral area $= 2\pi r h = 2 \times \pi \times 3 \times 9$

$= 54\pi \text{ cm}^2$.

[b] $\because 5 < 2x + 3 < 9 \quad \therefore 2 < 2x < 6$

$\therefore 1 < x < 3 \quad \therefore$ The S.S. $=]1 , 3[$

4

[a]

1 $X \cap Y =]-1 , 2[$

2 $X \cup Y =]-3 , 3[$

3 $X - Y =]-3 , -1]$

[b] $4\sqrt{4 \times 3} + 2\sqrt{16 \times 3} - \frac{2}{5}\sqrt{25 \times 3}$

$= 8\sqrt{3} + 8\sqrt{3} - 2\sqrt{3} = 14\sqrt{3}$

5

[a] $\because$ The slope of $\overleftrightarrow{AC} = \frac{2-6}{1-3} = 2$

, the slope of $\overleftrightarrow{BC} = \frac{2-8}{1-4} = 2$

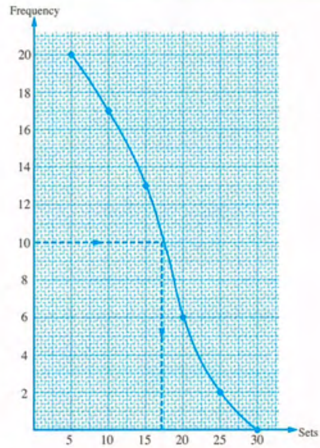
$\therefore$ The slope of $\overleftrightarrow{AC}$ = the slope of $\overleftrightarrow{BC}$

, $\because$ the point C is a common point

$\therefore C \in \overleftrightarrow{AB}$

[b]

The lower limits of sets	Descending cumulative frequency
5 and more	20
10 and more	17
15 and more	13
20 and more	6
25 and more	2
30 and more	0



$\because$ The order of the median $= \frac{20}{2} = 10$

$\therefore$ The median $\simeq 17$ marks

11 El-Dakahlia

1 1 $\{8\}$ 2 -6 3 10

4 7 5 2

---

2 1 b 2 d 3 c

4 c 5 d 6 b

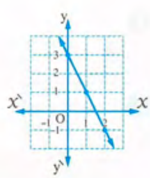
3

[a] $\because 3x + 8 < 14 \quad \therefore 3x < 6$

$\therefore x < 2 \quad \therefore$ The S.S. $=]-\infty , 2[$

[b] $y = 3 - 2x$

x	0	1	2
y	3	1	-1



4

[a] 1 $\because x = \frac{3}{\sqrt{5} - \sqrt{2}} \times \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}} = \frac{3(\sqrt{5} + \sqrt{2})}{5 - 2}$

$= \sqrt{5} + \sqrt{2}$

, $y = \sqrt{5} - \sqrt{2}$

$\therefore$ x and y are two conjugate numbers.

2 $\frac{x+y}{xy} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{2\sqrt{5}}{5-2}$

$= \frac{2\sqrt{5}}{3}$

[b] 1 $\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore \pi \times r^2 \times 10 = 250\pi$

$\therefore r^2 = 25$ $\therefore r = 5$ cm.

2 The lateral area $= 2\pi r h = 2 \times \pi \times 5 \times 10$

$= 100\pi$ cm^2.

5

[a] $\sqrt{25\times5} + 2\sqrt[3]{27\times3} - \sqrt{4\times5} - \sqrt[3]{8\times3}$

$= 5\sqrt{5} + 6\sqrt[3]{3} - 2\sqrt{5} - 2\sqrt[3]{3} = 3\sqrt{5} + 4\sqrt[3]{3}$

[b]

Sets	x	f	$x \times f$
2 –	3	2	6
4 –	5	1	5
6 –	7	3	21
8 –	9	3	27
10 –	11	1	11
Total		**10**	**70**

$\therefore$ The mean $= \frac{70}{10} = 7$

12 Ismailia

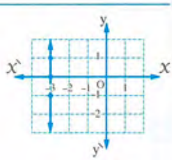
1 1 b 2 d 3 a
4 c 5 c 6 a

2 1 9 2 $2\sqrt{5}$ 3 12
4 125 5 10

3

[a] $x = -3$

x	−3	−3	−3
y	−1	0	1



[b] The slope of $\overleftrightarrow{AB} = \frac{12-5}{5-3} = \frac{7}{2}$

[c] $\sqrt{9\times3} + 2\sqrt{4\times3} - 2\sqrt{3}$

$= 3\sqrt{3} + 4\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$

4

[a] $\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3} \times \frac{22}{7} \times r^3 = 38808$

$\therefore r^3 = 9261$ $\therefore r = 21$ cm.

$\therefore d = 42$ cm.

[b]

1 $X \cup Y = [-3, \infty[$

2 $X \cap Y = [2, 5[$

3 $X - Y = [5, \infty[$

5

[a] $\because 8 < 3x - 1 \le 14$

$\therefore 9 < 3x \le 15$ $\therefore 3 < x \le 5$

$\therefore$ The S.S. $=]3, 5]$

[b]

Sets	x	f	$x \times f$
4 –	6	4	24
8 –	10	7	70
12 –	14	6	84
16 –	18	5	90
20 –	22	8	176
Total		**30**	**444**

$\therefore$ The mean $= \frac{444}{30} = 14.8$

13 El-Beheira

1 1 c 2 a 3 b
4 c 5 b 6 d

2 1 $4a^2b$ 2 8 3 $[2, 7[$
4 zero 5 24

3

[a] $\because y = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$

$\therefore \frac{x+y}{xy} = \frac{\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)} = \frac{2\sqrt{3}}{3-2} = 2\sqrt{3}$

[b] $\because \frac{2-k}{3-4} = 5$ $\qquad \therefore \frac{2-k}{-1} = 5$

$\therefore 2-k = -5$

$\therefore k = 7$

4

[a] $\because -1 \le 2x+3 < 5 \quad \therefore -4 \le 2x < 2$

$\therefore -2 \le x < 1 \qquad \therefore$ The S.S. $= [-2 , 1[$

[b] $\sqrt{25\times 2} + 2\sqrt{9\times 2} - \sqrt{16\times 2} - 4\sqrt{4\times\frac{1}{2}}$

$= 5\sqrt{2} + 6\sqrt{2} - 4\sqrt{2} - 4\sqrt{2} = 3\sqrt{2}$

5

[a] $\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = \frac{500}{3}\pi \quad \therefore r^3 = 125$

$\therefore r = 5$ cm. $\qquad \therefore d = 10$ cm.

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		**20**	**540**

$\therefore$ The mean $= \frac{540}{20} = 27$

14 El-Menia

1 [1] $3\sqrt{2}$ [2] $\varnothing$ [3] 3

[4] $\frac{-6}{5}$ [5] $]3 , 6]$

2 [1] c [2] d [3] d

[4] b [5] b [6] c

3

[a] $\sqrt{16\times 3} - 2\sqrt{4\times 3} + 2\sqrt{9\times\frac{1}{3}}$

$= 4\sqrt{3} - 4\sqrt{3} + 2\sqrt{3} = 2\sqrt{3}$

[b] $\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = 36\pi \quad \therefore r^3 = 27$

$\therefore r = 3$ cm. $\qquad \therefore d = 6$ cm.

[c] $(a+b)^2 = \left(\sqrt{5}-\sqrt{2}+\sqrt{5}+\sqrt{2}\right)^2$

$= \left(2\sqrt{5}\right)^2 = 20$

4

[a]

[1] $A \cap B = [1 , 3]$ [2] $A \cup B =]-2 , 5]$

[b] $\because (1.4)^2 = 1.96 \quad , \quad (1.5)^2 = 2.25 \quad , \quad \left(\sqrt{2}\right)^2 = 2$

$\therefore \sqrt{2}$ lies between 1.4 and 1.5

5

[a] [1] $\because 8x^3 - 20 = 7 \qquad \therefore 8x^3 = 27$

$\therefore x^3 = \frac{27}{8} \qquad \therefore x = \frac{3}{2}$

$\therefore$ The S.S. $= \left\{\frac{3}{2}\right\}$

[2] $\because -2 < 3x + 7 \le 10$

$\therefore -9 < 3x \le 3 \qquad \therefore -3 < x \le 1$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , -3 < x \le 1\}$

[b]

Sets	x	f	$x \times f$
0 –	2	2	4
4 –	6	10	60
8 –	10	8	80
12 –	14	7	98
16 –	18	3	54
Total		**30**	**296**

$\therefore$ The mean $= \frac{296}{30} = 9\,\frac{13}{15}$

15 Qena

1 [1] d [2] a [3] c

[4] a [5] d [6] c

2 1 25   2 3   3 9

4 $[2, 7]$ 5 $\sqrt{5} - \sqrt{2}$

3

[a] $\sqrt{9 \times 2} + \sqrt{25 \times 2} - \sqrt{9 \times 6}$

$= 3\sqrt{2} + 5\sqrt{2} - 3\sqrt{6} = 8\sqrt{2} - 3\sqrt{6}$

[b] $\because X = \frac{4}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{4\left(3 + \sqrt{5}\right)}{9 - 5} = 3 + \sqrt{5}$

, $y = 3 - \sqrt{5}$

$\therefore$ X and y are conjugate numbers.

, $(X + y)^2 = \left(3 + \sqrt{5} + 3 - \sqrt{5}\right)^2 = 6^2 = 36$

4

[a] $\because -2 < 3X + 7 < 10$

$\therefore -9 < 3X < 3$ $\therefore -3 < X < 1$

$\therefore$ The S.S. $=]-3, 1[$

[b]

1 $X \cup Y = [-1, 7]$ 2 $X \cap Y = [2, 4]$

5

[a] $\because$ (3 m , 2 m) satisfies the relation.

$\therefore 2\text{ m} = 6\text{ m} - 8$ $\therefore 2\text{ m} - 6\text{ m} = -8$

$\therefore -4\text{ m} = -8$ $\therefore \text{m} = 2$

[b]

Sets	X	f	$X \times f$
30 –	35	3	105
40 –	45	4	180
50 –	55	12	660
60 –	65	8	520
70 –	75	7	525
80 –	85	6	510
Total		**40**	**2500**

$\therefore$ The mean $= \frac{2500}{40} = 62.5$

Answers of the accumulative tests on Geometry

Accumulative test 1

1 1 c 2 a 3 c 4 c
5 b 6 a 7 a 8 c

2 [a] 6 cm. , 6 cm. [b] 6 cm.

3 15 cm.

Accumulative test 2

1 1 b 2 b 3 a 4 b
5 a 6 c 7 a 8 c

2 [a] 5 cm. [b] Prove by yourself.

3 1 12 cm. 2 4 cm. 3 26 cm.

Accumulative test 3

1 1 c 2 b 3 c 4 a
5 a 6 d 7 d 8 b

2 [a] 24 cm. [b] Prove by yourself.

3 [a] 18 cm. [b] 70°

Accumulative test 4

1 1 a 2 a 3 a 4 d
5 a

2 [a] Prove by yourself. [b] Prove by yourself.

3 [a] 90° [b] Prove by yourself.

4 Prove by yourself.

Accumulative test 5

1 1 d 2 b 3 c 4 d
5 a 6 b 7 b 8 a

2 [a] Prove by yourself. [b] Prove by yourself.

3 [a] 6 cm. [b] Prove by yourself.

Accumulative test 6

1 1 b 2 a 3 c 4 d
5 a

2 [a] 19 cm. [b] Prove by yourself.

3 [a] 105° [b] 30 cm.

4 [a] 1 25° 2 2 cm.
[b] Prove by yourself.

Accumulative test 7

1 1 b 2 c 3 c 4 c 5 b

2 [a] Prove by yourself. [b] Prove by yourself.

3 [a] Prove by yourself. [b] Prove by yourself.

4 [a] Prove by yourself. [b] Prove by yourself.

Accumulative test 8

1 1 c 2 a 3 a 4 b
5 a 6 d 7 b 8 b

2 [a] $AB < BC < AC$ [b] Prove by yourself.

3 [a] Prove by yourself. [b] Prove by yourself.

Accumulative test 9

1 1 a 2 d 3 b 4 c
5 c 6 a 7 b 8 c

2 [a] $AC > AB > BC$ [b] Prove by yourself.

3 10°

Answers of important questions on Geometry

Unit four

First Answers of multiple choice questions

1 (c)	2 (c)	3 (c)	4 (c)	5 (a)
6 (d)	7 (c)	8 (b)	9 (b)	10 (a)
11 (d)	12 (b)	13 (a)	14 (c)	15 (d)
16 (b)	17 (b)	18 (d)		

Second Answers of complete questions

1 one point   2 congruent

3 perpendicular, bisects it

4 perpendicular to the base , bisects the vertex angle

5 the base , the vertex angle

6 its axis of symmetry

7 equilateral   8 isosceles   9 zero

10 right   11 36°   12 90°

Third Answers of essay questions

1 In Δ ABC :

$\because$ m ($\angle$ ABC) = 90°

, m ($\angle$ C) = 30°

$\therefore$ AC = 2 AB = 2 × 5 = 10 cm. (First req.)

, $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BD}$ is a median from the vertex of the right angle

$\therefore$ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm. (Second req.)

2 $\because$ M is the intersection point of the medians of Δ ABC

$\therefore$ $\overline{BE}$ and $\overline{CD}$ are two medians of Δ ABC

$\therefore$ MD = $\frac{1}{3}$ DC = $\frac{1}{3}$ × 15 = 5 cm.

, ME = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 6 = 3 cm.

, $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 14 = 7 cm.

$\therefore$ The perimeter of Δ MDE = 5 + 3 + 7 = 15 cm.

(The req.)

3 $\because$ F and N are the midpoints of $\overline{AB}$ and $\overline{AC}$

$\therefore$ $\overline{BN}$ and $\overline{CF}$ are two medians of Δ ABC

, $\because$ $\overline{BN} \cap \overline{CF} = \{M\}$

$\therefore$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ MF = $\frac{1}{3}$ CF = $\frac{1}{3}$ × 9 = 3 cm.

, MN = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 4 = 2 cm.

, $\because$ F is the midpoint of $\overline{AB}$

$\therefore$ AF = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 6 = 3 cm.

, $\because$ N is the midpoint of $\overline{AC}$

$\therefore$ AN = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm.

$\therefore$ The perimeter of the figure AFMN

= 3 + 2 + 3 + 5 = 13 cm. (The req.)

4 $\because$ Δ EBD is an equilateral triangle.

$\therefore$ m ($\angle$ EBD) = 60°

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ C) = $\frac{180° - 50°}{2}$ = 65°

$\therefore$ m ($\angle$ DBC) = 60° + 65° = 125° (The req.)

5 $\because$ $\overrightarrow{AD}$ // $\overline{CB}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ B) = m ($\angle$ BAD) = 30° (alternate angles)

In Δ ABC : $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ C) = $\frac{180° - 30°}{2}$ = 75°

(The req.)

6 $\because$ $\overrightarrow{AE}$ // $\overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore$ m ($\angle$ B) = m ($\angle$ DAE) (corresponding angles)

, $\because$ $\overrightarrow{AE}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ C) = m ($\angle$ CAE) (alternate angles)

In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

because : AB = AC

$\therefore$ m ($\angle$ DAE) = m ($\angle$ CAE)

$\therefore$ $\overrightarrow{AE}$ bisects $\angle$ DAC (Q.E.D.)

7 In Δ ABC which is right-angled at B

$\because$ m ($\angle$ ACB) = 30° $\therefore$ AB = $\frac{1}{2}$ AC

, $\because$ AB = DE $\therefore$ DE = $\frac{1}{2}$ AC   (1)

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{DE}$ is a median in Δ ADC (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

8 In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 20 = 10$ cm.

In Δ XBY which is right-angled at B

$\because$ D is the midpoint of $\overline{XY}$

$\therefore \overline{BD}$ is a median

$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 10 = 5$ cm. (The req.)

9 In Δ ABD which is right-angled at D

$\because$ F is the midpoint of $\overline{AB}$

$\therefore \overline{DF}$ is a median

$\therefore DF = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8$ cm.

Similarly : $DE = \frac{1}{2} AC = \frac{1}{2} \times 18 = 9$ cm.

, $\because$ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore FE = \frac{1}{2} BC = \frac{1}{2} \times 20 = 10$ cm.

$\therefore$ The perimeter of Δ DEF = 8 + 9 + 10

= 27 cm. (The req.)

10 $\because \overline{ML} \parallel \overline{YZ}$, $\overleftrightarrow{YL}$ is a transversal to them

$\therefore$ m ($\angle$ LYZ) = m ($\angle$ MLY) (alternate angles)

, $\because$ m ($\angle$ LYZ) = m ($\angle$ LYM)

$\therefore$ m ($\angle$ MLY) = m ($\angle$ LYM)

$\therefore$ MY = ML

$\therefore \Delta$ LMY is an isosceles triangle. (Q.E.D.)

11 In Δ MXY : $\because$ MX = MY

$\therefore$ m ($\angle$ X) = m ($\angle$ Y) (1)

, $\because \overline{XY} \parallel \overline{ZL}$, $\overleftrightarrow{XL}$ is a transversal to them

$\therefore$ m ($\angle$ L) = m ($\angle$ X) (alternate angles) (2)

, $\because \overline{XY} \parallel \overline{ZL}$, $\overleftrightarrow{YZ}$ is a transversal to them

$\therefore$ m ($\angle$ Z) = m ($\angle$ Y) (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ L) = m ($\angle$ Z)

$\therefore$ ML = MZ

$\therefore \Delta$ MLZ is an isosceles triangle. (Q.E.D.)

12 In Δ ACD : $\because$ X is the midpoint of $\overline{AD}$

, Y is the midpoint of $\overline{CD}$

$\therefore XY = \frac{1}{2} AC$ (1)

, In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore AB = \frac{1}{2} AC$ (2)

From (1) and (2) : $\therefore$ AB = XY (Q.E.D.)

13 In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ In $\Delta\Delta$ ABD and ACE :

$$\begin{cases} m(\angle B) = m(\angle C) \\ m(\angle BAD) = m(\angle CAE) \\ AB = AC \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

AD = AE (Q.E.D.)

14 In $\Delta\Delta$ ABD and ACE :

$$\begin{cases} m(\angle D) = m(\angle E) = 90^\circ \\ m(\angle ABD) = m(\angle ACE) \\ BD = CE \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (Q.E.D.)

15 In Δ ABC : $\because$ AB = AC

, $\overrightarrow{AD}$ bisects $\angle$ A

$\therefore \overline{AD} \perp \overline{BC}$ (First req.)

, D is the midpoint of $\overline{BC}$

$\therefore CD = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.

, $\because$ m ($\angle$ BAD) = 180° – (65° + 90°) = 25°

$\therefore$ m ($\angle$ DAC) = m ($\angle$ BAD) = 25°(Second req.)

16 In Δ ADE : $\because$ AD = AE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ AED) (1)

, $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ ADE) = m ($\angle$ B) (corresponding angles) (2)

, $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ AED) = m ($\angle$ C) (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

17 In Δ ABC : $\because$ AB = AC

, $\overrightarrow{AE}$ bisects $\angle$ BAC

$\therefore$ BE = $\frac{1}{2}$ BC (Q.E.D. 1)

, $\overleftrightarrow{AE} \perp \overline{BC}$

$\therefore$ $\overleftrightarrow{AE}$ is the axis of symmetry of $\overline{BC}$

, $\because$ D $\in \overleftrightarrow{AE}$

$\therefore$ BD = CD (Q.E.D. 2)

18 In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\quad\therefore 2X - 1 = 7$

$\therefore 2X = 8 \quad\therefore X = 4$ (First req.)

$\therefore$ BC = 9 – 4 = 5 cm.

, $\because$ AB = AC = 7 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (Second req.)

19 In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\quad\therefore 2X - 1 = X + 3$

$\therefore 2X - X = 3 + 1$

$\therefore X = 4$ $\quad\therefore$ AB = AC = 4 + 3 = 7 cm.

, BC = 9 – 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (The req.)

20 $\because$ The sum of measures of the angles of the triangle = 180°

$\therefore 2X + 3X - 10° + X + 40° = 180°$

$\therefore 6X + 30° = 180°$

$\therefore 6X = 150° \quad\therefore X = \frac{150°}{6} = 25°$

$\therefore$ m ($\angle$ A) = 2 × 25° = 50°

, m ($\angle$ B) = 25° + 40° = 65°

, m ($\angle$ C) = 3 × 25° – 10° = 65°

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

21 In Δ ABC : $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BD}$ is a median from the vertex of the right angle

$\therefore$ BD = $\frac{1}{2}$ AC (1)

In Δ BDE : $\because$ m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30° $\quad\therefore$ BD = $\frac{1}{2}$ BE (2)

From (1) and (2) : $\therefore$ AC = BE (Q.E.D.)

22 In $\Delta\Delta$ ADC and BDC :

$\left\{\begin{array}{l} AC = BC \\ \overline{CD} \text{ is a common side} \\ m(\angle ACD) = m(\angle BCD) \end{array}\right.$

$\therefore$ Δ ADC $\equiv$ Δ BDC and we deduce that :

AD = BD

, $\because$ ED = BD $\quad\therefore$ AD = ED

$\therefore$ m ($\angle$ E) = m ($\angle$ EAD) (Q.E.D.)

23 $\because$ m ($\angle$ BAD) = 90° , m ($\angle$ BAE) = 30°

$\therefore$ m ($\angle$ DAF) = 60°

In Δ AFD :

$\because$ m ($\angle$ AFD) = 90° , m ($\angle$ DAF) = 60°

$\therefore$ m ($\angle$ ADF) = 180° – (90° + 60°) = 30°

$\therefore$ AD = 2 AF = 8 cm.

$\therefore$ The area of the square = 8 × 8 = 64 cm^2. (The req.)

24 In Δ ABC : $\because$ D is the midpoint of $\overline{BC}$

$\therefore$ $\overline{AD}$ is a median

, $\because$ AM = 2 MD

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because$ M $\in \overline{CE}$ $\quad\therefore$ $\overline{CE}$ is a median in Δ ABC

$\therefore$ EM = $\frac{1}{3}$ EC = $\frac{1}{3}$ × 12 = 4 cm. (The req.)

Unit five

First Answers of multiple choice questions

1 (c) **2** (d) **3** (c) **4** (a) **5** (d)
6 (b) **7** (b) **8** (a) **9** (d) **10** (b)
11 (d) **12** (b)

Second Answers of complete questions

1 opposite to a side greater in length than that opposite to the other angle.

2 the angle of the greater measure

3 $\overline{AB}$ 4 $\overline{BC}$

5 $>$ 6 $]4 , 14[$

7 the perpendicular line segment

8 the hypotenuse 9 AC 10 A + X

Third Answers of essay questions

1 In Δ ABC :

$\because$ m ($\angle$ C) = 180° – (80° + 40°) = 60°

$\therefore$ m ($\angle$ B) > m ($\angle$ C) > m ($\angle$ A)

$\therefore$ AC > AB > BC (The req.)

2 In Δ ABC : $\because$ AB < BC < AC

$\therefore$ m ($\angle$ C) < m ($\angle$ A) < m ($\angle$ B) (The req.)

3 In Δ ABC : $\because$ AC = AB

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) (1)

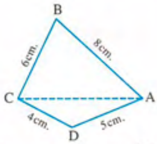
In Δ BCD : $\because$ BD > CD

$\therefore$ m ($\angle$ BCD) > m ($\angle$ CBD) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ ACD) > m ($\angle$ ABD) (Q.E.D.)

4



Const : Draw $\overline{AC}$

Proof : In Δ ABC :

$\because$ AB > BC

$\therefore$ m ($\angle$ BCA) > m ($\angle$ BAC) (1)

In Δ ADC : $\because$ AD > CD

$\therefore$ m ($\angle$ DCA) > m ($\angle$ DAC) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BCD) > m ($\angle$ BAD) (Q.E.D.)

5 In Δ ABC : $\because$ AB < AC

$\therefore$ m ($\angle$ ACB) < m ($\angle$ ABC)

, $\because$ $\overrightarrow{CM}$ bisects $\angle$ ACB , $\overrightarrow{BM}$ bisects $\angle$ ABC

$\therefore$ m ($\angle$ MCB) = $\frac{1}{2}$ m ($\angle$ ACB)

, m ($\angle$ MBC) = $\frac{1}{2}$ m ($\angle$ ABC)

$\therefore$ m ($\angle$ MCB) < m ($\angle$ MBC)

$\therefore$ BM < CM (Q.E.D.)

6 In Δ ABC : $\because$ AB > BC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ BAC) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ XYA) = m ($\angle$ ACB)

(corresponding angles) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ XYA) > m ($\angle$ BAC)

$\therefore$ AX > XY (Q.E.D.)

7 $\because$ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ B) = m ($\angle$ BAD) = 40° (alternate angles)

In Δ ABC :

$\therefore$ m ($\angle$ C) = 180° – (80° + 40°) = 60°

$\therefore$ m ($\angle$ C) > m ($\angle$ B)

$\therefore$ AB > AC (Q.E.D.)

8 In Δ AXY :

$\because$ m ($\angle$ AXY) = m ($\angle$ AYX)

$\therefore$ AY = AX (1)

, $\because$ AC > AB (2)

Subtracting (1) from (2) :

$\therefore$ AC – AY > AB – AX

$\therefore$ YC > XB (Q.E.D.)

9 In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = 65°

$\therefore$ m ($\angle$ BAC) = 180° – 65° × 2 = 50°

, $\because$ $\angle$ BAC is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ ADC) = 50° – 20° = 30°

$\therefore$ In Δ ADC : m ($\angle$ ADC) > m ($\angle$ ACD)

$\therefore$ AC > AD

, $\because$ AB = AC

$\therefore$ AB > AD (Q.E.D.)

10 In Δ ABD : $\because$ BD = AD

$\therefore$ m ($\angle$ BAD) = m ($\angle$ B)

$\therefore$ m ($\angle$ BAD) + m ($\angle$ DAC) > m ($\angle$ B)

$\therefore$ m ($\angle$ BAC) > m ($\angle$ B)

$\therefore$ BC > AC (Q.E.D.)

11 In ΔABC : $\because AB = AC$

$\therefore m(\angle B) = m(\angle C)$ (1)

, $\because D \in \overline{BC}$

$\therefore \angle ADC$ is an exterior angle of ΔADB

$\therefore m(\angle ADC) > m(\angle B)$ (2)

From (1) and (2) : $\therefore m(\angle ADC) > m(\angle C)$

$\therefore AC > AD$ (Q.E.D.)

12 In ΔABD : $\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AD}$

$\therefore XY = \frac{1}{2} BD$ (1)

In ΔDBC : $\because m(\angle BDC) = 90^\circ$

, E is the midpoint of $\overline{BC}$

$\therefore \overline{DE}$ is a median from the vertex of the right angle

$\therefore DE = \frac{1}{2} BC$ (2)

, $\because \overline{BC}$ is a hypotenuse of ΔBDC

$\therefore BD < BC$

$\therefore \frac{1}{2} BD < \frac{1}{2} BC$ (3)

From (1) , (2) and (3) :

$\therefore XY < DE$ (Q.E.D.)

13 $\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) > m(\angle BAD)$ (1)

, $\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle DAC)$ (2)

From (1) and (2) :

$\therefore m(\angle ADC) > m(\angle DAC)$

$\therefore AC > DC$ (Q.E.D.)

14 In ΔACE :

$\because AE = EC$

$\therefore m(\angle ECA) = m(\angle CAE) = 40^\circ$

$\therefore m(\angle AEC) = 180^\circ - (40^\circ + 40^\circ) = 100^\circ$

$\therefore m(\angle AEC) > m(\angle ACE)$

$\therefore AC > AE$ (Q.E.D. 1)

In ΔACD :

$\because \overline{AE}$ is a median , $AE = \frac{1}{2} CD$

$\therefore m(\angle DAC) = 90^\circ$

$\therefore$ In ΔABC : $\overline{BC}$ is a hypotenuse

$\therefore BC > AC$ (Q.E.D. 2)

Answers of the school book model examinations on Geometry

Model 1

1

1 the hypotenuse 2 5 cm. , 9 cm.

3 a side greater in length than that opposite to the other angle.

4 the angle at this vertex is right 5 equilateral

2

1 c	2 a	3 b
4 b	5 a	6 d

3

[a] >

[b] $\because \Delta$ DBC is equilateral

$\therefore m(\angle DBC) = 60°$

In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

$\therefore m(\angle ABD) = 60° + 65° = 125°$ (The req.)

[c] $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle ACB) = m(\angle DAC) = 50°$ (alternate angles)

$\therefore$ In ΔABC : $m(\angle B) = 180° - (70° + 50°) = 60°$

$\therefore m(\angle BAC) > m(\angle B)$

$\therefore BC > AC$ (Q.E.D.)

4

[a] Theoretical.

[b] In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore \Delta$ DBC is isosceles. (Q.E.D.)

5

[a] $\because \overline{AC}$ is the longest side.

$\therefore \angle B$ is the greatest angle in measure.

, $\because \overline{AB}$ is the shortest side.

$\therefore \angle C$ is the smallest angle in measure.

$\therefore$ The descending order of measures of the angles is $m(\angle B)$, $m(\angle A)$ and $m(\angle C)$ (The req.)

[b] In ΔABC : $\because AB > BC$

$\therefore m(\angle ACB) > m(\angle BAC)$ (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle XYA) = m(\angle ACB)$

(corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle XYA) > m(\angle BAC)$

$\therefore AX > XY$ (Q.E.D.)

Model 2

1

1 d	2 a	3 b
4 b	5 d	6 d

2

1 isosceles. 2 is less than

3 XY 4 $\frac{1}{2}$

5 is perpendicular to it.

3

[a] $\because \overline{AB}$ is the longest side.

$\therefore \angle C$ is the greatest angle in measure.

, $\because \overline{CB}$ is the shortest side.

$\therefore \angle A$ is the smallest angle in measure.

$\therefore$ The ascending order of measures of the angles is $m(\angle A)$, $m(\angle B)$ and $m(\angle C)$ (The req.)

[b] In ΔABC : $\because m(\angle B) = 90°$

, $\because$ D is the midpoint of $\overline{AC}$

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{BD}$, $\overline{AE}$ are two medians in ΔABC

$\therefore$ M is the intersection point of the medians of ΔABC

$\therefore BD = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm.

, $BM = \frac{2}{3} BD = \frac{2}{3} \times 4.5 = 3$ cm.

, $\because m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm. (The req.)

4

[a] In ΔABC : $\because$ D is the midpoint of $\overline{AC}$

$\therefore \overline{BD}$ is a median.

, $\because m(\angle ABC) = 90°$ $\therefore BD = \frac{1}{2} AC$ (1)

In ΔBDE : $\because m(\angle BDE) = 90°$, $m(\angle E) = 30°$

$\therefore BD = \frac{1}{2} BE$ (2)

From (1) and (2) : $\therefore AC = BE$ (Q.E.D.)

[b] $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) = 30° (alternate angles)

$\therefore$ In Δ ABC :

m ($\angle$ B) = 180° – (70° + 30°) = 80°

$\therefore$ m ($\angle$ B) > m ($\angle$ BAC)

$\therefore$ AC > BC (Q.E.D.)

5

[a] a side greater in length than that opposite to the other angle.

[b] $\because \overrightarrow{AB}$ bisects $\angle$ YAZ

$\therefore$ m ($\angle$ YAB) = m ($\angle$ BAZ) (1)

, $\because \overline{AB} \parallel \overline{XY}$, $\overleftrightarrow{AY}$ is a transversal.

$\therefore$ m ($\angle$ BAY) = m ($\angle$ AYX) (alternate angles) (2)

, $\because \overline{AB} \parallel \overline{XY}$, $\overleftrightarrow{ZX}$ is a transversal.

$\therefore$ m ($\angle$ X) = m ($\angle$ BAZ) (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ AYX) = m ($\angle$ X)

$\therefore$ m ($\angle$ AYX) + m ($\angle$ AYZ) > m ($\angle$ X)

$\therefore$ m ($\angle$ ZYX) > m ($\angle$ X)

$\therefore$ XZ > YZ (Q.E.D.)

Model for the merge students

1

1 1 : 2

2 half the length of the hypotenuse

3 congurent

4 >

5 bisects it , perpendicular to the base.

2

1 b 2 a 3 d 4 a 5 a

3

$\because$ m ($\angle$ B) = **90°** , m ($\angle$ C) = **30°**

$\therefore$ AB = $\frac{1}{2}$ × **AC**

$\therefore$ AC = **10** cm.

4

[a] AC , AB , BC

[b] 1 40° 2 $\overline{AB}$

5

1 ✔ 2 ✘ 3 ✘ 4 ✔ 5 ✔

Answers of the schools examinations on Geometry

1 Cairo

1

1 (c)   2 (d)   3 (b)   4 (c)   5 (b)   6 (b)

2

1 the hypotenuse   2 equal , 60°
3 $\overline{DH}$ 4 zero
5 BC , AC

3

[a] In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore \frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB)

$\therefore \Delta$ DBC is an isosceles triangle. (Q.E.D.)

[b] In Δ ADC :

$\because$ O is the midpoint of $\overline{AD}$

, E is the midpoint of $\overline{CD}$

$\therefore$ OE = $\frac{1}{2}$ AC

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC

$\therefore$ AB = OE = 5 cm. (The req.)

4

[a] $\because$ AB < AC < BC

$\therefore$ m ($\angle$ C) < m ($\angle$ B) < m ($\angle$ A) (The req.)

[b] $\because \overleftrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ B) = m ($\angle$ EAD) = 70° (corresponding angles)

, $\because \overleftrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30° (alternate angles)

$\therefore$ m ($\angle$ B) > m ($\angle$ C)

$\therefore$ AC > AB (Q.E.D.)

5

[a] $\because$ ABD is an equilateral triangle

$\therefore$ m ($\angle$ ADB) = 60°

In Δ BDC :

$\because$ CB = CD

$\therefore$ m ($\angle$ BDC) = $\frac{180° - 40°}{2}$ = 70°

$\therefore$ m ($\angle$ ADC) = 60° + 70° = 130° (The req.)

[b] $\because \overline{BF}$, $\overline{CE}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians of Δ ABC

$\therefore$ ME = $\frac{1}{2}$ CM = $\frac{1}{2} \times 6$ = 3 cm.

, MF = $\frac{1}{2}$ BM = $\frac{1}{2} \times 4$ = 2 cm.

, $\because$ E is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{AC}$

$\therefore$ EF = $\frac{1}{2}$ BC = $\frac{1}{2} \times 8$ = 4 cm.

$\therefore$ The perimeter of Δ MEF = 3 + 2 + 4
= 9 cm. (The req.)

2 Cairo

1

1 (b)   2 (b)   3 (d)   4 (d)   5 (d)   6 (b)

2

1 a median   2 3   3 half
4 5 , 9   5 BC , AC

3

[a] $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC = $\frac{1}{2} \times 10$ = 5 cm.

, $\because \overline{BE}$, $\overline{CD}$ are two medians in Δ ABC

$\therefore$ ME = $\frac{1}{2}$ MB = $\frac{1}{2} \times 5$ = 2.5 cm.

, MD = $\frac{1}{2}$ MC = $\frac{1}{2} \times 6$ = 3 cm.

$\therefore$ The perimeter of Δ MDE = 5 + 2.5 + 3
= 10.5 cm. (The req.)

[b] In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC

In Δ ACD :

$\because$ X is the midpoint of $\overline{AD}$

, Y is the midpoint of $\overline{CD}$

$\therefore$ XY = $\frac{1}{2}$ AC

$\therefore$ XY = AB (Q.E.D.)

4

[a] $\because$ ABC is an equilateral triangle

$\therefore m(\angle BAC) = 60°$

In Δ ABD :

$\because AB = BD$

$\therefore m(\angle BAD) = m(\angle D) = \frac{180° - 24°}{2} = 78°$

$\therefore m(\angle CAD) = 60° + 78° = 138°$ (The req.)

[b] $\because \overline{ED} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore m(\angle EDB) = m(\angle DBC)$ (alternate angles)

, $\because m(\angle EBD) = m(\angle DBC)$

$\therefore m(\angle EDB) = m(\angle EBD)$

$\therefore \Delta$ EBD is an isosceles triangle. (Q.E.D.)

5

[a] $\because AB = AC$ $\therefore A \in$ the axis of $\overline{BC}$

, $\because EB = EC$ $\therefore E \in$ the axis of $\overline{BC}$

$\therefore \overrightarrow{AD}$ is the axis of $\overline{BC}$

$\therefore \overrightarrow{AD} \perp \overline{BC}$ (Q.E.D. 1)

, $BD = \frac{1}{2} BC$ (Q.E.D. 2)

[b] In Δ ABC :

$\because AB > AC$

$\therefore m(\angle C) > m(\angle B)$ (1)

$\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles) (2)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle AXY) = m(\angle B)$ (corresponding angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle AYX) > m(\angle AXY)$ (Q.E.D.)

3 Cairo

1

1 (c) 2 (d) 3 (a) 4 (c) 5 (b) 6 (a)

2

1 the hypotenuse
2 greater measure
3 congruent
4 5 , 9
5 bisects the vertex angle , is perpendicular to the base

3

[a] $\because \overline{AD}$, $\overline{CE}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians

$\therefore AD = 3\ MD = 3 \times 2 = 6$ cm.

, $ME = \frac{1}{2} MC = \frac{1}{2} \times 5 = 2.5$ cm. (The req.)

[b] $\because$ BCD is an equilateral triangle

$\therefore m(\angle CBD) = 60°$

In Δ ABD :

$\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = \frac{180° - 30°}{2} = 75°$

$\therefore m(\angle CBA) = 60° + 75° = 135°$ (The req.)

4

[a] In Δ ABC : $\because AB = AC$

$\therefore m(\angle B) = m(\angle C)$ (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore m(\angle AXY) = m(\angle B)$ (corresponding angles) (2)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle AXY) = m(\angle AYX)$

$\therefore AX = AY$

$\therefore \Delta$ AXY is an isosceles triangle. (Q.E.D.)

[b] In Δ ACD :

$\because AD > CD$ $\therefore m(\angle ACD) > m(\angle CAD)$ (1)

In Δ ABC :

$\because AB > BC$ $\therefore m(\angle ACB) > m(\angle BAC)$ (2)

Adding (1) , (2) :

$\therefore m(\angle BCD) > m(\angle DAB)$ (Q.E.D.)

5

[a] In Δ ABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC$ (1)

, in Δ ACD :

$\because$ E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

$\therefore EF = \frac{1}{2} AC$ (2)

From (1) , (2) : $\therefore AB = EF$ (Q.E.D.)

[b] $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle ACB) = m(\angle DAC) = 30°$ (alternate angles)

In Δ ABC :

$\therefore$ m ($\angle$ BAC) > m ($\angle$ ACB)

$\therefore$ BC > AB (Q.E.D.)

4 Giza

1

1 1   2 300°   3 1 : 2

4 $\overline{BC}$ 5 10°

2

1 (c)   2 (b)   3 (c)   4 (a)   5 (c)   6 (a)

3

[a] $\because$ $\overline{BE}$, $\overline{CD}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians

$\therefore$ BM = $\frac{2}{3}$ BE = $\frac{2}{3} \times 15 = 10$ cm.

, CD = 3 MD = 3 × 4 = 12 cm. (The req.)

[b] In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ A) = 30°

$\therefore$ AC = 2 BC = 16 cm.

, $\because$ $\overline{BD}$ is a median

$\therefore$ BD = $\frac{1}{2}$ AC = 8 cm. (First req.)

In Δ ABD :

$\because$ BD = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ A) = 30° (Second req.)

4

[a] $\because$ m ($\angle$ ACB) = 180° – 120° = 60°

, m ($\angle$ ABC) = 180° – 130° = 50°

$\therefore$ m ($\angle$ ACB) > m ($\angle$ ABC)

$\therefore$ AB > AC (Q.E.D.)

[b] $\because$ $\overleftrightarrow{ED}$ // $\overline{AC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ A) = m ($\angle$ EBA) = 60° (alternate angles)

, $\because$ m ($\angle$ ABC) = 180° – (60° + 40°) = 80°

$\therefore$ m ($\angle$ A) < m ($\angle$ ABC)

$\therefore$ BC < AC (Q.E.D.)

5

[a] In Δ ABC :

$\because$ AC = BC $\therefore$ m ($\angle$ A) = m ($\angle$ B) = 25°

, $\because$ $\angle$ BCD is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ BCD) = 25° + 25° = 50°

, $\because$ $\overrightarrow{BE}$ // $\overrightarrow{DF}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore$ m ($\angle$ D) = m ($\angle$ BCD) = 50° (alternate angles)

(The req.)

[b] In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 90°}{2}$ = 45°

, $\because$ BCD is an equilateral triangle

$\therefore$ m ($\angle$ CBD) = 60°

$\therefore$ m ($\angle$ ABD) = 45° + 60° = 105° (The req.)

5 Giza

1

1 (a)   2 (a)   3 (c)   4 (c)   5 (c)   6 (b)

2

1 12   2 congruent   3 10

4 $\overline{AB}$ 5 the greater angle in measure

3

[a] $\because$ ACD is an equilateral triangle

$\therefore$ m ($\angle$ ACD) = 60°

In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = $\frac{180° - 40°}{2}$ = 70°

$\therefore$ m ($\angle$ BCD) = 60° + 70° = 130° (The req.)

[b] In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC (1)

In Δ ADC :

$\because$ E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

$\therefore$ EF = $\frac{1}{2}$ AC (2)

From (1) , (2) : $\therefore$ AB = EF (Q.E.D.)

4

[a] In Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (1)

In Δ BCD : $\because$ BD < CD

$\therefore$ m ($\angle$ CBD) > m ($\angle$ BCD) (2)

Adding (1) , (2) :

$\therefore m(\angle ABD) > m(\angle ACD)$ (Q.E.D.)

[b] In Δ ABC :

$\because m(\angle ABC) = 90°$, $m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC = \frac{1}{2} \times 24 = 12$ cm. (First req.)

, $\because \overline{BD}$ is a median.

$\therefore BD = \frac{1}{2} AC = \frac{1}{2} \times 24 = 12$ cm. (Second req.)

, $\because \overline{AH}$, $\overline{BD}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians.

$\therefore BM = \frac{2}{3} BD = \frac{2}{3} \times 12 = 8$ cm. (Third req.)

5

[a] In Δ ABC :

$\because m(\angle B) = 90°$, $m(\angle ACB) = 60°$

$\therefore m(\angle BAC) = 180° - (90° + 60°) = 30°$

$\therefore BC = \frac{1}{2} AC$

, $\because DE = BC$ $\therefore DE = \frac{1}{2} AC$

, $\because \overline{DE}$ is a median in Δ ADC

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

[b] In Δ ABD :

$\because AB < AD$

$\therefore m(\angle ABD) > m(\angle ADB)$ (1)

In Δ BCD : $\because BC < CD$

$\therefore m(\angle CBD) > m(\angle BDC)$ (2)

Adding (1) , (2) :

$\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.)

6 Alexandria

1

1 (c) 2 (a) 3 (c) 4 (d) 5 (b) 6 (d)

2

1 greater than 2 = 3 2 , 8

4 parallel 5 XZ

3

[a] $\because m(\angle Y) = 180° - (55° + 65°) = 60°$

$\therefore m(\angle X) < m(\angle Y) < m(\angle Z)$

$\therefore YZ < XZ < XY$ (The req.)

[b] In Δ ABC :

$\because m(\angle B) = 90°$, $m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC = 6$ cm.

, $\because \overline{BD}$ is a median

$\therefore BD = \frac{1}{2} AC = 6$ cm.

, $\because AD = \frac{1}{2} AC = 6$ cm.

$\therefore$ The perimeter of Δ ABD = 6 + 6 + 6 = 18 cm.

(The req.)

4

[a] In Δ ABC :

$\because AC = AB$ $\therefore m(\angle C) = m(\angle B)$

$\therefore$ In $\Delta\Delta$ ACD , ABE :

$$\begin{cases} m(\angle CAD) = m(\angle BAE) \\ AC = AB \\ m(\angle C) = m(\angle B) \end{cases}$$

$\therefore \Delta ACD \equiv \Delta ABE$

$\therefore AD = AE$ (Q.E.D.)

[b] $\because$ ABC is an equilateral triangle

$\therefore m(\angle ABC) = 60°$

, $\because \angle ABC$ is an exterior angle of Δ BCD

$\therefore m(\angle D) = 60° - 30° = 30°$

$\therefore m(\angle BCD) = m(\angle D)$

$\therefore BC = BD$

$\therefore \Delta$ CBD is an isosceles triangle. (Q.E.D.)

5

[a] $\because$ F is the midpoint of $\overline{XY}$

, L is the midpoint of $\overline{XZ}$

$\therefore FL = \frac{1}{2} YZ = 4$ cm.

, $\because \overline{YL}$, $\overline{ZF}$ are two medians in Δ XYZ

$\therefore$ M is the point of intersection of the medians

$\therefore MF = \frac{1}{2} MZ = 2$ cm.

, $ML = \frac{1}{3} YL = 3$ cm.

$\therefore$ The perimeter of Δ MFL = 4 + 2 + 3 = 9 cm.

(The req.)

[b] In Δ AMB :

$\because MB > MA$ $\therefore m(\angle A) > m(\angle B)$ (1)

, $\because \overline{AB} \parallel \overline{DE}$, $\overleftrightarrow{AE}$ is a transversal

$\therefore m(\angle E) = m(\angle A)$ (alternate angles) (2)

, $\because \overline{AB} \parallel \overline{DE}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore m(\angle D) = m(\angle B)$ (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle E) > m(\angle D)$

$\therefore MD > ME$ (Q.E.D.)

7 Alexandria

1

1 (a) 2 (b) 3 (a) 4 (c) 5 (b) 6 (a)

2

1 $\varnothing$ 2 bisects the base , is perpendicular to it

3 2 4 1 5 4

3

[a] $\because$ H is the midpoint of $\overline{AB}$

, D is the midpoint of $\overline{BC}$

$\therefore AC = 2\,HD = 12$ cm.

, $\because \overline{AD}$, $\overline{CH}$ are two medians in ΔABC

$\therefore$ O is the intersection point of the medians

$\therefore AO = 2\,OD = 6$ cm.

, $OC = \frac{2}{3}\,HC = 8$ cm.

$\therefore$ The perimeter of $\Delta AOC = 12 + 6 + 8 = 26$ cm. (The req.)

[b] $\because BC = AD = 12$ cm. (properties of parallelogram)

$\therefore CE = 12 - 8 = 4$ cm.

In ΔDEC :

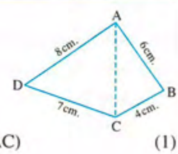
$\because m(\angle CDE) = 180° - (90° + 60°) = 30°$

, $m(\angle CED) = 90°$

$\therefore CD = 2\,CE = 8$ cm.

$\therefore AB = CD = 8$ cm. (The req.)

4



[a] Const. : Draw $\overline{AC}$

Proof : In ΔABC :

$\because AB > BC$

$\therefore m(\angle ACB) > m(\angle BAC)$ (1)

In ΔACD :

$\because AD > DC$

$\therefore m(\angle ACD) > m(\angle DAC)$ (2)

Adding (1) , (2) :

$\therefore m(\angle BCD) > m(\angle BAD)$ (Q.E.D.)

[b] $\because \overleftrightarrow{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle C) = m(\angle DAC) = x$ (alternate angles)

$\therefore m(\angle C) < m(\angle BAC)$

$\therefore AB < BC$ (Q.E.D.)

5

[a] In ΔADC : $\because AD = CD$

$\therefore m(\angle DAC) = m(\angle C) = 30°$

, $\because \angle ADB$ is an exterior angle at D

$\therefore m(\angle ADB) = 30° + 30° = 60°$

$\therefore$ In ΔADB : $\because AD = BD$

, $m(\angle ADB) = 60°$

$\therefore \Delta ADB$ is an equilateral triangle. (Q.E.D. 1)

, $\because \overline{AD}$ is a median in ΔBAC

, $AD = \frac{1}{2}\,BC$ $\therefore m(\angle BAC) = 90°$

$\therefore \Delta BAC$ is a right-angled triangle. (Q.E.D. 2)

[b] $\because$ ABC is an equilateral triangle

$\therefore m(\angle BCA) = 60°$

In ΔACD :

$\because AD = CD$

$\therefore m(\angle DCA) = m(\angle DAC) = \frac{180° - 40°}{2} = 70°$

$\therefore m(\angle DCB) = 60° + 70° = 130°$ (The req.)

8 El-Kalyoubia

1

1 (c) 2 (b) 3 (b) 4 (c) 5 (b) 6 (d)

2

1 45° 2 290° 3 1 : 2 4 3

5 bisects the base , is perpendicular to it

3

[a] $\because AB = AC$

$\therefore m(\angle B) = m(\angle C) = 2x - 10$

$\therefore x + 40 + 2x - 10 + 2x - 10 = 180°$

$\therefore 5x + 20 = 180°$ $\therefore 5x = 160°$

$\therefore x = 32°$

$\therefore m(\angle A) = 32° + 40° = 72°$

, $m(\angle B) = m(\angle C) = 2 \times 32 - 10 = 54°$ (The req.)

[b] $\because \overrightarrow{DE} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m ($\angle$ DBC) = m ($\angle$ EDB) (alternate angles) (1)

, $\because \overrightarrow{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ C) = m ($\angle$ ADE) (corresponding angles) (2)

, $\because$ m ($\angle$ ADE) = m ($\angle$ EDB) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ DBC) = m ($\angle$ C)

$\therefore$ DB = CD

$\therefore \Delta$ DBC is an isosceles triangle. (Q.E.D.)

4

[a] $\because \overline{ED}$ is a median in Δ EBC

, ED = $\frac{1}{2}$ BC

$\therefore$ m ($\angle$ BEC) = 90° (Q.E.D. 1)

, $\because$ ECD is an equilateral triangle

$\therefore$ m ($\angle$ C) = 60°

, $\because$ BC = BA

$\therefore$ ABC is an equilateral triangle. (Q.E.D. 2)

[b] In Δ ABC :

$\because$ m ($\angle$ B) = 90° , $\overline{BD}$ is a median

$\therefore$ BD = $\frac{1}{2}$ AC = 6 cm. (First req.)

, $\because$ m ($\angle$ ADB) = 180° – 120° = 60°

, BD = AD = $\frac{1}{2}$ AC

$\therefore \Delta$ ABD is equilateral

$\therefore$ AB = BD = 6 cm. (Second req.)

, $\because \overline{BD}$, $\overline{AE}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians

$\therefore$ BM = $\frac{2}{3}$ BD = 4 cm. (Third req.)

, MD = $\frac{1}{3}$ BD = 2 cm. (Fourth req.)

5

[a] In Δ ABD :

$\because$ m ($\angle$ B) = 90° $\therefore$ AD > BD

$\therefore$ AD > BD – CD

$\therefore$ AD > BC (Q.E.D.)

[b] Assuming that ABC is a triangle

$\therefore$ AB < AC + BC (adding AB to both sides)

$\therefore$ 2 AB < AC + BC + AB

$\therefore$ AB < $\frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in a triangle is less than half of the perimeter. (Q.E.D.)

9 El-Sharkia

1

1 (a) 2 (d) 3 (d) 4 (a) 5 (c) 6 (b)

2

1 2 , 8 2 equilateral 3 $\frac{1}{2}$

4 120° 5 6

3

[a] In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm. (First req.)

, $\because \overline{BD}$ is a median.

$\therefore$ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm.

, $\because \overline{AE}$, $\overline{BD}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians.

$\therefore$ BM = $\frac{2}{3}$ BD = $\frac{2}{3}$ × 6 = 4 cm. (Second req.)

[b] In Δ ABC :

$\because$ m ($\angle$ BAC) = 180° – (70° + 30°) = 80°

$\therefore$ m ($\angle$ CAD) = m ($\angle$ BAD) = $\frac{80°}{2}$ = 40°

$\therefore$ In Δ CAD :

m ($\angle$ ADC) = 180° – (70° + 40°) = 70°

$\therefore$ m ($\angle$ C) = m ($\angle$ ADC)

$\therefore$ AC = AD

$\therefore \Delta$ ADC is an isosceles triangle. (Q.E.D.)

4

[a] $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ ACB) = m ($\angle$ DAC)

= 32° (alternate angles)

$\therefore$ In Δ ABC :

m ($\angle$ B) = 180° – (80° + 32°) = 68°

$\therefore$ m ($\angle$ B) > m ($\angle$ ACB)

$\therefore$ AC > AB (Q.E.D.)

[b] $\because$ ABC is an equilateral triangle

$\therefore$ m ($\angle$ BCA) = 60°

In Δ ACD :

$\because$ AD = CD

$\therefore$ m ($\angle$ DCA) = m ($\angle$ DAC) = $\frac{180° - 40°}{2}$ = 70°

$\therefore$ m ($\angle$ BCD) = 60° + 70° = 130° (The req.)

5

[a] $\because$ D is the midpoint of $\overline{BC}$

, E is the midpoint of $\overline{AC}$

$\therefore$ AB = 2 DE = 8 cm.

, $\because$ $\overline{AD}$, $\overline{BE}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians

$\therefore$ MB = 2 ME = 4 cm. , AM = 2 MD = 6 cm.

$\therefore$ The perimeter of Δ MAB = 8 + 4 + 6 = 18 cm. (The req.)

[b] $\because$ ZX > YZ > XY

$\therefore$ m ($\angle$ Y) > m ($\angle$ X) > m ($\angle$ Z) (The req.)

10 El-Gharbia

1

1 (b) 2 (b) 3 (b) 4 (b) 5 (d) 6 (b)

2

1 equilateral 2 1 : 2 3 $\frac{1}{2}$

4 120° 5 $\frac{1}{2}$, //

3

[a] In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , $\overline{BD}$ is a median

$\therefore$ BD = $\frac{1}{2}$ AC (1)

In Δ BDE :

$\because$ m ($\angle$ BDE) = 90° , m ($\angle$ E) = 30°

$\therefore$ BD = $\frac{1}{2}$ BE (2)

From (1) , (2) :

$\therefore$ AC = BE (Q.E.D.)

[b] In Δ ABC : $\because$ AB > BC

$\therefore$ m ($\angle$ C) > m ($\angle$ A) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C) (corresponding angles) (2)

From (1) , (2) :

$\therefore$ m ($\angle$ AYX) > m ($\angle$ A)

$\therefore$ AX > XY (Q.E.D.)

4

[a] $\because$ DBC is an equilateral triangle

$\therefore$ m ($\angle$ DBC) = 60°

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$ = 65°

$\therefore$ m ($\angle$ ABD) = 60° + 65° = 125° (The req.)

[b] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m ($\angle$ ADB) = m ($\angle$ DBC) (alternate angles)

, $\because$ m ($\angle$ ABD) = m ($\angle$ DBC)

$\therefore$ m ($\angle$ ADB) = m ($\angle$ ABD)

$\therefore$ In Δ ADB : AB = AD (Q.E.D. 1)

, $\because$ $\overrightarrow{AE}$ bisects $\angle$ BAD

$\therefore$ $\overline{AE}$ $\perp$ $\overline{BD}$ (Q.E.D. 2)

5

[a] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ ACB) = m ($\angle$ DAC) = 30° (alternate angles)

In Δ ABC : $\therefore$ m ($\angle$ BAC) > m ($\angle$ ACB)

$\therefore$ BC > AB (Q.E.D.)

[b] $\because$ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30° (alternate angles)

In Δ ABC : $\because$ AC = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ B) = $\frac{180° - 30°}{2}$ = 75° (The req.)

11 El-Dakahlia

1

1 (a) 2 (c) 3 (c) 4 (c) 5 (b) 6 (a)

2

1 the hypotenuse 2 130° 3 zero

4 bisects the base , bisects the vertex angle

5 2 , 10

3

[a] $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ BC = 2 DE = 10 cm.

$\because$ $\overline{BE}$, $\overline{CD}$ are two medians in Δ ABC

, M is the point of intersection of the medians

$\therefore$ MB = 2 ME = 6 cm. , MC = 2 MD = 8 cm.

$\therefore$ The perimeter of Δ MBC = 10 + 6 + 8 = 24 cm. (The req.)

[b] $\because$ AB < AC < BC

$\therefore$ m ($\angle$ C) < m ($\angle$ B) < m ($\angle$ A) (The req.)

4

[a] In ΔABC : $\because m(\angle ABC) = 90°$, $\overline{BM}$ is a median

$\therefore BM = \frac{1}{2} AC$ (1)

In ΔBMD : $\because m(\angle BMD) = 90°$, $m(\angle D) = 30°$

$\therefore BM = \frac{1}{2} BD$ (2)

From (1) , (2) : $\therefore AC = BD$ (Q.E.D.)

[b] $\because$ ABC is an equilateral triangle

$\therefore m(\angle ABC) = 60°$

In ΔBCD : $\because BD = CD$

$\therefore m(\angle CBD) = m(\angle BCD) = \frac{180° - 40°}{2} = 70°$

$\therefore m(\angle ABD) = 60° + 70° = 130°$ (The req.)

5

[a] In ΔBCD : $\because BD = BC$

$\therefore m(\angle C) = m(\angle BDC) = 70°$

$\therefore m(\angle CBD) = 180° - 2 \times 70° = 40°$

, $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore m(\angle ADB) = m(\angle CBD) = 40°$ (alternate angles)

$\therefore$ In ΔADB :

$m(\angle ABD) = 180° - (100° + 40°) = 40°$

$\therefore m(\angle ABD) = m(\angle ADB)$

$\therefore AB = AD$

$\therefore \Delta ABD$ is an isosceles triangle. (Q.E.D.)

[b] In ΔXYL : $\because XY + YL > XL$ (1)

In ΔXZL : $\because ZL + XZ > XL$ (2)

Adding (1) , (2) : $\therefore XY + YL + ZL + XZ > 2\,XL$

$\therefore$ The perimeter of $\Delta XYZ > 2\,XL$ (Q.E.D.)

12 Suez

1

1 (b) 2 (a) 3 (a) 4 (c) 5 (c) 6 (a)

2

1 greater than 2 the hypotenuse

3 congruent 4 60°

5 bisects the base , is perpendicular to it

3

[a] $\because \overrightarrow{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle C) = m(\angle DAC) = 40°$ (alternate angles)

$\therefore m(\angle BAC) > m(\angle C)$

$\therefore BC > AB$ (Q.E.D.)

[b] In ΔABC : $\because AB > BC$

$\therefore m(\angle ACB) > m(\angle CAB)$ (1)

In ΔACD : $\because AD > DC$

$\therefore m(\angle ACD) > m(\angle CAD)$ (2)

Adding (1) , (2) :

$\therefore m(\angle BCD) > m(\angle BAD)$ (Q.E.D.)

4

[a] In ΔABC : $\because m(\angle ABC) = 90°$, $\overline{BD}$ is a median

$\therefore AC = 2\,BD = 10$ cm. (First req.)

In ΔBDE : $\because m(\angle BDE) = 90°$, $m(\angle E) = 30°$

$\therefore BE = 2\,BD = 10$ cm. (Second req.)

[b] $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore BC = 2\,DE = 12$ cm.

, $\because \overline{BE}$, $\overline{CD}$ are two medians in ΔABC

$\therefore$ M is the point of intersection of the medians

$\therefore CM = 2\,MD = 8$ cm. , $BM = 2\,ME = 6$ cm.

$\therefore$ The perimeter of $\Delta MBC = 12 + 8 + 6 = 26$ cm.

(The req.)

5

[a] $\because$ BCD is an equilateral triangle

$\therefore m(\angle CBD) = 60°$

In ΔABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = \frac{180° - 80°}{2} = 50°$

$\therefore m(\angle ABC) = 60° + 50° = 110°$ (The req.)

[b] $\because m(\angle B) = m(\angle C)$ $\therefore AB = AC$

$\therefore 2x - 1 = x + 3$ $\therefore x = 4$

$\therefore AB = AC = 2 \times 4 - 1 = 7$ cm.

, $BC = 9 - 4 = 5$ cm.

$\therefore$ The perimeter of $\Delta ABC = 7 + 7 + 5 = 19$ cm.

(The req.)

13 Kafr El-Sheikh

1

1 (c) 2 (c) 3 (c) 4 (d) 5 (b) 6 (a)

2

1 1 : 2 2 its end points

3 half the length of the hypotenuse

4 half 5 $\overline{AB}$

3

[a] In Δ ABC : $\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC (1)

In Δ ADC : $\because$ m ($\angle$ D) = 90° , $\overline{DE}$ is a median

$\therefore$ DE = $\frac{1}{2}$ AC (2)

From (1) , (2) : $\therefore$ AB = DE (Q.E.D.)

[b] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ B) + m ($\angle$ BAD) = 180° (interior angles)

$\therefore$ m ($\angle$ B) = 180° – 100° = 80°

In Δ ABC : $\therefore$ m ($\angle$ B) > m ($\angle$ BAC)

$\therefore$ AC > BC (Q.E.D.)

4

[a] $\because$ m ($\angle$ B) = 90° , $\overline{BD}$ is a median in Δ ABC

$\therefore$ BD = $\frac{1}{2}$ AC = 6 cm. (First req.)

, $\because$ $\overline{BD}$, $\overline{AE}$ are two medians intersecting at M

$\therefore$ MD = $\frac{1}{3}$ BD = 2 cm. (Second req.)

[b] In Δ ABD : $\because$ AD > AB

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ADB) (1)

, in Δ BCD : $\because$ BC = CD

$\therefore$ m ($\angle$ CBD) = m ($\angle$ BDC) (2)

Adding (1) , (2) :

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ADC) (Q.E.D.)

5

[a] $\because$ ACD is an equilateral triangle

$\therefore$ m ($\angle$ ACD) = 60°

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) = $\frac{180° - 50°}{2}$ = 65°

$\therefore$ m ($\angle$ BCD) = 60° + 65° = 125° (The req.)

[b] $\because$ m ($\angle$ A) + m ($\angle$ B) + m ($\angle$ C) = 180°

$\therefore$ $5X + 2 + 6X - 10 + X + 20 = 180°$

$\therefore$ $12X = 168$ $\therefore$ $X = 14$

$\therefore$ m ($\angle$ A) = 5 × 14 + 2 = 72°

, m ($\angle$ B) = 6 × 14 – 10 = 74°

, m ($\angle$ C) = 14 + 20 = 34°

$\therefore$ m ($\angle$ C) < m ($\angle$ A) < m ($\angle$ B)

$\therefore$ AB < BC < AC (The req.)

14 Beni Suef

1

1 (b) 2 (b) 3 (c) 4 (a) 5 (b) 6 (b)

2

1 8 2 3 3 an isosceles

4 its end points 5 $\angle$ E

3

[a] In Δ ABC : $\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC (1)

In Δ ADC : $\because$ m ($\angle$ D) = 90° , $\overline{DE}$ is a median

$\therefore$ DE = $\frac{1}{2}$ AC (2)

From (1) , (2) : $\therefore$ AB = DE (Q.E.D.)

[b] In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = 70°

$\therefore$ m ($\angle$ BAC) = 180° – 2 × 70° = 40°

, $\because$ m ($\angle$ ACD) = 180° – 70° = 110°

In Δ ACD : $\because$ AC = CD

$\therefore$ m ($\angle$ CAD) = m ($\angle$ D) = $\frac{180° - 110°}{2}$ = 35°

$\therefore$ m ($\angle$ BAD) = 40° + 35° = 75° (The req.)

4

[a] In Δ ABC : m ($\angle$ A) = 180° – (90° + 30°) = 60°

In Δ BCD : $\because$ BD = CD

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) = 30°

$\therefore$ m ($\angle$ ABD) = 90° – 30° = 60°

In Δ ABD :

$\therefore$ m ($\angle$ ADB) = 180° – (60° + 60°) = 60°

$\therefore$ m ($\angle$ A) = m ($\angle$ ABD) = m ($\angle$ ADB)

$\therefore$ Δ ABD is an equilateral triangle. (Q.E.D.)

[b] $\because$ AB = AC , $\overline{AD} \perp \overline{BC}$ $\therefore$ BD = CD = 3 cm.

$\therefore$ BC = 6 cm. (First req.)

, m ($\angle$ BAC) = 2 m ($\angle$ CAD) = 50° (Second req.)

5

[a] In Δ ABD : $\because$ AD > AB

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ADB) (1)

In Δ BCD : $\because$ CD > CB

$\therefore$ m ($\angle$ DBC) > m ($\angle$ BDC) (2)

Adding (1) , (2) :

$\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.)

[b] $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle ADE) = m(\angle B)$ (corresponding angles)

, $\because \angle B$ is an obtuse angle

$\therefore \angle ADE$ is an obtuse angle

In ΔADE : $\overline{AE}$ is the longest side

$\therefore AE > AD$ (Q.E.D.)

15 Aswan

1

1 (c) 2 (b) 3 (d) 4 (a) 5 (b) 6 (c)

2

1 $\overline{DF}$ 2 1 3 12

4 half the length of the hypotenuse 5 1 , 9

3

[a] $\because \overline{AY}$, $\overline{CX}$ are two medians in ΔABC

$\therefore$ M is the point of intersection of the medians.

$\therefore AM = 2\,YM = 2 \times 3 = 6$ cm.

, $\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore AC = 2\,XY = 2 \times 5 = 10$ cm.

$\therefore$ The perimeter of $\Delta MAC = 6 + 10 + 8 = 24$ cm.

(The req.)

[b] $\because \angle ADC$ is an exterior angle of ΔADB

$\therefore m(\angle ADC) = 30° + 40° = 70°$

In ΔACD : $\because AC = AD$

$\therefore m(\angle C) = m(\angle ADC) = 70°$

$\therefore m(\angle CAD) = 180° - 2 \times 70° = 40°$

$\therefore m(\angle CAB) = 40° + 30° = 70°$

$\therefore m(\angle C) = m(\angle CAB)$

$\therefore AB = CB$ (Q.E.D.)

4

[a] From ΔABM : $MA + MB > AB$

(Triangle inequality) (1)

From ΔBMC : $MB + MC > BC$

(Triangle inequality) (2)

From ΔAMC : $MA + MC > AC$

(Triangle inequality) (3)

Adding (1) , (2) , (3) :

$\therefore 2\,MA + 2\,MB + 2\,MC > AB + BC + AC$

$\therefore MA + MB + MC > \frac{1}{2}$ the perimeter of ΔABC

(Q.E.D.)

[b] $\because m(\angle Z) = 180° - (40° + 80°) = 60°$

$\therefore m(\angle X) < m(\angle Z) < m(\angle Y)$

$\therefore YZ < XY < XZ$ (The req.)

5

[a] In ΔABC : $m(\angle A) = 180° - (90° + 30°) = 60°$

In ΔBCD : $\because BD = CD$

$\therefore m(\angle DBC) = m(\angle DCB) = 30°$

$\therefore m(\angle ABD) = 90° - 30° = 60°$

In ΔABD :

$\therefore m(\angle ADB) = 180° - (60° + 60°) = 60°$

$\therefore m(\angle A) = m(\angle ABD) = m(\angle ADB)$

$\therefore \Delta ABD$ is an equilateral triangle. (Q.E.D.)

[b] $\because AB = AC$ $\therefore A \in$ the axis of $\overline{BC}$

, $\because EB = EC$ $\therefore E \in$ the axis of $\overline{BC}$

$\therefore \overleftrightarrow{AE}$ is the axis of $\overline{BC}$ (Q.E.D. 1)

$\therefore BD = DC$ (Q.E.D. 2)

Some Schools Examinations of 2020 on Algebra and Statistics

1 Cairo Governorate

Shoubra Educational Zone
St. Catherine Language School

Answer the following questions :

1 Choose the correct answer :

1. The multiplicative inverse of $\sqrt{3}$ is ....

   (a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{3}{\sqrt{3}}$

2. The S.S. of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

   (a) $\varnothing$ (b) $\{3, -3\}$ (c) $\{3\}$ (d) $\{-3\}$

3. If (k , 3) satisfies the relation : $y = 2x + 5$, then k = ...

   (a) 1 (b) – 1 (c) 2 (d) 3

4. The volume of a cube is 27 cm^3 , then its lateral area = ... cm^2

   (a) 12 (b) 54 (c) 36 (d) 27

5. If $2x + 1 = 7$, then $3x$ = ...

   (a) 6 (b) 9 (c) 12 (d) – 12

6. The mean of the values : 3 , 2 , 4 , 7 is ....

   (a) 2 (b) 3 (c) 7 (d) 4

---

2 Complete :

1. $3a^2 b \times$ .... $= 12a^4 b^2$
2. If the mode of the values : 6 , 9 , $x - 2$, 10 is 6 , then x = ...
3. $[2, 7] - \{7\}$ = ...
4. The slope of the straight line parallel to x-axis is ....
5. The median of : 24 , 20 , 11 , 36 , 40 is ....

---

3 [a] If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$ **, find the value of :** $\frac{x + y}{x y}$

[b] If the slope of the straight line passing through the two points A (4 , k) , B (3 , 2) is 5 , find the value of k

---

4 [a] **Find in $\mathbb{R}$ the S.S. of the inequality :**

$-1 \le 2x + 3 < 5$ and represent the S.S. on the number line.

[b] **Simplify :** $\sqrt{50} + 2\sqrt{18} - \sqrt{32} - 8\sqrt{\frac{1}{2}}$

5 [a] If the volume of a sphere is $\frac{500}{3}\pi$ cm³, find the length of its diameter.

[b] **Find the mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

2 Cairo Governorate

East Nasr City Educational Zone
Alson Lang. School

Answer the following questions :

1 **Choose the correct answer :**

1 $\left(\sqrt{5}+\sqrt{3}\right)^2\left(\sqrt{5}-\sqrt{3}\right)^2 = \cdots\cdots$

(a) 2 (b) 3 (c) 4 (d) 8

2 The lower limit of a set is 4 and the upper limit is 8, then its centre is ……………

(a) 8 (b) 6 (c) 4 (d) 2

3 $5 \in \cdots\cdots$

(a) $\{55\}$ (b) $]1, 5[$ (c) $]-\infty, 4]$ (d) $]-1, \infty[$

4 The mode of the values : 4, 11, 8, $2X$ is 8, then $X =$ ……………

(a) 2 (b) 4 (c) 9 (d) 11

5 If the volume of a cube is 27 cm³, then the perimeter of one of its faces is …………… cm.

(a) 12 (b) 9 (c) 15 (d) 40

6 If $(-1, 5)$ satisfies the equation : $3X + ky = 7$, then $k =$ ……………

(a) 2 (b) 0.8 (c) 3 (d) 5

2 **Complete :**

1 If the volume of a sphere is $\frac{9}{2}\pi$ cm³, then its radius length is ……………

2 $(2X-3)(3X+5) = 6X^2 +$ ……………

3 $[3, 4] - \{3, 5\} =$ ……………

4 If A $(1, -2)$, B $(5, -4)$, then the slope of $\overleftrightarrow{AB}$ is ……………

5 The mean of the values : 7, 11, 21, 10 and 16 is ……………

3 [a] **Simplify to the simplest form :**

1 $6\sqrt[3]{16}+\sqrt[3]{54}-6\sqrt[3]{\frac{1}{4}}$ 2 $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)$

[b] If $X = \frac{4}{\sqrt{7}-\sqrt{3}}$, $y = \sqrt{7}-\sqrt{3}$

, **prove that :** X and y are conjugate numbers , **then find the value of :** $(X+y)^2$

4 [a] Find the total area of a right circular cylinder of volume 72π cm^3 and height 8 cm. (in terms of π)

[b] **Find in $\mathbb{R}$ the S.S. of :**

1 $5 - 3X > 11$, then represent the solution set on the number line.

2 $8X^3 + 7 = 8$

---

5 [a] **Graph the relation :** $y = 3X + 1$ and if (2 , a) satisfies the relation , find the value of a

[b] **Find the arithmetic mean of the following frequency distribution :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	4	6	8	7	5	30

3 Cairo Governorate

Helwan Educational Zone
Saint Mary Lang. School

Answer the following questions :

1 **Choose the correct answer :**

1 The slope of the straight line passing through (4 , 1) , (6 , – 3) is ....

(a) – 1 (b) 0 (c) 2 (d) – 2

2 The solution set of : $2X^3 + 54 = 0$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\{-3\}$ (c) $\{-3 , 3\}$ (d) $\varnothing$

3 If (6 k , 4 k) satisfies the relation : $X + y = 50$, then k = ...

(a) 0 (b) 10 (c) 15 (d) 5

4 If the order of the median of some values is tenth , then the number of these values is ....

(a) 19 (b) 20 (c) 21 (d) 22

5 If $2X = 14$, then $6X =$ ....

(a) 12 (b) 28 (c) 36 (d) 42

6 $]-1 , 3] \cup \{0 , -1\} =$ ....

(a) $]0 , 3]$ (b) $]-1 , 3[$ (c) $[-1 , 3]$ (d) $[0 , 3]$

---

2 **Complete each of the following :**

1 The volume of the sphere whose radius length equals 14 cm. is .... $\left(\pi \approx \frac{22}{7}\right)$

2 If the mode of the values : 16 , 18 , $X - 3$, 14 is 16 , then $X =$ ....

3 The median of the values : 29 , 24 , 30 , 23 , 18 , 28 is ....

4 If the slope of a straight line equals zero , then the line is parallel to ....

5 If the lower limit of a set is 28 and the upper limit of it is 32 , then the centre of the set equals ....

3 **[a]** If $X =]-\infty , 4]$ and $Y =]2 , \infty[$, **find using the number line :**

1 $X \cap Y$ 2 $X \cup Y$ 3 $\grave{X}$

[b] A right circular cylinder whose volume is 704 cm^3. and its diameter length is 8 cm. , then find its height. $\left(\pi \approx \frac{22}{7}\right)$

4 **[a] Find the solution set in $\mathbb{R}$ of the inequality :**

$-4 \leq 5x + 1 < 11$ and represent it on the number line.

[b] Simplify : $\sqrt[3]{54} + \sqrt{50} + \sqrt[3]{16} + \sqrt{8}$

5 **[a] Graph the relation :** $y = 2x + 2$

[b] Find the arithmetic mean of the following data :

Sets	20 –	22 –	24 –	26 –	Total
Frequency	16	12	14	8	50

4 Giza Governorate

El-Dokki Zone
Math. Inspection

Answer the following questions :

1 **Choose the correct answer :**

1 $2\sqrt{x} \times 3\sqrt{x} =$ .... (where $x > 0$)

(a) $6x^2$ (b) $6x$ (c) $5x^2$ (d) $5x$

2 If (m , 2) satisfies the relation : $x + 2y = 7$, then m = ...

(a) -4 (b) -3 (c) 3 (d) 4

3 $\left(\sqrt{5} - 2\right) + \left(\sqrt{5} + 2\right) =$ ....

(a) 1 (b) 2 (c) 4 (d) $2\sqrt{5}$

4 The volume of a cube is 27 cm^3. , then the area of one of its faces is .... cm^2.

(a) 3 (b) 6 (c) 9 (d) 12

5 If $a = \frac{2}{\sqrt{3} - 1}$, $b = \sqrt{3} - 1$, then 2 a b = ...

(a) 1 (b) 2 (c) 3 (d) 4

6 The arithmetic mean of the values : 7 , 4 , 9 , 10 , 11 , 16 , 13 is ....

(a) 13 (b) 11 (c) 10 (d) 9

2 **Complete the following :**

1 Let A (1 , 3) , B (2 , 5) , then the slope of $\overleftrightarrow{AB}$ equals ....

2 The S.S. of the equation : $(x + 3)(x - 1) = 0$ in $\mathbb{R}$ is ....

3 The median of the values : 6 , 7 , 9 , 10 , 8 , 5 , 4 is ....

4 The mode of the values : 5 , 6 , 7 , 6 , 9 , 5 , 7 , 5 , 9 , 4 , 6 , 9 , 5 is ....

5 $[1 , 5] - \{1 , 5\} =$ ....

---

3 [a] If $X = [2 , 8]$, $Y =]-3 , 4[$, **find each of the following using the number line :**

1 $X \cap Y$ 2 $X \cup Y$

[b] **Find the S.S. of the inequality :** $5x + 1 \geq 21$ in $\mathbb{R}$ and represent the solution set on the number line.

---

4 [a] **Find the value of :** $\sqrt{20} + \sqrt{45} - \sqrt{80}$ (showing the steps of your answer)

[b] Find the volume of a right circular cylinder of height 10 cm. and its radius length is 7 cm.

---

5 [a] **Represent graphically the relation :** $y = 3 - x$

[b] **Find the arithmetic mean of the following frequency distribution :**

The set	0 –	10 –	20 –	30 –	40 –	Total
Frequency	4	5	6	3	2	20

5 Giza Governorate — 6th October Directorate

Answer the following questions :

1 **Choose the correct answer :**

1 The S.S. of the equation : $x^2 + 5 = 0$ in $\mathbb{R}$ is ....

(a) 5 (b) $\{\sqrt{5} , -\sqrt{5}\}$ (c) $\{\sqrt{5}\}$ (d) $\varnothing$

2 If the point (a , 1) satisfies the relation : $x + y = 5$, then a = ...

(a) – 4 (b) 1 (c) 4 (d) 5

3 If four times a number is 48 , then third of this number is ....

(a) 12 (b) 6 (c) 4 (d) 8

4 $[-1 , 5] -]-1 , 5[=$ ....

(a) $\varnothing$ (b) $\{-1 , 5\}$ (c) $[-1 , 5]$ (d) $]-1 , 5[$

5 The irrational number between 3 and 4 is ....

(a) $\sqrt{17}$ (b) $\sqrt{6}$ (c) $\sqrt[3]{29}$ (d) 3.6

6 A cube the sum of its edge lengths is 48 cm. , then its volume is .... cm^3.

(a) 64 (b) 6 (c) 4 (d) 46

2 Complete :

1 If the lower limit of a set is 4 and its centre is 6 , then its upper limit is ....

2 If $\frac{1}{x} = \sqrt{5} - 2$, then $x =$ .... (in its simplest form)

3 A sphere its diameter length is 6 cm. , then its volume is .... cm^3.

4 If A $(-1 , 4)$, B $(x , 2)$ and the slope of $\overleftrightarrow{AB} = -2$, then $x =$ ....

5 The S.S. of : $\sqrt{5}\, x \le 5$ is .... in $\mathbb{R}$

3 [a] A right circular cylinder , its radius length equals its height and its volume is $216\,\pi$ cm^3. Find the height of the right cylinder.

[b] **Find the S.S. in $\mathbb{R}$:**

1 $5 > 2x - 3 > -1$ (represent it on the number line)

2 $(2x - 1)^3 = 125$

4 [a] If $X =]-\infty , 1]$ and $Y = [-2 , 4[$, **find :**

1 $X \cap Y$ 2 $Y - X$ 3 $\grave{X}$

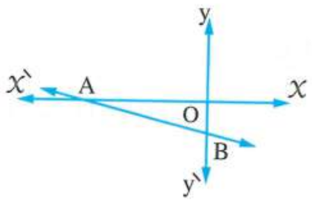
[b] **Simplify :** $5\sqrt{8} + 2\sqrt[3]{2} - 2\sqrt{50} - \sqrt[3]{16}$

[c] If $x = \sqrt{7} + \sqrt{4}$, $y = \frac{3}{x}$

1 **Prove that :** x and y are two conjugate numbers.

2 **Find :** $x^2 + 2xy + y^2$

5 [a] If the relation : $x + 4y = -4$ is represented in the opposite figure where A is the intersection point with x-axis and B is the intersection point with y-axis , **then find :**



1 The coordinates of A and B

2 The area of Δ ABO where O is the origin point.

3 The slope of $\overleftrightarrow{AB}$

[b] **From the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	**Total**
Frequency	7	10	12	13	k	**50**

1 Find k 2 Find the arithmetic mean.

6 Alexandria Governorate

El-Montazah Educational Zone
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1 $\sqrt{4}$ .... $]-2, \infty[$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 $\sqrt{\frac{x}{y}}$ = ... (where $y > 0$)

(a) $\frac{1}{y}\sqrt{x}$ (b) $\frac{1}{x}\sqrt{y}$ (c) $\frac{1}{y}\sqrt{xy}$ (d) $\frac{x}{y}$

3 The order of the median of the values : 4 , 5 , 6 , 7 and 8 is the ....

(a) third. (b) fourth. (c) fifth. (d) sixth.

4 If $x = (-2)^4$, $y = -2^4$, then ....

(a) $x = y$ (b) $x > y$ (c) $x < y$ (d) $x \leq y$

5 If $(2k, k)$ satisfies the relation : $y + 2x = 5$, then k = ...

(a) 5 (b) 4 (c) 2 (d) 1

6 If the mean of the values : 9 , 5 , 6 , x , 14 is 7 , then x = ...

(a) 3 (b) 2 (c) 1 (d) 5

2 Complete :

1 The additive inverse of the number $-5 + \sqrt{3}$ is ....

2 If the mode of the values : 4 , 11 , 8 , $2x$ is 4 , then x = ...

3 The cube whose volume is 8 cm^3 , then the sum of all edge lengths is .... cm.

4 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

5 The straight line which represents the relation : $2x + 7y = 14$ intersects x-axis at the point (.............. , ...)

3 [a] If $x = \sqrt{7} - \sqrt{6}$, $y = \frac{1}{x}$, **prove that :** $(x + y)^2 = 28$

[b] If A (3 , 4) , B (5 , a) and the slope of $\overleftrightarrow{AB} = 3$, find the value of a

[c] Find the lateral area of a right circular cylinder of volume 72π cm^3 and height 8 cm.

4 [a] **Graph the relation :** $y = 2 - x$

[b] **Simplify :** 1 $\sqrt{32} - 6\sqrt{\frac{1}{2}}$ 2 $\sqrt[3]{128} + \sqrt[3]{16}$

[c] If $X =]-\infty, 2[$ and $Y = [-1, 5]$, **find using the number line :**

1 $X \cap Y$ 2 $X \cup Y$ 3 $\grave{X}$

5 **[a] Complete :** The S.S. of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....

[b] Find in $\mathbb{R}$ the S.S. of the inequality :

$5 - 3x > 11$, then represent the S.S. on the number line.

[c] Find the mean of the following data :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

7 Alexandria Governorate

East Educational Zone
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. The mode for the values : 3 , 5 , 3 , 4 , 3 is ....

(a) 3 (b) 4 (c) 5 (d) 12

2. Let A (3 , 5) and B (5 , – 1) , then the slope of $\overleftrightarrow{AB}$ = ...

(a) $-\frac{1}{3}$ (b) – 3 (c) 3 (d) $\frac{1}{3}$

3. If the point (a , 1) satisfies the relation : $x + y = 5$, then a = ...

(a) 1 (b) – 4 (c) 4 (d) 5

4. The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{-3\}$ (c) $\{3\}$ (d) $\{3 , -3\}$

5. $4.274 \simeq$ .... $\left(\text{to the nearest } \frac{1}{10}\right)$

(a) 4 (b) 4.2 (c) 4.3 (d) 4.27

6. The lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

2 **Complete the following :**

1. The surface area of a sphere of diameter length 14 cm. equals ....
2. $\left(\sqrt{8}+\sqrt{2}\right)\left(\sqrt{8}-\sqrt{2}\right) =$ ....
3. The conjugate of the number $\frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$ is ....
4. A cube whose volume is 8 cm.^{3} , then the sum of lengths of all its edges equals ....
5. The S.S. of the equation : $x(x^3 - 1) = 0$ in $\mathbb{R}$ is ....

3 [a] **Find in the simplest form :** $6\sqrt{\frac{1}{2}} + \frac{1}{3}\sqrt[3]{54} - \sqrt{18} - \sqrt[3]{2}$

[b] If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$ **, find the value of :** $\frac{x + y}{x y - 1}$

---

4 [a] **Find the S.S. in $\mathbb{R}$ of the inequality :** $2x + 1 \le 7$, then represent it on the number line.

[b] Find the volume of the sphere whose diameter length is 4.2 cm. $\left(\pi = \frac{22}{7}\right)$

---

5 [a] If the slope of $\overleftrightarrow{AB}$ is 3 where $A = (3, 4)$, $B = (4, y)$, find the value of y

[b] **Find the arithmetic mean of the following distribution :**

Sets	4 –	8 –	12 –	16 –	20 –	**Total**
Frequency	2	4	8	6	4	**24**

8 El-Kalyoubia Governorate — Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. The solution set of the equation : $x + 5 = 5$ in $\mathbb{N}$ is ....
   (a) $\{0\}$ (b) $\{10\}$ (c) $\{-10\}$ (d) $\varnothing$
2. The rational number that lies between 0.2 , 0.3 is ....
   (a) 0.21 (b) 0.11 (c) 0.31 (d) 0.33
3. $\sqrt[3]{x^6} = \sqrt{\cdots\cdots}$
   (a) x^3 (b) x^2 (c) x (d) x^4
4. If $(2, -5)$ satisfies the relation : $3x - y + c = 0$, then c = ...
   (a) 1 (b) – 1 (c) 11 (d) – 11
5. If the arithmetic mean of the set of values : 18 , 23 , 29 , 2 k – 1 , k is 18 , then k = ...
   (a) 1 (b) 7 (c) 29 (d) 19
6. The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is ....
   (a) 22 (b) 23 (c) 24 (d) 25

---

2 **Complete :**

1. $0.3 = \cdots\cdots$ (in the form of $\frac{a}{b}$)
2. $\sqrt[3]{343} = \cdots\cdots$
3. The slope of any line parallel to x-axis is ....

4 The mode is the .... common value in the set.

5 If the order of the median of some values is the fourth , then the number of the values is ....

3 **[a] Find the solution set of :** $5x - 3 < 2x + 9$ in $\mathbb{R}$

[b] Find the value of : $\sqrt{18} + \sqrt{54} - 3\sqrt{2} - \frac{1}{2}\sqrt{24}$

4 **[a]** The radius length of the base of a right circular cylinder is 4 cm. and its height is 9 cm. Find the volume in terms of π

[b] If A (2 , – 1) , B (10 , 3) and C (2 , 3) , find the slope of each of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

5 **[a] Find :** $[-1 , 4] - [-3 , 2[$ by using the number line.

[b] The following table shows the frequency distribution for the score of 50 students in an examination :

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	3	5	9	10	12	7	4	50

Find the mean of the students score.

9 El-Monofia Governorate

Shiben Elkom Directorate
Supervisor of Math

Answer the following questions :

1 **Choose the correct answer :**

1 The degree of the algebraic term $2x^3y^2$ is the ....

(a) second. (b) third. (c) fourth. (d) fifth.

2 If the radius length of a sphere is 6 cm. , then its volume is .... cm^3.

(a) 6π (b) 36π (c) 72π (d) 288π

3 If x is a negative number , then the number .... is positive.

(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{1}{2}x$

4 $\sqrt{8} - 2\sqrt{2} =$ ....

(a) 4 (b) 8 (c) zero (d) 2

5 If $|x| = 7$, then $x =$ ....

(a) 7 (b) – 7 (c) ± 7 (d) 8

6 The arithmetic mean for five values is 13 , then the sum of these values is ....

(a) 70 (b) 56 (c) 65 (d) 13

2 **Complete :**

1. The slope of the straight line parallel to X-axis is ....
2. If the mode of the values : 18 , 11 , 4 , 2 X is 18 , then $X =$ ....
3. If (k , 2) represents the relation : $X + 2y = 5$, then k = ...
4. If the order of the median of some values is the seventh , then the number of these values is ....
5. The median of : a + 2 , a , a − 2 , a − 1 , a + 1 is ....

---

3 [a] **Simplify :** $\sqrt{75} - 6\sqrt{\frac{1}{3}} - 3\sqrt{12}$

[b] If $A = [-2 , 3]$, $B = [1 , \infty[$, **find using the number line :**

1. $A \cap B$
2. $A \cup B$

[c] The diameter length of a cylinder is 7 cm. and its height is 10 cm. Find the lateral area of the cylinder.

---

4 [a] **Represent the relation :** $2X + y = 4$, then find the slope of the straight line representing this relation.

[b] If $X = \frac{1}{\sqrt{7}+\sqrt{6}}$, $y = \sqrt{7} + \sqrt{6}$, **prove that :** X and y are two conjugate numbers , **then find :** $(X + y)^2$ in the simplest form.

---

5 [a] **Find the S.S. in $\mathbb{R}$ for the inequality :**

$\sqrt[3]{-8} \le X + 1 \le \sqrt{9}$, then represent it on the number line.

[b] **From the following frequency distribution :**

The set	10 –	20 –	30 –	40 –	50 –	Total
Frequency	10	20	25	k	15	100

Find : 1. The value of k 2. The arithmetic mean.

10 El-Gharbia Governorate

Central Mathematics Supervision
Official Language Schools

Answer the following questions :

1 **Choose the correct answer :**

1. The S.S. in $\mathbb{R}$ for the equation : $X^3 + 27 = 0$ is ....
(a) $\{-3\}$ (b) $\{2\}$ (c) $\{3\}$ (d) $\varnothing$

2 If the mode of the values : 3 , 6 , $X + 1$, 6 , 3 , 1 is 6 , then $X =$ ....

(a) 1 (b) 2 (c) 5 (d) 0

3 The cube whose volume is 64 cm^3, the length of one of its edges is .... cm.

(a) 8 (b) 3 (c) 16 (d) 4

4 If $X < \sqrt{51} < X + 1$, $X \in \mathbb{Z}$, then $X =$ ....

(a) 8 (b) 7 (c) 6 (d) 5

5 $\sqrt{7} + \sqrt{7} =$ ....

(a) $\sqrt{28}$ (b) 7 (c) 14 (d) $\sqrt{14}$

6 If the point (a , 1) satisfies the relation $X + y = 5$, then a = ...

(a) 1 (b) 2 (c) 5 (d) 4

2 Complete :

1 $\sqrt[3]{\cdots\cdots} = -\sqrt{4}$

2 If the order of the median of some values is seventh , then the number of these values is ....

3 If the lower limit of a set is 8 and the upper limit of the same set is 10 , then the centre of this set is ....

4 $[-3 , 6] \cap [3 , 9] =$ ....

5 The slope of X-axis is ....

3 [a] Reduce to the simplest form : $\dfrac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

[b] Prove that : $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

[c] Find in $\mathbb{R}$ the solution set of the inequality : $-3 < 4X - 7 < 5$

4 [a] A right circular cylinder whose height is 10 cm. and its volume is 90 π cm^3.
Find the length of the radius of its base.

[b] If X = [– 3 , 4] , Y =]1 , ∞[**, find each of the following using the number line :**

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

5 [a] Simplify : $\sqrt{50} + \sqrt{18} - \sqrt{32}$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

11 El-Dakahlia Governorate — Maths Supervision

Answer the following questions :

1 Choose the correct answer from those given :

1. $[3, 5] -]3, 5[=$ ....
   (a) $\varnothing$ (b) $[3, 5]$ (c) $]3, 5[$ (d) $\{3, 5\}$
2. If the point (a , 1) satisfies the relation : $x + y = 5$, then a = ...
   (a) -4 (b) 1 (c) 4 (d) 5
3. If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....
   (a) 2 (b) 4 (c) 6 (d) 8
4. If the radius length of a sphere is 6 cm. , then its volume is .... cm^3.
   (a) 6π (b) 36π (c) 72π (d) 288π
5. $\sqrt{100-36} = 10 -$ ....
   (a) -6 (b) 2 (c) 4 (d) 6
6. The intersection point of the ascending and descending cumulative curves determines the .... on the sets axis.
   (a) order of the median (b) median
   (c) mean (d) mode

2 Complete each of the following :

1. $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$, ... (in the same pattern)
2. The slope of any straight line parallel to x-axis is ....
3. If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then n = ...
4. The arithmetic mean of the set of values : $3 - x$, $5 + x$, 4 equals ....
5. If the mode of the values : 4 , 11 , 8 , $2x$ is 4 , then $x =$ ....

3 **[a]** Find the slope of $\overleftrightarrow{AB}$ where A (– 1 , 3) and B (2 , 5) , is the point C (8 , 1) $\in \overleftrightarrow{AB}$?

[b] If $x = \sqrt{7} + \sqrt{5}$, $xy = 2$, **find the value of :** $\frac{x + y}{xy}$

4 **[a] Find the S.S. of the inequality :** $-2 \le 3x + 7 < 10$ in $\mathbb{R}$, then represent the interval of solution on the number line.

[b] Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 72π cm^3.

5 **[a] Simplify to the simplest form :** $\sqrt{18}+\sqrt[3]{54}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{16}$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

12 Ismailia Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The slope of y-axis is ....

(a) 0 (b) $\frac{1}{2}$ (c) undefined. (d) – 1

2 The mean of 8 , 19 , 11 , 12 , 10 is ....

(a) 12 (b) 15 (c) 20 (d) 11

3 The multiplicative inverse of $\frac{\sqrt{6}}{2}$ is ....

(a) $-\frac{\sqrt{6}}{2}$ (b) $\frac{\sqrt{6}}{3}$ (c) $\frac{\sqrt{6}}{2}$ (d) $2\sqrt{6}$

4 If the age of Ali now is X years , then his age after 12 years is .... years.

(a) $X + 12$ (b) $X - 12$ (c) $X + 15$ (d) $12\,X$

5 $\sqrt[3]{125}=\sqrt{\dots\dots\dots}$

(a) 5 (b) 100 (c) 10 (d) 25

6 If the mode of : 7 , 10 , k + 3 , 9 is 7 , then k = ...

(a) 3 (b) 10 (c) 4 (d) 9

2 **Complete :**

1 $4\,a^5 \times 5\,a^2 =$ ....

2 The median of : 15 , 7 , 16 , 9 , 4 , 20 is ....

3 $[2 , 7] - \{2 , 7\} =$ ....

4 If (3 , k) satisfies the relation : $2\,X + y = 10$, then k = ...

5 $\{1 , 2 , 3\} \cap \{2 , 4 , 5\} =$ ....

3 [a] The area of a sphere is 616 cm². Find its diameter length $\left(\pi = \frac{22}{7}\right)$

[b] Graph the relation : $y = 2\,X$

[c] Find the slope of $\overleftrightarrow{AB}$ where A (– 1 , 5) , B (2 , 6)

4 **[a] Simplify :** $\sqrt{72} + 2\sqrt{32} - 3\sqrt{2}$

[b] Find the S.S. in $\mathbb{R}$ and represent it on the number line of : $1 < 3 - 2X \leq 11$

5 **[a]** If $A = [-2, 3]$, $B =]0, 5[$ **, using the number line find :**

1. $A \cup B$ 2. $A \cap B$ 3. $A - B$

[b] From the following frequency distribution :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	7	10	8	6	9	**40**

Find the mean.

13 Kafr El-Sheikh Governorate — Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. The S.S. of the equation : $X(X^2 + 4) = 0$ in $\mathbb{R}$ is ....
   (a) $\{4\}$ (b) $\{0\}$ (c) $\{-4, 0\}$ (d) $\{4, -4\}$
2. The slope of the straight line which is perpendicular to X-axis is ....
   (a) 1 (b) zero (c) – 1 (d) undefined.
3. If the arithmetic mean of the numbers : $5, 4, X - 3, 6, 4$ is 4, then $X =$ ....
   (a) 5 (b) 4 (c) 6 (d) 3
4. If the mode of the numbers : $5, 2, 4, X - 2$ is 5, then $X =$ ....
   (a) 4 (b) 6 (c) 7 (d) 5
5. If $-2X < 6$, then X ....
   (a) < 6 (b) > -3 (c) > 6 (d) > -6
6. $\mathbb{Z} \cap \mathbb{N} =$ ....
   (a) $\{0\}$ (b) $\mathbb{Z}_-$ (c) $\mathbb{N}$ (d) $\mathbb{Q}$

2 **Complete the following :**

1. The multiplicative inverse of the number $\sqrt{10} - 3$ is ....
2. $[3, 5] -]3, 5[=$ ....
3. The median of the numbers : 41, 19, 15, 30, 20 is ....
4. $\sqrt{18} - \sqrt{2} =$ ....
5. If the slope of the straight line passing through $(2, k)$, $(3, -1)$ is 2, then $k =$ ....

3 [a] Find the lateral area of the right circular cylinder of volume $150\,\pi$ cm^3 and height 6 cm.

[b] **Find in the simplest form :** $3\sqrt{2}+\sqrt{8}-\sqrt{18}$

---

4 [a] **Find in $\mathbb{R}$ the S.S. of the inequality :** $x < 2x - 1 < x + 3$

[b] If $x = \sqrt{7}-\sqrt{5}$, $y = \frac{2}{x}$, **find :** $\frac{x+y}{xy}$ in the simplest form.

---

5 [a] If $(-1 , 5)$ satisfies the relation : $3x + k\,y = 7$, then find k

[b] **The following table shows the frequency of marks of 50 students :**

Sets	2 –	6 –	10 –	14 –	ℓ –	22 –	26 –	**Total**
Frequency	3	6	8	10	11	k	4	**50**

Find : 1 The value of each of ℓ and k

2 The arithmetic mean for the marks of students.

14 Souhag Governorate

Akhmeem Educational Management
Private Future Generation Language School

Answer the following questions :

1 Choose the correct answer :

1 The simplest form of $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$ is ....

(a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $\sqrt[2]{3}$

2 The volume of a cube is 64 cm^3, then its edge length is .... cm.

(a) 4 (b) 8 (c) 16 (d) 64

3 The mean of the values : 34 , 23 , 25 , 40 , 22 , 12 is ....

(a) 22 (b) 23 (c) 24 (d) 26

4 If the point (k , 1) satisfies the relation : $x + y = 5$, then k = ...

(a) 1 (b) – 4 (c) 4 (d) 5

5 $\left(2\sqrt[3]{2}\right)^3$ = ...

(a) 4 (b) 8 (c) 16 (d) 40

6 If the mode of the values : 4 , 11 , 8 , $2x$ is 4 , then x = ...

(a) 2 (b) 4 (c) 6 (d) 8

---

2 Complete :

1 The S.S. of : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

2 $\sqrt{8}+\sqrt{18}-3\sqrt{2}$ = ...

3 The mode of : 3 , 5 , 3 , 4 , 3 is ....

4 $]-2 , 2[\cup \{-2 , 2\} =$ ....

5 If the volume of a sphere = $\frac{9}{2}\pi$ cm^3, then its diameter length equals .... cm.

---

3 [a] **Find in the simplest form :** $\sqrt{18} + \sqrt{32} - 3\sqrt{2} - \frac{1}{2}\sqrt{8}$

[b] If $X = \sqrt{5} - \sqrt{2}$, $y = \frac{3}{\sqrt{5} - \sqrt{2}}$, **prove that :** X and y are two conjugate numbers.

---

4 [a] **Represent graphically the linear relation :** $y = 2 - X$

[b] **Find the solution set of the inequality :**

$-2 < 3X + 7 \leq 10$ in $\mathbb{R}$, then represent the S.S. on the number line.

---

5 [a] A right circular cylinder of radius length 4 cm. and its height is 9 cm.

Find its volume in terms of π

[b] **Find the arithmetic mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

15 Aswan Governorate

Aswan Educational Directorate
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The multiplicative inverse of $\frac{\sqrt{3}}{5}$ is ....

(a) $-\frac{\sqrt{3}}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5\sqrt{3}}{3}$

2 If $X = \sqrt{6} - \sqrt{2}$, $y = \frac{4}{X}$, then y = ...

(a) 4 (b) $\sqrt{6} + \sqrt{2}$ (c) 10 (d) $\sqrt{8}$

3 If the ordered pair (2 k , k) satisfies the relation : $y + 2X = 5$, then k = ...

(a) 1 (b) 2 (c) 3 (d) 4

4 If the lower boundary of a set is 4 and the upper boundary is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

5 $[1 , 5] - \{1 , 5\} =$ ....

(a) $[2 , 4]$ (b) $]1 , 5[$ (c) $]0 , \infty[$ (d) $]1 , 5]$

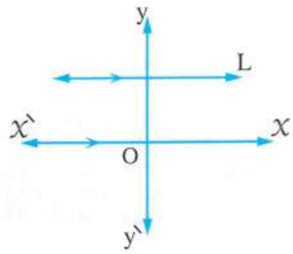
6 **In the opposite figure :**

The slope of the straight line

L is ....

(a) positive. (b) negative.

(c) zero. (d) undefined.



2 **Complete each of the following :**

1 $\sqrt[3]{64} = \sqrt{\ldots\ldots}$

2 In the relation : $y = 3x + 4$, if $y = 1$, then $x =$ ....

3 If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then $x =$ ....

4 $[-2 , 5[\cap \mathbb{R}_+ =$ ....

5 The median of the set of values : 34 , 23 , 25 , 40 , 22 , 4 is ....

3 **[a] Find in the simplest form the value of :** $\sqrt[3]{128} + \sqrt[3]{16} + 2\sqrt[3]{-54}$

[b] If $x = \sqrt{3} + 1$ and $y = \frac{2}{\sqrt{3} + 1}$

1 **Prove that :** x and y are conjugate.

2 **Find the value of :** $\frac{x + y}{x y}$ in the simplest form.

4 **[a]** If $X =]-1 , 4]$ and $Y = [3 , \infty[$ **, using the number line find each of the following :**

1 $X \cup Y$ **2** $X - Y$ **3** $X \cap Y$

[b] Find the S.S. in $\mathbb{R}$ of : $-2 \leq 3x + 7 \leq 10$ and represent it on the number line.

5 **[a]** Represent graphically the relation $y = 2 - x$ and if $(-4 , b)$ satisfies the relation , find the value of b

[b] Find the arithmetic mean of the following frequency distribution :

Sets of marks	5 –	15 –	25 –	35 –	45 –	**Total**
Number of pupils	7	10	12	13	8	**50**

Some Schools Examinations of 2020 on Geometry

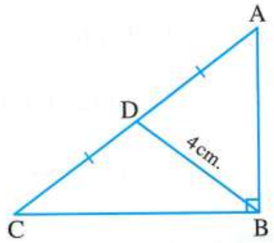
1 Cairo Governorate

Western Cairo Educational Zone
Mathematics Inspection

Answer the following questions :

1 Choose the correct answer :

1 In the opposite figure :



AC = ... cm.

(a) 4 (b) 6

(c) 8 (d) 2

2 If Δ ABC is right-angled at A and AB = AC , then m ($\angle$ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

3 In Δ ABC , if AB = 6 cm. , AC = 7 cm. , then BC $\in$ ....

(a)]6 , 13] (b) [6 , 7] (c)]1 , 13[(d) [1 , 7[

4 In Δ XYZ , if XY < XZ , then ....

(a) m ($\angle$ Y) $\leq$ m ($\angle$ Z) (b) m ($\angle$ Y) > m ($\angle$ Z)

(c) m ($\angle$ Y) = m ($\angle$ Z) (d) m ($\angle$ Z) > m ($\angle$ Y)

5 If Δ ABC is right-angled at B , m ($\angle$ A) = 55° , then the number of axes of symmetry of Δ ABC equals ....

(a) 1 (b) 2 (c) 3 (d) zero

6 The triangle in which the measures of two angles of it are 42° and 69° is .... triangle.

(a) an isosceles (b) an equilateral (c) a scalene (d) a right-angled

2 Complete the following :

1 Any point on the axis of symmetry of a line segment is .... from its terminals.

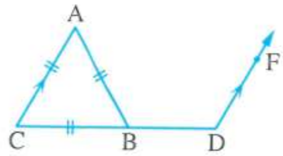
2 The longest side in the right-angled triangle is ....

3 The point of intersection of the medians of the triangle divides each of them by the ratio .... : .... from the vertex.

4 The measure of any exterior angle of an equilateral triangle equals ....°

5 The sum of the lengths of any two sides in a triangle is .... the length of the third side.

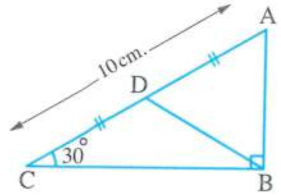
3 **[a] In the opposite figure :**



Δ ABC is an equilateral triangle , $\overrightarrow{DF} \parallel \overline{AC}$

Find by proof : m ($\angle$ D)

[b] In the opposite figure :

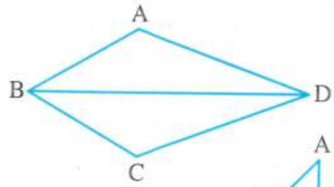


m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

, AC = 10 cm. , AD = DC

Find : The perimeter of Δ ABD

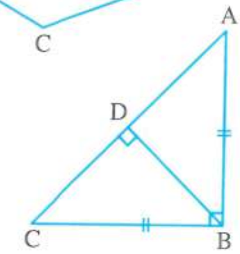
4 **[a] In the opposite figure :**



AB < AD , BC < CD

Prove that : m ($\angle$ ABC) > m ($\angle$ ADC)

[b] In the opposite figure :



m ($\angle$ ABC) = 90° , $\overline{BD} \perp \overline{AC}$

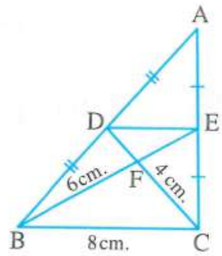
, AB = BC

Prove that : Δ DCB is an isosceles triangle.

5 **[a]** XYZ is a triangle in which m ($\angle$ X) = 60° , m ($\angle$ Y) = 50°

Order the lengths of the sides of the triangle descendingly.

[b] In the opposite figure :



ABC is a triangle in which D , E are the midpoints of $\overline{AB}$, $\overline{AC}$

, FC = 4 cm. , FB = 6 cm.

, BC = 8 cm.

Find : The perimeter of Δ DFE

2 Cairo Governorate

Hadayeq El-Koba Zone
Al Nokrashy Governmental Lang. School

Answer the following questions :

1 **Choose the correct answer from those given :**

1 A triangle has one line of symmetry , the lengths of two sides are 4 cm. and 8 cm. , then the length of the third side is .... cm.

(a) 3 (b) 4 (c) 8 (d) 6

2 The point of intersection of the medians of the triangle divides each median in the ratio of .... from the base.

(a) 2 : 1 (b) 2 : 3 (c) 1 : 2 (d) 1 : 3

3 If m (∠ A) = 50° , then the measure of its reflex angle is ....

(a) 40° (b) 130° (c) 310° (d) 180°

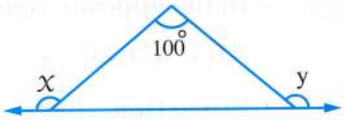
4 If the length of the side of an equilateral triangle is 10 cm. , then the length of its height is .... cm.

(a) 10 (b) 5 (c) $5\sqrt{3}$ (d) 6

5 In Δ ABC , if AB = 6 cm. , AC = 7 cm. , then the length of $\overline{BC} \in$ ....

(a) [6 , 7] (b)]1 , 7[(c) [1 , 13] (d)]1 , 13[

6 **In the opposite figure :**



$x + y =$ ....

(a) 180° (b) 360°

(c) 240° (d) 280°

---

2 **Complete :**

1 If the measures of two angles in a triangle are different , then the greater angle in measure of them is ....

2 In the triangle ABC , if m (∠ A) = 50° , m (∠ B) = 60° , then the longest side is ....

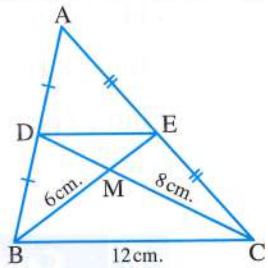
3 The median drawn from the vertex angle of an isosceles triangle .... and ....

4 In Δ ABC , if m (∠ A) = 30° , m (∠ B) = 90° , then AC = ... BC

5 The perpendicular bisector of a line segment is called ....

---

3 **[a]** **In the opposite figure :**

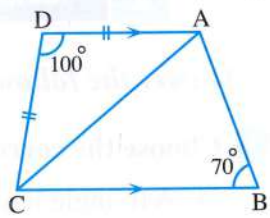


In Δ ABC : $\overline{BE}$, $\overline{CD}$ are two medians , MB = 6 cm.

, BC = 12 cm. , MC = 8 cm.

Find : The perimeter of Δ MDE

[b] **In the opposite figure :**



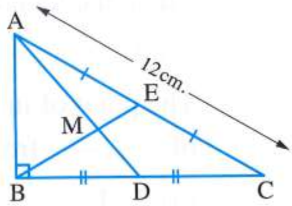
$\overline{AD}$ // $\overline{BC}$, AD = DC

, m (∠ D) = 100° , m (∠ B) = 70°

Prove that : **1** AC > AB **2** Δ ABC is isosceles.

---

4 **[a]** **In the opposite figure :**



Δ ABC is right-angled at B

, E and D are the midpoints of $\overline{AC}$, $\overline{BC}$ respectively

, AC = 12 cm.

Find : The length of each of $\overline{BE}$, $\overline{ME}$

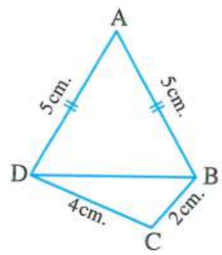
[b] In the opposite figure :

ABCD is a quadrilateral

, AB = AD = 5 cm.

, BC = 2 cm. , DC = 4 cm.

Prove that : m (∠ ABC) > m (∠ ADC)

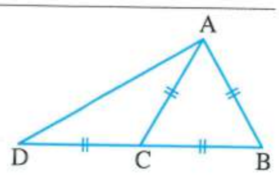


---

5 **[a] In the opposite figure :**

AB = BC = AC = CD

Prove that : m (∠ BAD) = 90°

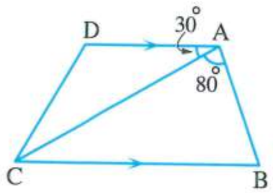


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 80°

, m (∠ DAC) = 30°

Prove that : BC > AB



3 Cairo Governorate

New Cairo Educational Zone
Dr. Nermien Ismail Schools

Answer the following questions :

1 **Choose the correct answer :**

1. In Δ ABC , if AB = AC , m (∠ B) = 40° , then m (∠ A) = ...
   (a) 70° (b) 55° (c) 100° (d) 40°

2. The point of concurrence of the medians of the triangle divides each median at the ratio .... from the vertex.
   (a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 1 : 3

3. In Δ ABC , if AB = 7 cm. , BC = 10 cm. , then the length of $\overline{AC}$ must satisfy which of the following inequalities ?
   (a) 3 ≤ AC ≤ 17 (b) 3 < AC < 17 (c) 10 < AC < 20 (d) 14 < AC < 20

4. If Δ ABD is obtuse-angled at B and C is the midpoint of $\overline{BD}$, then the longest side in Δ ABD is ....
   (a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{BD}$

5. In Δ ABC , if m (∠ A) = 64° , m (∠ B) = 35° , then the longest side of the triangle is ....
   (a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) otherwise.

6 ABCD is a rectangle , M is the point of intersection of its diagonals , if the length of the diagonal is 6 cm. , then the length of the median $\overline{AM}$ is .... cm.

(a) 3 (b) 6 (c) 9 (d) 12

2 Complete each of the following :

1 The length of the side which is opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

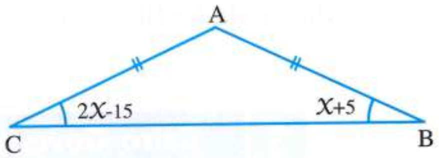
2 In the right-angled triangle , the longest side is the ....

3 The straight line drawn from the vertex of the isosceles triangle , perpendicular to the base .... this vertex.

4 The measure of the exterior angle of the equilateral triangle equals ....°

5 The number of axes of symmetry of the isosceles triangle is ....

3 [a] In the opposite figure :

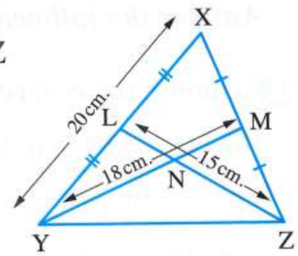


ABC is a triangle , AB = AC , m (∠ B) = $(X + 5)°$

, m (∠ C) = $(2X - 15)°$

Find : m (∠ A) (show all of your work)

[b] In the opposite figure :



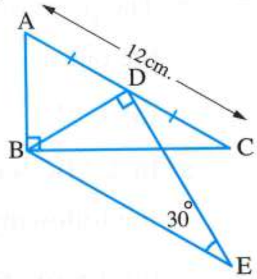
N is the point of concurrence of the medians of the triangle XYZ

, LZ = 15 cm. , YM = 18 cm.

, XY = 20 cm.

Find : The perimeter of the triangle NLY

[c] In the opposite figure :



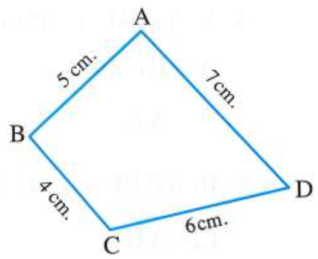
m (∠ ABC) = m (∠ BDE) = 90°

, D is the midpoint of $\overline{AC}$

, m (∠ E) = 30° , AC = 12 cm.

Find with proof : The length of $\overline{BE}$

4 [a] In the opposite figure :



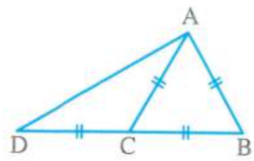
ABCD is a quadrilateral in which :

AB = 5 cm. , BC = 4 cm. , CD = 6 cm.

, AD = 7 cm.

Prove that : m (∠ ABC) > m (∠ ADC)

[b] In the opposite figure :



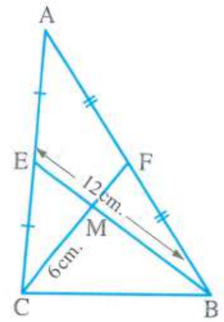
$AB = AC = CB = CD$

Prove that : $\overline{AB} \perp \overline{AD}$

[c] XYZ is a triangle in which : $XY = 10$ cm. , $YZ = 6$ cm. and $XZ = 8$ cm.

Arrange the measures of the angles of the triangle.

[d] In the opposite figure :



ABC is a triangle in which : F , E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

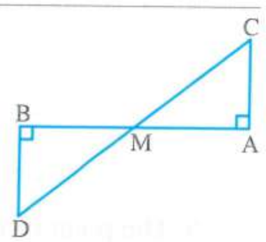
, $EB = 12$ cm.

, $MC = 6$ cm.

Find with proof : The length of each of $\overline{EM}$ and $\overline{MF}$

---

5 **[a] In the opposite figure :**



$\overline{DC} \cap \overline{AB} = \{M\}$

, $m(\angle A) = m(\angle B) = 90°$

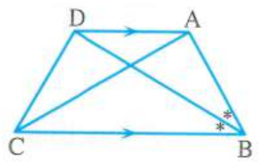
Prove that : $DC > AB$

[b] ABC is a triangle in which : $m(\angle A) = (6\,X)°$

, $m(\angle B) = (4\,X - 9)°$, $m(\angle C) = 3\,(X - 2)°$

Arrange the lengths of the sides of the triangle.

[c] In the opposite figure :



$\overline{AD} \,//\, \overline{BC}$, $\overrightarrow{BD}$ bisects $\angle ABC$

Prove that : Δ BAD is an isosceles triangle.

4 Giza Governorate — South Giza Administration

Answer the following questions :

1 **Choose the correct answer :**

1. If the measures of two angles of a triangle are $40°$, $100°$, then the triangle is .... triangle.

   (a) an isosceles (b) an equilateral (c) a scalene (d) a right-angled

2. The angle whose measure is more than $90°$ and less than $180°$ is .... angle.

   (a) an acute (b) an obtuse (c) a straight (d) a reflex

3 If the lengths of two sides in an isosceles triangle are 7 cm. and 3 cm. , then the length of the third side is .... cm.

(a) 3 (b) 10 (c) 7 (d) 4

4 In Δ ABC , if m ($\angle$ B) = 120° , then the longest side in it is ....

(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

5 If Δ ABC is right-angled at B , AB = 3 cm. , BC = 4 cm. , then the length of the median from B is .... cm.

(a) 5 (b) 4 (c) 2.5 (d) 6

6 In Δ ABC , if m ($\angle$ A) = 30° , m ($\angle$ B) = 90° and AC = 10 cm. , then BC = ...

(a) 20 cm. (b) 15 cm. (c) 10 cm. (d) 5 cm.

2 Complete each of the following :

1 The angle of measure 70° complements an angle of measure ....°

2 In Δ ABC , if AB = 3 cm. , BC = 5 cm. , then AC $\in$].............. , ...[

3 If $\overline{AB} \equiv \overline{CD}$ and AB = 6 cm. , then AB + CD = ... cm.

4 The bisector of the vertex angle of an isosceles triangle .... and ....

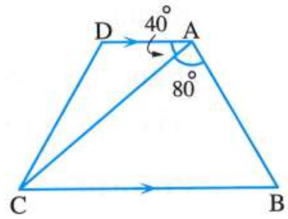
5 The point of intersection of the medians of the triangle divides each median in the ratio .... : .... from the vertex.

3 [a] **In the opposite figure :**

$\overline{AD} // \overline{BC}$, m ($\angle$ BAC) = 80°

and m ($\angle$ DAC) = 40°

Prove that : BC > AC



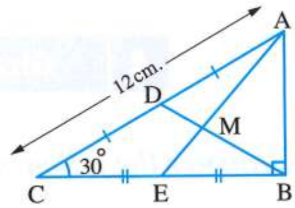
[b] **In the opposite figure :**

Δ ABC is right-angled at B , m ($\angle$ C) = 30°

, D is the midpoint of $\overline{AC}$

, E is the midpoint of $\overline{BC}$, AC = 12 cm.

Find : The length of each of $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$

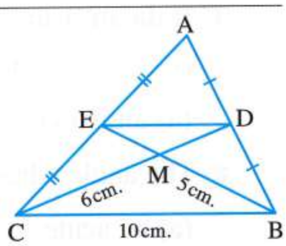


4 [a] **In the opposite figure :**

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, BC = 10 cm. , MB = 5 cm. and MC = 6 cm.

Find : The perimeter of Δ MDE

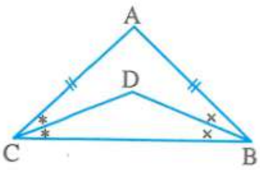


[b] In the opposite figure :

AB = AC , $\overrightarrow{BD}$ bisects ∠ ABC

and $\overrightarrow{CD}$ bisects ∠ ACB

Prove that : Δ DBC is an isosceles triangle.

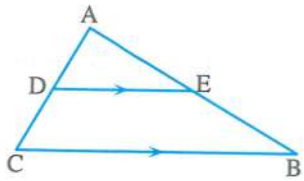


5 **[a] In the opposite figure :**

ABC is a triangle in which :

AB > AC and $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ ADE) > m (∠ AED)

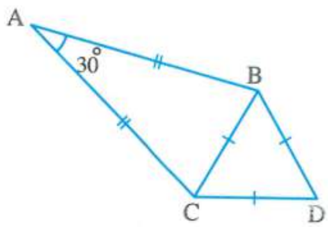


[b] In the opposite figure :

m (∠ A) = 30° , AB = AC

and Δ DBC is equilateral.

Find : m (∠ ABD)



5 Giza Governorate

Boulaq El Dakrour Directorate
Dar El Hanan Lang. Sch. For Girls

Answer the following questions :

1 **Choose the correct answer :**

1 The lengths 9 cm. , 4 cm. and .... may be the side lengths of an isosceles triangle.

(a) 9 cm. (b) 13 cm. (c) 5 cm. (d) 4 cm.

2 In Δ ABC , if m (∠ B) = 130° , then the longest side of it is ....

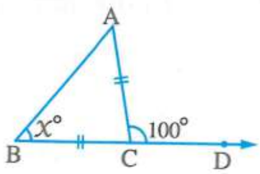
(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

3 **In the opposite figure :**

CA = CB , m (∠ B) = x°

, m (∠ ACD) = 100° where C ∈ $\overline{BD}$

, then x = ...



(a) 50° (b) 100° (c) 150° (d) 200°

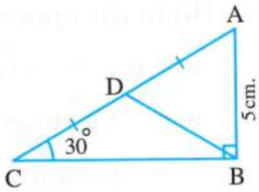
4 The measure of the exterior angle of an equilateral triangle equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

5 In Δ ABC , if AB = 6 cm. and AC = 7 cm. , then BC ∈ ....

(a)]6 , 13] (b) [6 , 7] (c)]1 , 13[(d) [1 , 7[

6 **In the opposite figure :**



AD = DC , m (∠ C) = 30°

, m (∠ ABC) = 90° , AB = 5 cm.

, then the perimeter of Δ ABD = ... cm.

(a) 5 (b) 15 (c) 20 (d) 25

2 **Complete the following :**

1 ABC is a triangle in which AB = AC and m (∠ A) = 60° , if its perimeter = 18 cm. , then BC = ... cm.

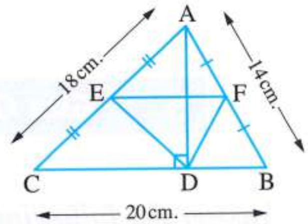
2 The number of the axes of symmetry of the equilateral triangle equals ....

3 The longest side of the right-angled triangle is the ....

4 If the angles of a triangle are congruent , then the triangle is ....

5 In Δ ABC , if AB > BC , then m (∠ A) .... m (∠ C)

3 **[a] In the opposite figure :**



ABC is a triangle in which AB = 14 cm.

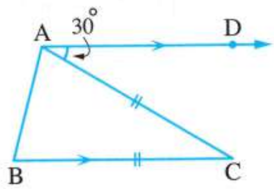
, AC = 18 cm. , BC = 20 cm.

, E is the midpoint of $\overline{AC}$

, F is the midpoint of $\overline{AB}$ and $\overline{AD} \perp \overline{BC}$

Find : The perimeter of Δ DEF

[b] In the opposite figure :

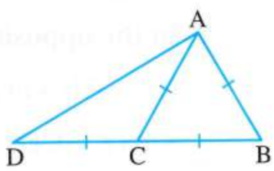


ABC is a triangle in which AC = BC

, $\overrightarrow{AD}$ // $\overline{BC}$, m (∠ DAC) = 30°

Find with proof : The measures of the angles of Δ ABC

4 **[a] In the opposite figure :**

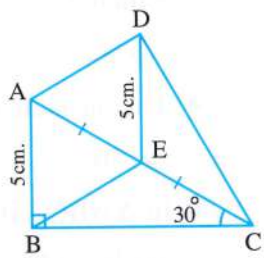


AB = BC = AC = DC

Prove that :

m (∠ BAD) = 90°

[b] In the opposite figure :



m (∠ ABC) = 90° , E is the midpoint of $\overline{AC}$

, m (∠ ACB) = 30°

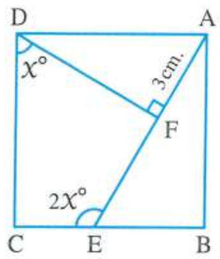
, AB = DE = 5 cm.

Prove that : m (∠ ADC) = 90°

5 **[a]** In Δ ABC , m ($\angle$ A) = 40° , m ($\angle$ B) = 75° , m ($\angle$ C) = 65°

, arrange the lengths of the sides of this triangle descendingly.

[b] In the opposite figure :



ABCD is a square , E $\in \overline{BC}$

where m ($\angle$ FDC) = x° and m ($\angle$ FEC) = 2 x°

, $\overline{DF} \perp \overline{AE}$, AF = 3 cm.

Calculate : The area of the square ABCD

6 Alexandria Governorate

El-Montazah Educational Zone
Leaders Language School

Answer the following questions :

1 **Complete :**

1. If Δ ABC is a right-angled triangle at B , m ($\angle$ A) = 30° , AC = 10 cm. , then CB = …………… cm.
2. In Δ ABC , m ($\angle$ A) = m ($\angle$ B) = m ($\angle$ C) , then the measure of the exterior angle equals ……………°
3. In Δ ABC , AB = AC , m ($\angle$ B) = x + 30° , m ($\angle$ C) = 2 x + 5° , then x = ……………°
4. In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to ……………
5. In any triangle , the sum of the lengths of any two sides …………… the length of the third side.

2 **Choose the correct answer :**

1. If $\overline{AD}$ is a median of Δ ABC and M is the point of concurrence of the medians , then AM = …………… AD

   (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2

2. The measure of one of the base angles of an isosceles triangle is 65° , then the measure of its vertex angle equals ……………°

   (a) 65 (b) 50 (c) 130 (d) 55

3. In the triangle ABC , if m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then the longest side is ……………

   (a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) 110 cm.

4. The numbers which can not be side lengths of a triangle are ……………

   (a) 3 , 3 , 3 (b) 3 , 3 , 4 (c) 3 , 3 , 5 (d) 3 , 3 , 6

5 The number of the axes of symmetry of the scalene triangle is ....

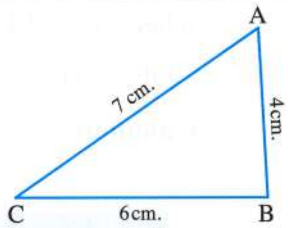
(a) 1 (b) 2 (c) 3 (d) 0

6 If Δ XYZ is right-angled at Y , then XZ .... YZ

(a) < (b) ≤ (c) > (d) =

3 **[a] In the opposite figure :**

Arrange the angles of Δ ABC descendingly due to their measures.

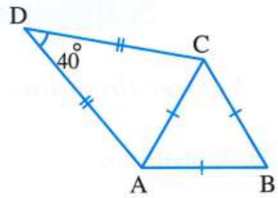


[b] In the opposite figure :

m (∠ D) = 40° , DA = DC

and Δ ABC is an equilateral triangle.

Find : m (∠ DCB)



4 **[a] In the opposite figure :**

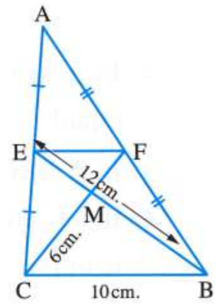
ABC is a triangle

, F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

If BE = 12 cm. , CM = 6 cm.

, BC = 10 cm.

, then find : The perimeter of Δ MEF

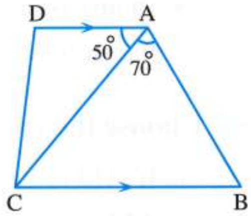


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, m (∠ CAB) = 70°

, m (∠ DAC) = 50°

Prove that : BC > AC



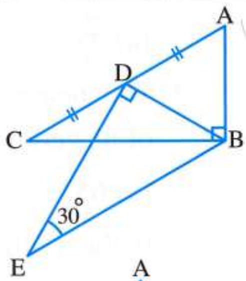
5 **[a] In the opposite figure :**

m (∠ ABC) = m (∠ BDE) = 90°

, m (∠ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE

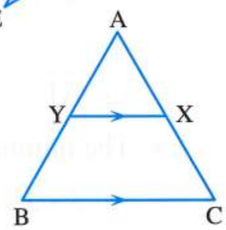


[b] In the opposite figure :

ABC is a triangle in which :

AB = AC , $\overline{XY}$ // $\overline{CB}$

Prove that : Δ AXY is an isosceles triangle.



7 Alexandria Governorate

Borg El-Arab Zone
Mathematical Supervisors

Answer the following questions :

1 Choose the correct answer :

1 An isosceles triangle has two sides of lengths 6 cm. and 12 cm. , then the length of the third side equals .... cm.

(a) 6 (b) 9 (c) 12 (d) 18

2 In Δ XYZ , if m ($\angle$ Y) = 115° , then the longest side is ....

(a) $\overline{XY}$ (b) $\overline{YZ}$
(c) $\overline{ZX}$ (d) the median of the triangle.

3 The lengths 5 cm. , 4 cm. and .... cm. are lengths of sides of a triangle.

(a) 8 (b) 9 (c) 12 (d) 10

4 The triangle having two angles of measures 74° and 53° is .... triangle.

(a) an isosceles (b) an equilateral (c) a scalene (d) a right-angled

5 The intersection point of the medians of a triangle divides each median by the ratio 1 : .... from the base.

(a) 1 (b) 2 (c) 3 (d) 4

6 If two sides of a triangle have unequal lengths , then the smaller side is opposite to the angle of the .... measure from that is opposite to the other side.

(a) greater (b) smaller (c) equal (d) otherwise

2 Complete each of the following :

1 The length of the median of the right-angled triangle drawn from the vertex of the right angle equals .... the length of the hypotenuse.

2 The number of the axes of symmetry of an isosceles triangle is ....

3 The measure of the exterior angle of the equilateral triangle equals ....°

4 The two angles of the base of an isosceles triangle are ....

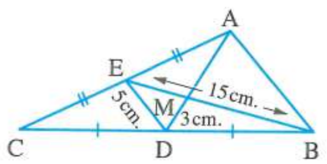
5 The sum of the measures of the accumulative angles at a point equals ....°

3 [a] In the opposite figure :

If E is the midpoint of $\overline{AC}$ and D is the midpoint of $\overline{BC}$

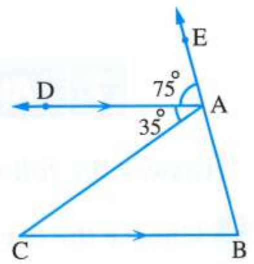
, ED = 5 cm. , MD = 3 cm. and BE = 15 cm.

, **find :** The perimeter of Δ AMB



[b] ABC is a triangle in which : m ($\angle$ B) = 40° , m ($\angle$ C) = 80°
Arrange its side lengths ascendingly.

4 **[a] In the opposite figure :**



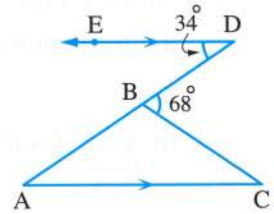
$\overrightarrow{AD} \parallel \overline{BC}$

, m (∠ EAD) = 75°

and m (∠ DAC) = 35°

Prove that : AC > AB

[b] In the opposite figure :



$\overrightarrow{DE} \parallel \overline{AC}$

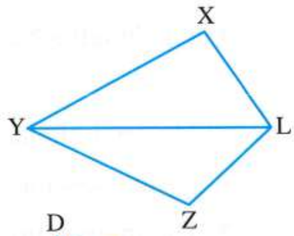
, m (∠ EDA) = 34°

and m (∠ DBC) = 68°

Prove that : Δ ABC is an isosceles triangle.

---

5 **[a] In the opposite figure :**

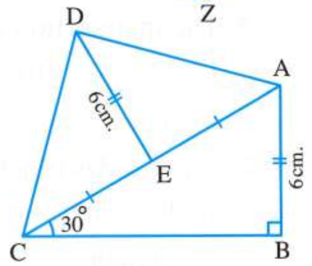


If XY > XL

, YZ > ZL

, **prove that :** m (∠ XLZ) > m (∠ XYZ)

[b] In the opposite figure :



m (∠ B) = 90° , m (∠ ACB) = 30°

, E is the midpoint of $\overline{AC}$ and AB = DE = 6 cm.

Find : 1 The length of $\overline{AC}$

2 m (∠ ADC)

8 El-Kalyoubia Governorate — Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 In any isosceles triangle , the type of the base angles is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

2 The medians of the triangle intersect at ....

(a) 4 points. (b) 3 points. (c) 2 points. (d) a point.

3 ABC is a triangle in which m (∠ A) = 100° , then the greatest side in length in the triangle is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) $\overline{BD}$

4 The numbers which can be lengths of sides of a triangle are ....

(a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

5 The triangle which has three axes of symmetry is ....

(a) scalene. (b) isosceles. (c) right-angled. (d) equilateral.

6 If ΔABC is an equilateral triangle , then $m(\angle B) =$ ....

(a) 30° (b) 60° (c) 70° (d) 90°

2 Complete :

1 In ΔABC , if the point D is the midpoint of $\overline{AB}$ and the point E is the midpoint of $\overline{AC}$, then DE = ... BC

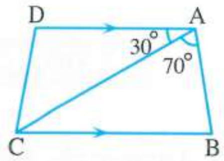
2 The base angles in the isosceles triangle are .... in measure.

3 In the triangle , the smallest angle in measure is opposite to .... side in length.

4 In the triangle ABC , if AB = AC , $m(\angle A) = 70°$, so $m(\angle C) =$ ....°

5 The point of concurrence of the medians of the triangle divides each median in the ratio of .... from the base.

3 [a] In the opposite figure :

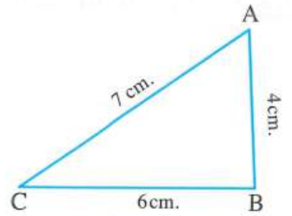


$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 70°$

, $m(\angle DAC) = 30°$

Prove that : AC > BC

[b] In the opposite figure :

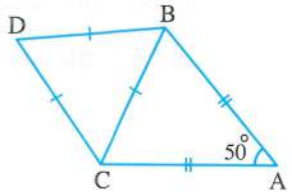


AB = 4 cm. , BC = 6 cm.

, AC = 7 cm.

Arrange the measures of the angles of the triangle ABC descendingly.

4 [a] In the opposite figure :

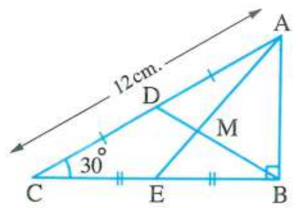


$m(\angle A) = 50°$, AB = AC

and ΔDBC is an equilateral triangle.

Find : $m(\angle ABD)$

[b] In the opposite figure :



ΔABC is right-angled at B , $m(\angle C) = 30°$

, D is the midpoint of $\overline{AC}$

, E is the midpoint of $\overline{BC}$, AC = 12 cm.

Find : The length of each of $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$

5 **[a] In the opposite figure :**

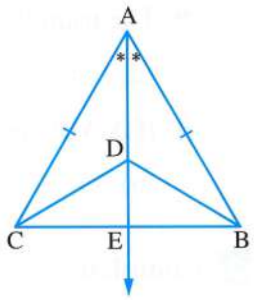
ABC is a triangle in which :

$AB = AC$, $\overrightarrow{AE}$ bisects $\angle BAC$

, $\overline{AE} \cap \overline{BC} = \{E\}$, $D \in \overline{AE}$

Prove that : **1** $BE = \frac{1}{2} BC$

2 $BD = CD$



[b] In the opposite figure :

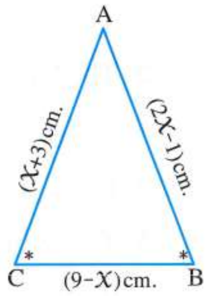
ABC is a triangle in which :

$m(\angle B) = m(\angle C)$

, $AB = (2X - 1)$ cm.

, $AC = (X + 3)$ cm. , $BC = (9 - X)$ cm.

Find : The perimeter of the triangle ABC



9 El-Sharkia Governorate

Hehia Educational Zone
Governmental Language Schools

Answer the following questions :

1 **Complete the following :**

1 The base angles of the isosceles triangle are ....

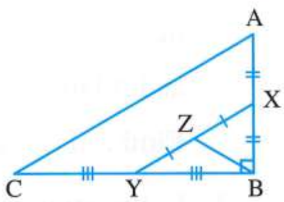
2 In ΔABC , if $\overline{AB} \perp \overline{BC}$ and $AB = BC$, then $m(\angle A) =$ ....°

3 In ΔABC , if $AB > AC$, then $m(\angle C)$ .... $m(\angle B)$

4 The triangle whose side lengths are $(2X - 1)$ cm. , $(X + 3)$ cm. , 7 cm. becomes an equilateral triangle when $X =$ .... cm.

5 **In the opposite figure :**

$AC =$ .... BZ



2 **Choose the correct answer from those given :**

1 The sum of lengths of any two sides in a triangle is .... the length of the third side.

(a) smaller than (b) greater than

(c) equal to (d) twice

2 The measure of the exterior angle of the equilateral triangle equals ....

(a) $30°$ (b) $60°$ (c) $90°$ (d) $120°$

3 The length of the hypotenuse of the right-angled triangle equals .... the length of the median drawn from the vertex of the right angle.

(a) third (b) quarter (c) half (d) twice

4 The lengths of two sides in a triangle are 4 cm. and 9 cm. and it has one axis of symmetry , then the length of the third side is ....

(a) 4 cm. (b) 5 cm. (c) 9 cm. (d) 13 cm.

5 The quadrilateral ABCD in which $\overline{BD}$ is an axis of symmetry of $\overline{AC}$ may be a ....

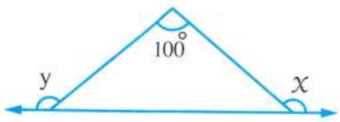
(a) rhombus. (b) rectangle. (c) parallelogram. (d) trapezium.

6 **In the opposite figure :**

$x + y =$ ....

(a) $100°$ (b) $280°$

(c) $140°$ (d) $80°$

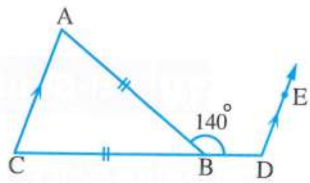


3 **[a] In the opposite figure :**

$AB = BC$, $m(\angle ABD) = 140°$

and $\overline{AC} \parallel \overrightarrow{DE}$

Find : $m(\angle EDC)$



[b] In the opposite figure :

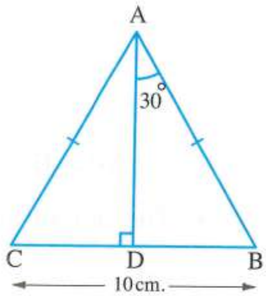
$AB = AC$, $BC = 10$ cm.

, $m(\angle BAD) = 30°$

and $\overline{AD} \perp \overline{BC}$

Find : **1** The length of each of $\overline{BD}$ and $\overline{AD}$

2 The area of ΔABC



4 **[a] In the opposite figure :**

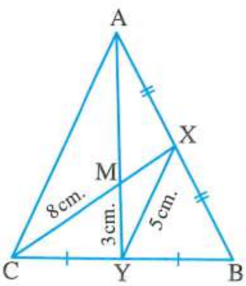
ABC is a triangle , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$, $XY = 5$ cm.

, $\overline{XC} \cap \overline{AY} = \{M\}$ where $CM = 8$ cm.

, $YM = 3$ cm.

Find : The perimeter of ΔMXY

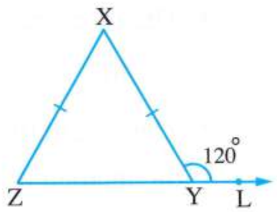


[b] In the opposite figure :

$XY = XZ$, $m(\angle XYL) = 120°$, $L \in \overrightarrow{ZY}$

Prove that :

ΔXYZ is an equilateral triangle.



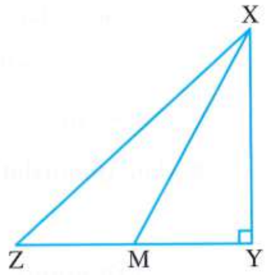
---

5 **[a] In the opposite figure :**

XYZ is a right-angled triangle

at Y and $M \in \overline{YZ}$

Prove that : $XZ > XM$



[b] In the opposite figure :

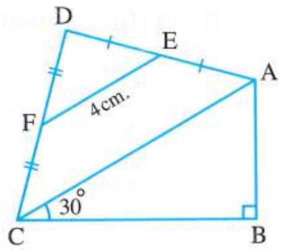
ABCD is a quadrilateral in which :

$m(\angle B) = 90°$, E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

, $m(\angle ACB) = 30°$ and $EF = 4$ cm.

Find by proof : The length of $\overline{AB}$



10 El-Gharbia Governorate

The Central Maths Supervision
Official Language Schools

Answer the following questions :

1 **Choose the correct answer :**

1 In ΔABC , if $m(\angle C) = 65°$, $m(\angle A) = 75°$, then ....

(a) $AB > BC$ (b) $AB < AC$ (c) $BC > AB$ (d) $AB = AC$

2 The sum of measures of two angles in the equilateral triangle equals ....

(a) 180° (b) 60° (c) 360° (d) 120°

3 The numbers 5 , 4 , ... can be lengths of sides of a triangle.

(a) 8 (b) 9 (c) 10 (d) 12

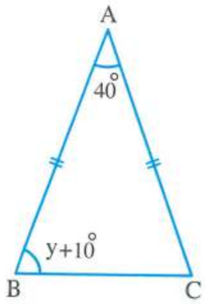
4 If M is the point of intersection of the medians of ΔABC and D is the midpoint of $\overline{BC}$, then $AD =$ ....

(a) 2 AM (b) 3 MD (c) $\frac{2}{3}$ MD (d) AM

5 If ΔABC is right-angled at B , then ....

(a) $AC < AB$ (b) $AC > BC$ (c) $AB = AC$ (d) $BC > AC$

6 **In the opposite figure :**



y = ...

(a) 30°

(b) 40°

(c) 60°

(d) 70°

---

2 Complete the following :

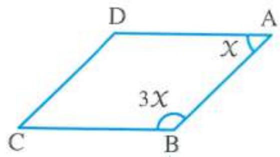
1 In Δ XYZ , if XY = XZ , $\overrightarrow{XL} \perp \overline{YZ}$, then $\overrightarrow{XL}$ bisects each of .... and ....

2 The number of axes of symmetry of the isosceles triangle is ....

3 If ABC is a right-angled triangle at B , AB = BC , then m ($\angle$ C) = ...°

4 The longest side of the right-angled triangle is ....

5 **In the opposite figure :**

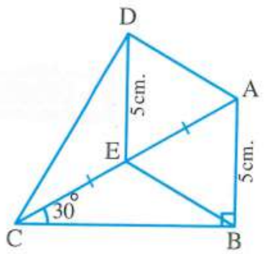


ABCD is a parallelogram

, then x = ...°

---

3 [a] In the opposite figure :



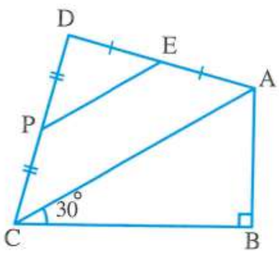
ABC is a right-angled triangle at B

, m ($\angle$ ACB) = 30° , AB = 5 cm.

and E is the midpoint of $\overline{AC}$

If DE = 5 cm. , **prove that :** m ($\angle$ ADC) = 90°

[b] In the opposite figure :



m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

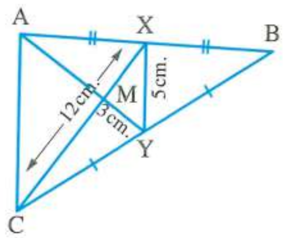
E is the midpoint of $\overline{AD}$

, P is the midpoint of $\overline{CD}$

Prove that : AB = EP

---

4 [a] In the opposite figure :



M is the intersection point of the medians

of Δ ABC , XY = 5 cm.

, CX = 12 cm. , MY = 3 cm.

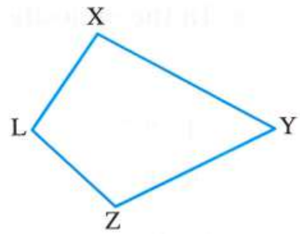
Find with proof : The perimeter of Δ MAC

[b] In the opposite figure :

$XY > XL$ and $YZ > ZL$

Prove that :

$m(\angle XLZ) > m(\angle XYZ)$



---

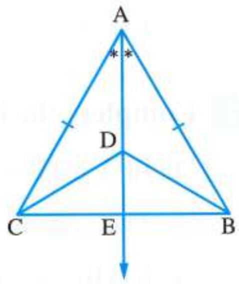
5 **[a] In the opposite figure :**

ABC is a triangle in which $AB = AC$

, $\overrightarrow{AE}$ bisects $\angle BAC$

Prove that :

1 $BE = \frac{1}{2} BC$ **2** $BD = CD$



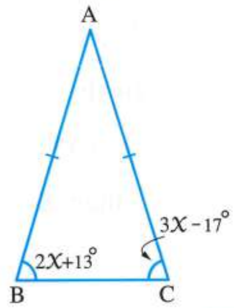
[b] In the opposite figure :

$AB = AC$, $m(\angle B) = 2X + 13°$

, $m(\angle C) = 3X - 17°$

Find :

The measures of the angles of ΔABC



11 Suez Governorate — Math Inspectorate

Answer the following questions :

1 **Choose the correct answer :**

1 In ΔABC , if $AB = 3$ cm. , $BC = 5$ cm. , then $AC \in$ ....

(a) $]3, 5[$ (b) $[3, 5]$ (c) $]2, 8[$ (d) $[2, 8]$

2 If the lengths of two sides of an isosceles triangle are 5 cm. and 10 cm. , then the length of the third side is .... cm.

(a) 10 (b) 5 (c) 15 (d) 4

3 In ΔABC , if $m(\angle A) = 100°$, then the longest side of it is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) its median.

4 In ΔABC , if $2m(\angle A) = m(\angle B) + m(\angle C)$, then $m(\angle A) =$ ....°

(a) 45 (b) 90 (c) 60 (d) 120

5 If $A \in$ the axis of symmetry of $\overline{BC}$, then $\overline{AB}$ .... $\overline{AC}$

(a) $\equiv$ (b) $=$ (c) // (d) $\perp$

6 The point of intersection of the medians of the triangle divides each of them in the ratio .... from the vertex.

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 1 : 2

2 Complete :

1 The base angles of an isosceles triangle are .... in measure.

2 If Δ ABC has one axis of symmetry and m ($\angle$ A) = 120° , then m ($\angle$ B) = ...°

3 In Δ ABC , if AB > AC , then m ($\angle$ C) > ....

4 The bisector of the vertex angle of an isosceles triangle .... and ....

5 In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to ....

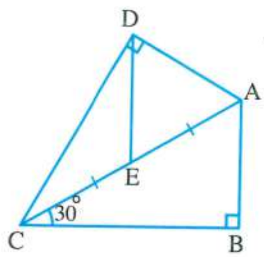
3 [a] In the opposite figure :

m ($\angle$ B) = 90° , m ($\angle$ ADC) = 90°

, m ($\angle$ ACB) = 30°

, $\overline{DE}$ is a median in Δ ADC

Prove that : AB = DE



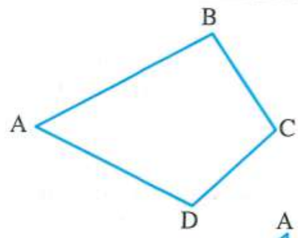
[b] In Δ ABC , if AB = 7 cm. , BC = 5 cm. , AC = 6 cm. , arrange the measures of the angles of the triangle ABC ascendingly.

4 [a] In the opposite figure :

AB > BC , AD > CD

Prove that :

m ($\angle$ C) > m ($\angle$ A)

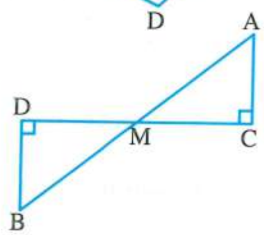


[b] In the opposite figure :

$\overline{AB} \cap \overline{CD}$ = {M}

, m ($\angle$ C) = m ($\angle$ D) = 90°

Prove that : AB > DC

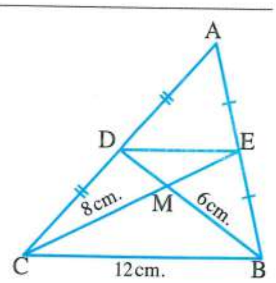


5 [a] In the opposite figure :

If D , E are the midpoints of $\overline{AC}$, $\overline{AB}$

, MB = 6 cm. , MC = 8 cm. , BC = 12 cm.

Find : The perimeter of Δ MDE

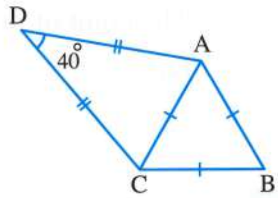


[b] In the opposite figure :

AB = BC = AC , DA = DC

, m (∠ D) = 40°

Find : m (∠ BAD)



12 Port Said Governorate

Educational Directorate
Math Department

Answer the following questions :

1 Choose the correct answer :

1 In Δ ABC , if AC = 4 cm. , BC = 3 cm. , then m (∠ B) .... m (∠ A)

(a) > (b) < (c) = (d) ≤

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) twice (c) third (d) quarter

3 In Δ ABC , if m (∠ A) = 100° and AB = AC , then m (∠ ABC) = ...

(a) 80° (b) 60° (c) 40° (d) 30°

4 The point of intersection of the medians of the triangle divides each of them in the ratio .... from the base.

(a) 1 : 3 (b) 3 : 1 (c) 1 : 2 (d) 2 : 1

5 If Δ ABD is obtuse-angled at B and C is the midpoint of $\overline{BD}$, then the longest side is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{BD}$

6 The triangle whose side lengths are 2 cm. , (X + 3) cm. and 5 cm. , becomes an isosceles triangle when X = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

---

2 Complete :

1 The median of an isosceles triangle from the vertex angle bisects .... and is perpendicular to ....

2 The measure of the exterior angle at any vertex of the equilateral triangle is ....°

3 The base angles of the isosceles triangle are ....

4 ABC is a triangle in which AB = 4 cm. , BC = 6 cm. , then AC ∈].............. , ...[

5 The longest side in the right-angled triangle is ....

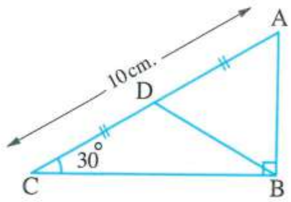
3 [a] In Δ ABC , if m ($\angle$ A) = $(6\,X)^\circ$, m ($\angle$ B) = $(4\,X - 9)^\circ$ and m ($\angle$ C) = $3\,(X - 2)^\circ$, arrange the side lengths of Δ ABC ascendingly.

[b] **In the opposite figure :**

m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

, AD = DC and AC = 10 cm.

Find : The perimeter of Δ ABD



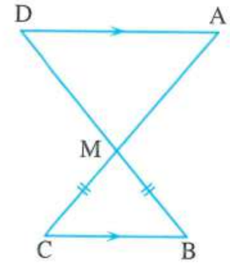
4 [a] **In the opposite figure :**

If $\overline{AC} \cap \overline{BD} = \{M\}$

, $\overline{AD}$ // $\overline{BC}$ and MB = MC

, prove that :

Δ MAD is isosceles.

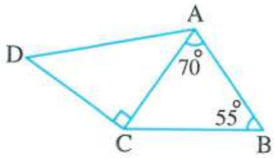


[b] **In the opposite figure :**

m ($\angle$ BAC) = 70° , m ($\angle$ B) = 55°

and m ($\angle$ ACD) = 90°

Prove that : AD > AB

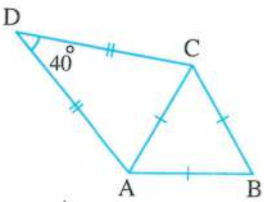


5 [a] **In the opposite figure :**

m ($\angle$ D) = 40° , DA = DC

and Δ ABC is equilateral

Find : m ($\angle$ DCB)

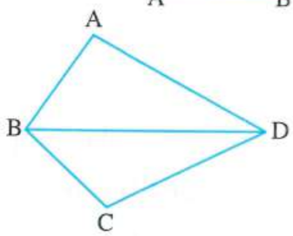


[b] **In the opposite figure :**

AB < AD and BC < CD

Prove that :

m ($\angle$ ABC) > m ($\angle$ ADC)



13 Damietta Governorate

Damietta Education Zone
Inspector of Math

Answer the following questions :

1 **Complete each of the following :**

1. If the measure of one of the base angles of an isosceles triangle equals 50° , then the measure of the vertex angle equals ....°
2. The supplementary of the obtuse angle is .... angle.

3 The longest side in the right-angled triangle is ....

4 The perpendicular straight line on a line segment from its midpoint is called ....

5 If 4 cm. , 7 cm. are the lengths of two sides in a triangle
, then .... < the length of the third side < ....

2 Choose the correct answer :

1 The point of intersection of the medians of the triangle divides each of them in the ratio of .... from the base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

2 In Δ ABC , if m ($\angle$ B) = 70° , m ($\angle$ C) = 50° , then AB .... AC

(a) > (b) < (c) = (d) ≥

3 The number of the quadrilaterals in the figure  is ....

(a) 3 (b) 4 (c) 5 (d) 6

4 In the right-angled triangle , the length of the median from the vertex of the right angle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) double (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

5 The sum of the measures of the accumulative angles at a point equals ....°

(a) 90 (b) 180 (c) 360 (d) 308

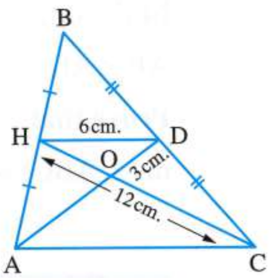
6 The number of lines of symmetry of Δ ABC in which AB = AC , m ($\angle$ B) = 60° is ....

(a) 3 (b) 2 (c) 1 (d) zero

3 [a] In the opposite figure :

HD = 6 cm. , HC = 12 cm.
, H is the midpoint of $\overline{AB}$
and D is the midpoint of $\overline{BC}$
, DO = 3 cm.

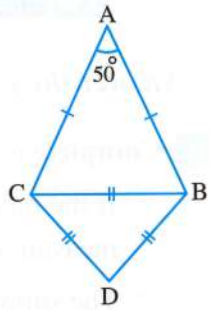
Calculate : The perimeter of the triangle AOC



[b] In the opposite figure :

AB = AC , m ($\angle$ A) = 50°
Δ CDB is equilateral.

Find with proof : m ($\angle$ ABD)

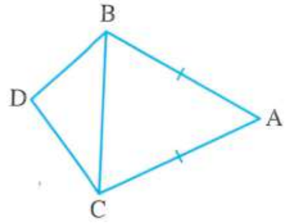


4 **[a] In the opposite figure :**

$AB = AC$, $BD < CD$

Prove that :

$m(\angle ABD) > m(\angle ACD)$



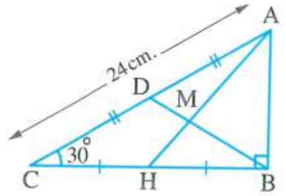
[b] In the opposite figure :

ΔABC is right-angled at B

, $\overline{AH}$, $\overline{BD}$ are two medians

, $m(\angle C) = 30°$, $AC = 24$ cm.

Find : The length of each of $\overline{AB}$, $\overline{BD}$, $\overline{BM}$



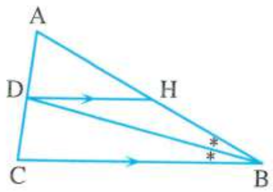
5 **[a] In the opposite figure :**

$\overrightarrow{BD}$ bisects $\angle ABC$

, $\overline{HD} // \overline{BC}$

Prove that :

ΔHBD is an isosceles triangle.

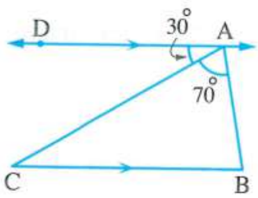


[b] In the opposite figure :

$\overleftrightarrow{AD} // \overline{BC}$, $m(\angle BAC) = 70°$

, $m(\angle DAC) = 30°$

Prove that : $AC > BC$



14 El-Fayoum Governorate

East El-Fayoum Zone
El-Eman Language School

Answer the following questions :

1 **Choose the correct answer from those given :**

1 In ΔABC , if $(AB)^2 = (BC)^2 - (AC)^2$, $m(\angle C) = 42°$, then $m(\angle B) =$ ....

(a) 40° (b) 90° (c) 48° (d) 110°

2 The scalene triangle has .... axes of symmetry.

(a) 3 (b) 2 (c) 1 (d) 0

3 If A lies on the axis of symmetry of $\overline{BC}$, then AB .... AC

(a) < (b) > (c) = (d) ≤

4 If $\overline{AD}$ is a median of ΔABC , M is the point of concurrence of the medians , then MD = ... AD

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

5 If 10 cm. , 5 cm. and X cm. are side lengths of an isosceles triangle , then X = ... cm.

(a) 5 (b) 10 (c) 15 (d) 4

6 The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 90° (c) 50° (d) 120°

2 **Complete the following :**

1 The total area of a cuboid = 120 cm^2 and its lateral area = 96 cm^2 , then the area of its base equals .... cm^2

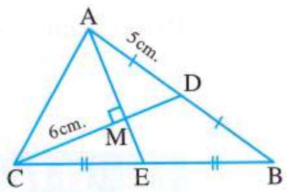
2 The base angles of the isosceles triangle are ....

3 ABC is a right-angled triangle at B , m ($\angle$ C) = 30° , AB = 5 cm. , then AC = ... cm.

4 In ΔABC , if m ($\angle$ C) = 30° , m ($\angle$ A) = 70° , then the smallest side in length is ....

5 In any triangle , if the lengths of two sides are not equal , then the greater side in length is opposite to ....

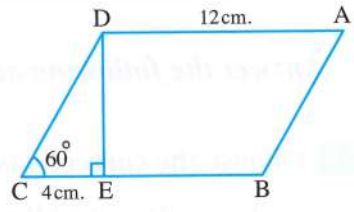
3 [a] **In the opposite figure :**



M is the concurrence point of the medians of ΔABC , $\overline{AM} \perp \overline{CD}$, AD = 5 cm. , MC = 6 cm.

Find with proof : The length of $\overline{ME}$

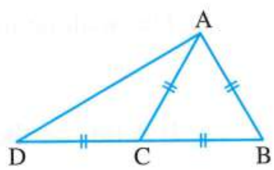
[b] **In the opposite figure :**



ABCD is a parallelogram , m ($\angle$ C) = 60° , $\overline{DE} \perp \overline{BC}$, AD = 12 cm. , CE = 4 cm.

Find with proof : The perimeter of the parallelogram ABCD

4 [a] **In the opposite figure :**



ABC is an equilateral triangle , D $\in \overrightarrow{BC}$, BC = CD

Prove that : $\overline{AB} \perp \overline{AD}$

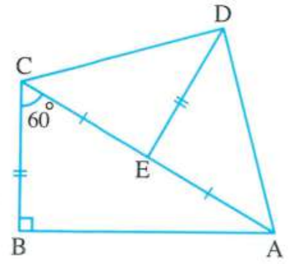
[b] In the opposite figure :

ABC is a right-angled triangle at B

, m ($\angle$ ACB) = 60° , E is the midpoint of $\overline{AC}$

, DE = BC

Prove that : m ($\angle$ ADC) = 90°

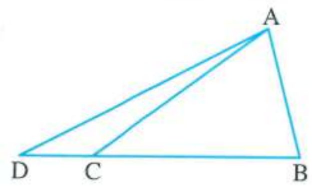


---

5 **[a] In the opposite figure :**

C $\in \overline{BD}$, AC > AB

Prove that : m ($\angle$ B) > m ($\angle$ D)

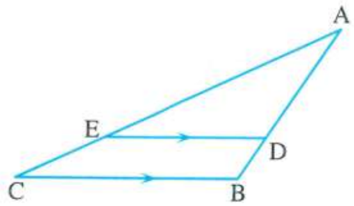


[b] In the opposite figure :

ABC is an obtuse-angled triangle at B

, $\overline{DE}$ // $\overline{BC}$

Prove that : AE > AD



15 Luxor Governorate

Armant Educational Directorate
Mohamed Raafat Lang. Sch.

Answer the following questions :

1 **Complete the following :**

1. In the right-angled triangle , the .... is the longest side.
2. In Δ ABC , if D is the midpoint of $\overline{BC}$ and AD = $\frac{1}{2}$ BC , then m ($\angle$ A) = ...°
3. In Δ ABC , if m ($\angle$ B) = 65° and m ($\angle$ C) = 50° , then the shortest side in Δ ABC is ....
4. In Δ ABC , if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called ....
5. The measure of the exterior angle of the equilateral triangle is ....°

---

2 **Choose the correct answer :**

1. In Δ ABC , if m ($\angle$ B) > m ($\angle$ C) , then ....

   (a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{AC}$

2. The point of concurrence of the medians of the triangle divides each median in the ratio of .... from the base.

   (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

3 The lengths of two sides in a triangle are 4 cm. , 9 cm. and it has one axis of symmetry , then the length of the third side is .... cm.

(a) 4 (b) 5 (c) 9 (d) 13

4 The number of axes of symmetry of the equilateral triangle equals ....

(a) 0 (b) 1 (c) 2 (d) 3

5 If Δ ABC is right-angled at B , AB = 6 cm. , BC = 8 cm. , then the length of the median drawn from B is .... cm.

(a) 10 (b) 8 (c) 6 (d) 5

6 The lengths which can be lengths of sides of a triangle are ....

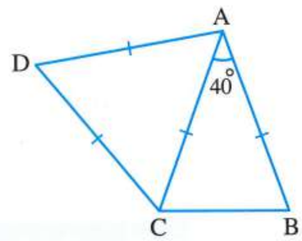
(a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

3 [a] **In the opposite figure :**

AB = AC = AD = CD

, m ($\angle$ BAC) = 40°

Find : m ($\angle$ BCD)



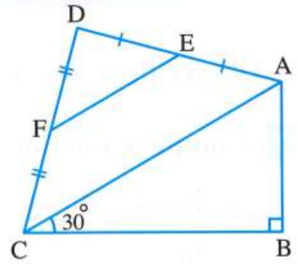
[b] **In the opposite figure :**

m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

, E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

Prove that : AB = EF



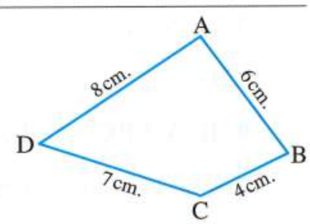
4 [a] **In the opposite figure :**

ABCD is a quadrilateral in which :

AB = 6 cm. , BC = 4 cm.

, CD = 7 cm. , DA = 8 cm.

Prove that : m ($\angle$ BCD) > m ($\angle$ BAD)



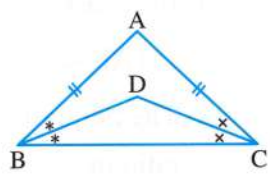
[b] **In the opposite figure :**

ABC is a triangle in which :

AB = AC , $\overrightarrow{BD}$ bisects $\angle$ ABC

, $\overrightarrow{CD}$ bisects $\angle$ ACB

Prove that : Δ DBC is an isosceles triangle.

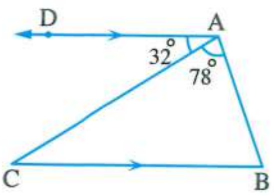


5 **[a] In the opposite figure :**

$\overrightarrow{AD} \parallel \overline{BC}$, m (∠ BAC) = 78°

, m (∠ CAD) = 32°

Prove that : AC > AB



[b] In the opposite figure :

ABC is a triangle , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

, $\overline{XC} \cap \overline{AY} = \{M\}$, XY = 5 cm.

, CM = 8 cm. , YM = 3 cm.

Find : The perimeter of Δ MAC